Effect of Turret Positioning Errors on the GMRT Primary Beam

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Abstract

The effect of small feed displacements on the primary beam is well studied for prime focus paraboloids eg. Ruze[1] (1966). Using these results Kapahi[2] (1992) has computed the loss in directivity expected for a GMRT dish due to axial displacements of the feed, and the squint introduced by a lateral dispacement of the feed.

Results quoted in the literature are for lateral displacements in which the feed continues to point at the vertex of the paraboloid. In case of turret positioning errors however, the feed no longer points at the vertex of the main dish.

In this report formulae are developed for the aperture plane phase errors introduced by a misalignment of the feed turret, i.e. for the feed not pointing towards the paraboloid vertex. As expected, the linear term agrees with that in the case of lateral feed displacement. Higher order terms (which lead to directivity loss) are however different from that quoted in the literature, particularly so in the case where radial focusing has been done after a lateral feed displacement.

Quantitative results (under simplifying assumptions) tuned to the GMRT dish are also presented.

[1]Ruze, J. "Antenna Tolerance Theory – A Review", Proc. IEEE **54**, 633 (1966)

[2]Kapahi V. K., "Reduction in directivity of a GMRT dish due to displacement of feed center from the focus point", 1992

1 Methodology

The first order aperture plane phase error introduced by a misalignment of the feed turret with the axis of a parabolic dish is straightforward to calculate under the simplifying assumptions of geometric optics and scalar fields. The method used here is to compute the deviation of the actual dish from a hypothetical dish of the same focal length and diameter whose axis coincides with the new axis of the misaligned feed (see Figure 5[A]).

The axial deviation of the actual paraboloid from the hypothetical perfectly aligned paraboloid is given by (see the Appendix for details)

$$\delta z'' = \frac{x''}{2f}\left[\frac{\rho''^2}{4f} + f - R\right]\epsilon \tag{11}$$

where:

$\delta z''$	$\equiv$	the axial deviation from the perfectly aligned paraboloid
f	$\equiv$	the focal length of the paraboloid
x''	$\equiv$	distance in aperture plane along the feed displacement
ρ''	$\equiv$	radial distance in the aperture plane
R	$\equiv$	distance from turret rotation axis to feed phase center
ϵ	$\equiv$	the misalignment angle of the turret

The relation between the axial deviation and the phase in the aperture plane is given by

$$\delta\phi = \left(\frac{2\pi}{\lambda}\right)\frac{2\delta z''}{1+(\frac{\rho''}{2f})^2} \tag{1}$$

For a shallow dish (i.e. $F/D >> 1$), the aperture plane phase is a simple linear gradient,

$$\delta\phi = \left(\frac{2\pi}{\lambda}\right)x''\left(1 - R/f\right)\epsilon \tag{2}$$

The factor 1 in the brackets simply realigns the beam with the axis of the original paraboloid (this phase gradient is in the aperture plane of the hypothetical perfectly aligned paraboloid). So the net effect is that there is a beam squint of magnitude $R\epsilon/f = \Delta x/f$ (where Δx is the lateral displacement of the feed) in the direction opposite to the feed displacement. This is, as is to be expected, in agreement with the standard formulae for beam squint introduced by lateral feed displacement.

2 Quantitative Results

Quantitative results were calculated for both 1D and 2D geometries. The feed radiation pattern is assumed to be symmetric with an edge taper (excluding space loss) of 10 dB. The size of the dish, and the distance between the turret rotation axis and the feed phase center were taken as the values corresponding to the GMRT dish. All the figures should be taken only as roughly indicative, the exact numbers are taper sensitive. The 325 MHz kildal feed in particular has a taper different from that assumed here.

Figure 1[A] shows the beam squint angle as a function of the turret misalignment angle. Figure 1[B] shows the aperture efficiency loss as a function of turret misalignment angle, while Figure 1[C] shows the same aperture efficiency loss, but now as a function of beam squint angle. Aperture efficiency loss was computed using the standard treatment for the loss due to small phase errors in the aperture plane. Figures 2 are a set of voltage patterns at a series of closely spaced frequencies for a beam squint angle of $30'$.

Figure 3 shows the beam for the full 2d calculation for the case of a $30'$ offset in the elevation beam (due entirely to turret alignment problems). Figure 4 shows cuts through the peak of the beam in the elevation (Figure 4[A]) and azimuth (Figure 4[B]) directions. Figure 4[C] shows an overlay of the central parts of the elevation (dotted) and azimuth beams (solid, note that the elevation beam has been shifted to make the comparison easier). Notice that in the central part of the beam, the cuts along the elevation and azimuth are still fairly symmetrical.

3 Appendix

Consider a paraboloidal antenna of focal length f illuminated by a feed which is supported on a turret whose rotation axis is a distance R above the phase center of the feed. It is assumed that when there is no misalignment the feed phase center is exactly coincident with the focal point of the paraboloid (see

Figure 5[A]).

In a co-ordinate frame with the origin at the focal point and the Z axis pointing toward the paraboloid vertex, the surface of the paraboloid satisfies

$$\rho^2 = 4f(f - z) \tag{3}$$

where ρ is radial distance in the aperture plane. Now let the feed turret be misaligned by an angle ϵ. Consider a new co-ordinate system, the double primed system in which the Z'' axis is along the new feed axis, (the misalignment angle is a rotation about the Y'' axis), and the origin is the new feed phase center. In computing the equation of the paraboloid in this co-ordinate frame it is convenient to consider an intermediate co-ordinate system, the primed system, in which the origin is shifted to the new feed phase center, but the axis are parallel to those in the original unprimed system (see Figure 5[B]). Then

$$\begin{aligned} x' &= x - \Delta x \\ y' &= y \\ z' &= z + \Delta z \end{aligned} \tag{4}$$

$$\tag{5}$$

where

$$\Delta x = R\sin(\epsilon) \qquad \text{and} \quad \Delta z = R(1 - \cos(\epsilon)) \tag{6}$$

and

$$\begin{aligned} x'' &= x'\cos(\epsilon) - z'\sin(\epsilon) \\ y'' &= y' \\ z'' &= z'\cos(\epsilon) + x'\sin(\epsilon) \end{aligned} \tag{7}$$

$$\tag{8}$$

These equations can be trivially inverted to give the equation of the paraboloid surface in this new co-ordinate system as

$$4f(f - z''\cos(\epsilon) + x''\sin(\epsilon) + \Delta z) = (x''\cos(\epsilon) + z''\sin(\epsilon) - \Delta x)^2 + y''^2 \tag{9}$$

Now define

$$\delta z'' = z'' - f + \frac{\rho''^2}{4f} \tag{10}$$

i.e. $\delta z''$ is the axial distance between the original paraboloid and an identical paraboloid that has been rotated so that its axis is aligned with the new feed axis Z''. Substituting for z'', Δx, and Δz in equation (9) and retaining only those terms linear in ϵ (and ignoring also the term $\epsilon\delta z''$) gives

$$\delta z'' = \frac{x''}{2f}\left[\frac{\rho''^2}{4f} + f - R\right]\epsilon \tag{11}$$

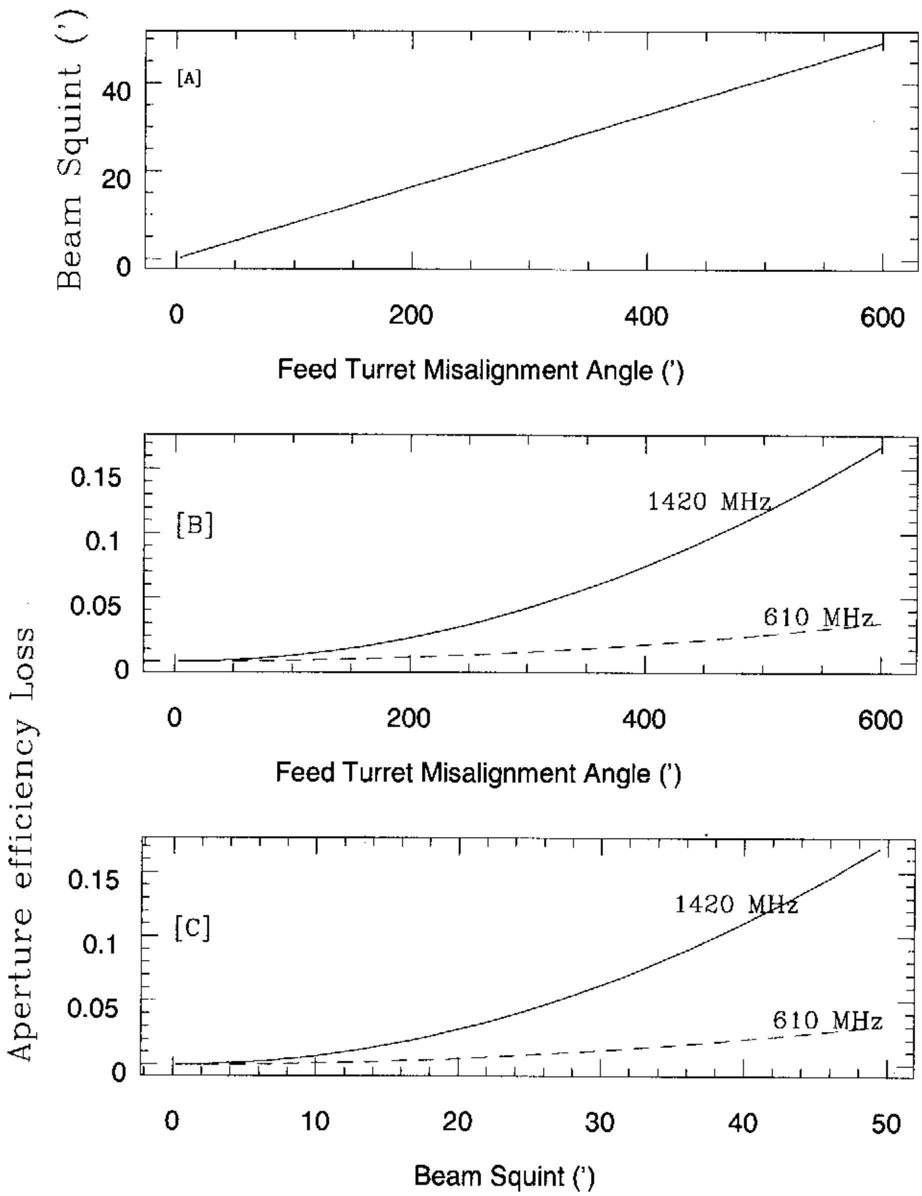


Figure 1: [A] The beam squint as a function of the turret misalignment value. [B] The loss in aperture efficiency as a function of turret misalignment angle. [C] The loss in aperture efficiency as a function of beam squint angle

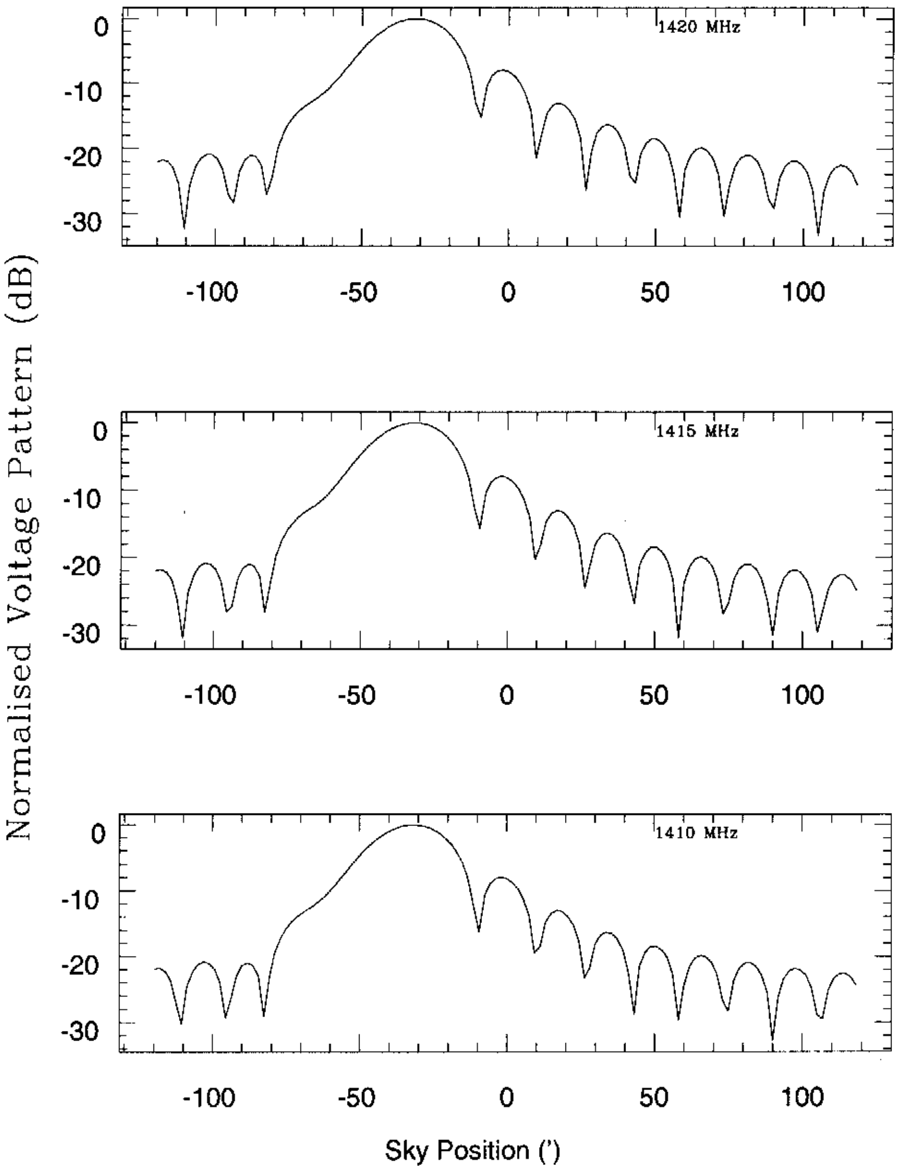


Figure 2: Elevation beam as a function of frequency for three closely spaced frequencies. The beam squint is $\sim 30'$.

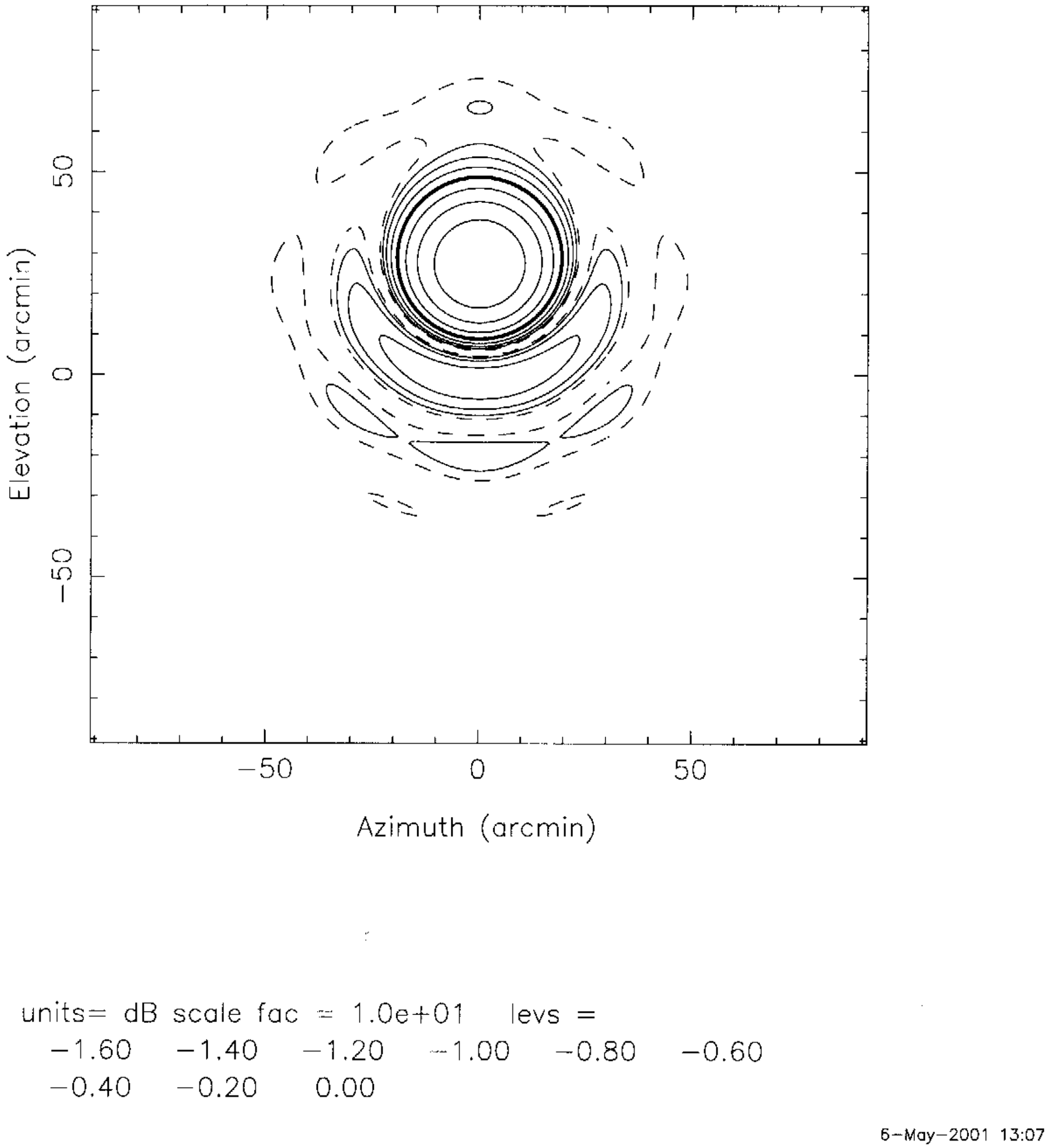


Figure 3: The 2d beam (power) shape at 1280 MHz when the beam squint is $\sim 30'$ in elevation.

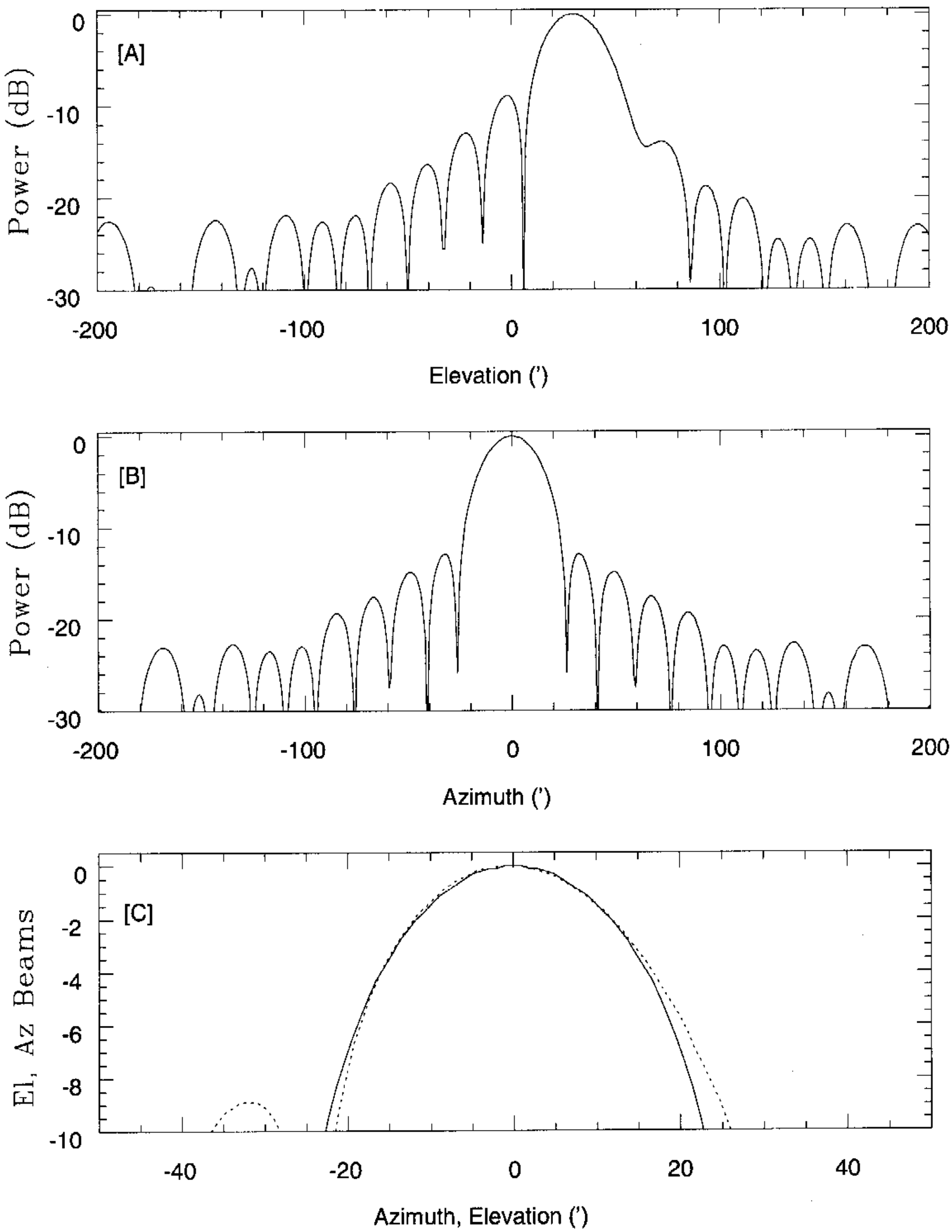
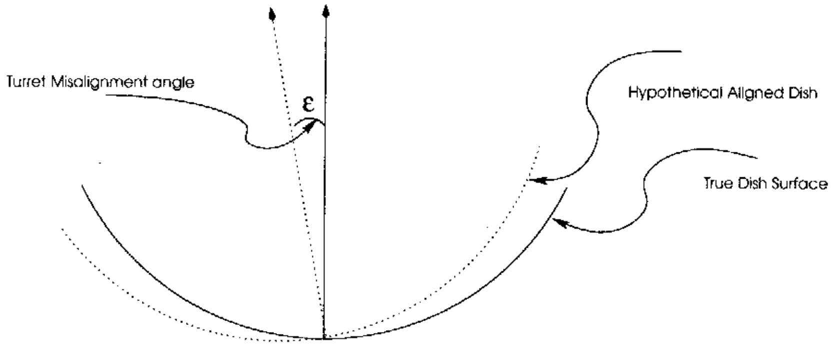


Figure 4: [A] A cut through the elevation direction when there is a beam squint of 30′ at 1280 MHz. [B] A cut through the azimuth axis for the same case. [C] Comparison of the elevation (dotted) and azimuth (solid) power beams. Note that the elevation beam has been offset back to the origin to allow easy comparison. Despite the sidelobe patterns in the azimuth and elevation axis being different, the central parts of the beam are fairly symmetrical.



[A]

[B]

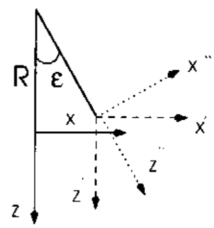


Figure 5: [A] Schematic illustrating how the aperture plane phase errors are computed when the feed turret is misaligned. Phase errors are computed using the path difference between the hypothetical aligned parabola (dotted lines) and the true parabola (solid lines). [B] The co-ordinate systems and symbols used in the text. The unprimed co-ordinate system is aligned with the dish axis. The primed co-ordinate system has the origin shifted to the phase center as the mis-aligned feed, but has no rotation wrt to the unprimed system. The double primed system has the same origin as the primed system but is rotated along the Y' axis so that the Z'' axis is along the axis of the misaligned feed.