CONFUSION

LIMIT

Introduction

Confusion due to weak radio sources can quite seriously limit the flux density below which reliable estimates of position and flux of the sources cannot be made. For when observing a source of flux density S_0, above the sensitivity limit of a telescope, the beam pattern looks also at a number of much weaker sources, which although individually may undetactable, can collectively contribute a power comparable to, or even greater than that due to the source being observed. Further, as these sources are not uniformly distributed in the sky but rather follow a *Poisson Distribution* (*i.e.* randomly disrtibuted without clustering), the total power collected by the telescope varies as the beam pattern is displaced in the sky. For a given beamwidth there is no way of avoiding this limitation due to confusion.

Formulae and Equations

Let N, the number of sources and S, the flux density be related as

$$N = constant S^m \tag{1}$$

where N = Number of sources with flux densities greater than a given value of S (in Jy) per Steradians.

$$N = n \frac{S^m}{{S_{lim}}^m} \tag{2}$$

where n = Average number of sources with flux densities greater than some limiting flux value S_{lim} per Steradians.

Equation (2) gives the "*Cumulative* logS - logN Plot". So we differentiate this equation to get "*Differential* logS - log(dN/dS) Plot".

$$dN = nm \frac{S^{m-1}}{{S_{lim}}^m} dS \tag{3}$$

where dN is the average number of sources with flux densities between S and $S + dS$ per Steradians.

Assuming that the sources are distributed randomly, this number follows the Poisson distribution with a standard deviation of $\sqrt{dN}$. Thus, Now the contribution of all these sources to the background radiation will therefore have a standard deviation of $\sqrt{S^2 dN}$.

$$\Delta S = \left(nm{S_{lim}}^{-m} \int_0^{S_{lim}} S^{m+1} dS\right)^{1/2} \tag{4}$$

where ΔS gives the total standard deviation of the background from all the sources weaker than S_{lim}, here ΔS is in Jy per Steradians and

$$\Delta S^2 = \int_0^{N_{lim}} S^2 dN = \int_0^{S_{lim}} S^2 \frac{dN}{dS} dS \tag{5}$$

$$ConfusionNoise = \sigma = \left(\int_{4\pi} P_n(\mathbf{n})^2 d\Omega \int_0^{S_{lim}} S^2 \frac{dN}{dS} dS\right)^{1/2} \tag{6}$$

where $P_n(\mathbf{n})$ is the true beam pattern.

Noise fluctuations which again are statistical, obey *Normal Distribution* (see Statistical Optics by GOODMAN - Central Limit Theorem) so, there's one chance in five hundred that a point will deviate from the average value by more that 3σ. Defining

$$\tilde{\Omega} = \int_{4\pi} P_n(\mathbf{n})^2 d\Omega \tag{7}$$

For a Gaussian beam

$$\tilde{\Omega} = \frac{\Omega}{2} \tag{8}$$

where Ω is the true beam Solid Angle, assuming beam shape to be Gaussian.

Assuming that the detection of a given source is valid if its flux density is five times the noise, (*ie* 5σ is the conservative limit) this limit is defined as the *Confusion Limit* of the Antenna (taking 5σ as the cut off, the probability of complete misinterpretations of source are one in twenty). This *Confusion* is always present and only means of improving this situation is to build larger Antennas with their corresponding smaller beamwidths and thus smallar value of σ

Since now we assume that a source with 5σ as it's flux density can be detected, hence we're not going to include all those sources which are above this flux limit in determining the confusion noise. Thus,

$$S_{lim} = Confusion Limit \tag{9}$$

So adopting this value of S_{lim}, the problem "Determination of *Confusion Limit*" is essntially reduced to that of solving the two integrals in equation (6).

Next with the provided Source - Count data (see Source - Count TableA) at 408 MHz, and using the relation

$$N_{327}(> S) = N_{408}(> S)(\frac{\nu_{327}}{\nu_{408}})^{-\alpha_0} \tag{10}$$

Where α_0 is the spectral index. Using the same equation (10) and

$$\alpha_0 = 0.75 \tag{11}$$

the source count at 327 MHz and various other frequencies (38, 150, 233, 610, 1420 all in MHz)were found. This gives "Cumulative logS - logN Plot".

Determination of $\tilde{\Omega}$

For a Gaussian beam

$$\tilde{\Omega} = \frac{2\pi\sigma_x\sigma_y}{2} \tag{12}$$

1. For Single Dish

$$\tilde{\Omega} = 2.485 \times 10^{-4} \tag{13}$$

Table A:

SOURCE - COUNT AT 408 MHz & 327 MHz

S (in Jy) (408)	N (>S)	S (in Jy) (327
100.000000	0.30	118.055
10.000000	15.60	11.8055
5.000000	56.20	5.90275
2.000000	288.40	2.36110
1.000000	907.00	1.18055
0.500000	2558.00	0.590275
0.200000	8332.00	0.236110
1.00000E-01	17798.00	0.118055
5.00000E-02	34128.00	5.90275E-02
2.00000E-02	69898.00	2.36110E-02
1.00000E-02	112168.00	1.18055E-02
5.00000E-03	178408.00	5.90275E-03
1.00000E-03	709778.00	1.18055E-03

2. For GMRT Synthesised beam

$$\tilde{\Omega} = 6.9656 \times 10^{-10} \tag{14}$$

3. For Central Square Synthesised beam

$$\tilde{\Omega} = 1.3623 \times 10^{-7} \tag{15}$$

Using equation(13), equation(14), equation(15) and using the relation

$$\nu^2 \tilde{\Omega} = constant \tag{16}$$

we determine the value of $\tilde{\Omega}$ at vorious other frequencies.

Calculations and *Confusion Limit* Determinations

Step1

Now the form of number - density relation (equation(1)) must be expressed in the form

$$N(>S) = kS^m \tag{17}$$

From "Cumulative logS - logN Plot at 327 MHz" we infer that the slope m increases continuously from about -1.8 for S>10Jy to about -0.8 at S$\sim$1mJy.

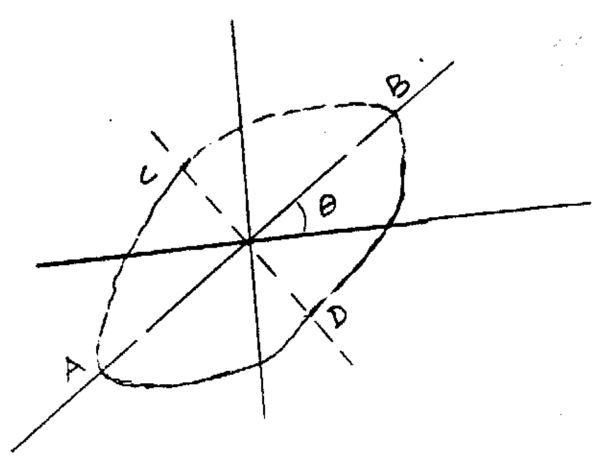
Step2

In order to evaluate the integral, we had approximated the logS - logN(>S) curve by a series of straight lines of different slopes in different flux density ranges. The slopes have been determined visually as the best fits with the actual curve.

Step3

	MAJOR AXIS (in Sec)	MINOR AXIS (in Sec)	θ
Full Synthesis GMRT Array	8.821	5.93	16.6°
Central Square full synthesis	117.784	86.854	-27.7°

At Declination



AB = Major Axis
CD = Minor Axis

Fig 1.

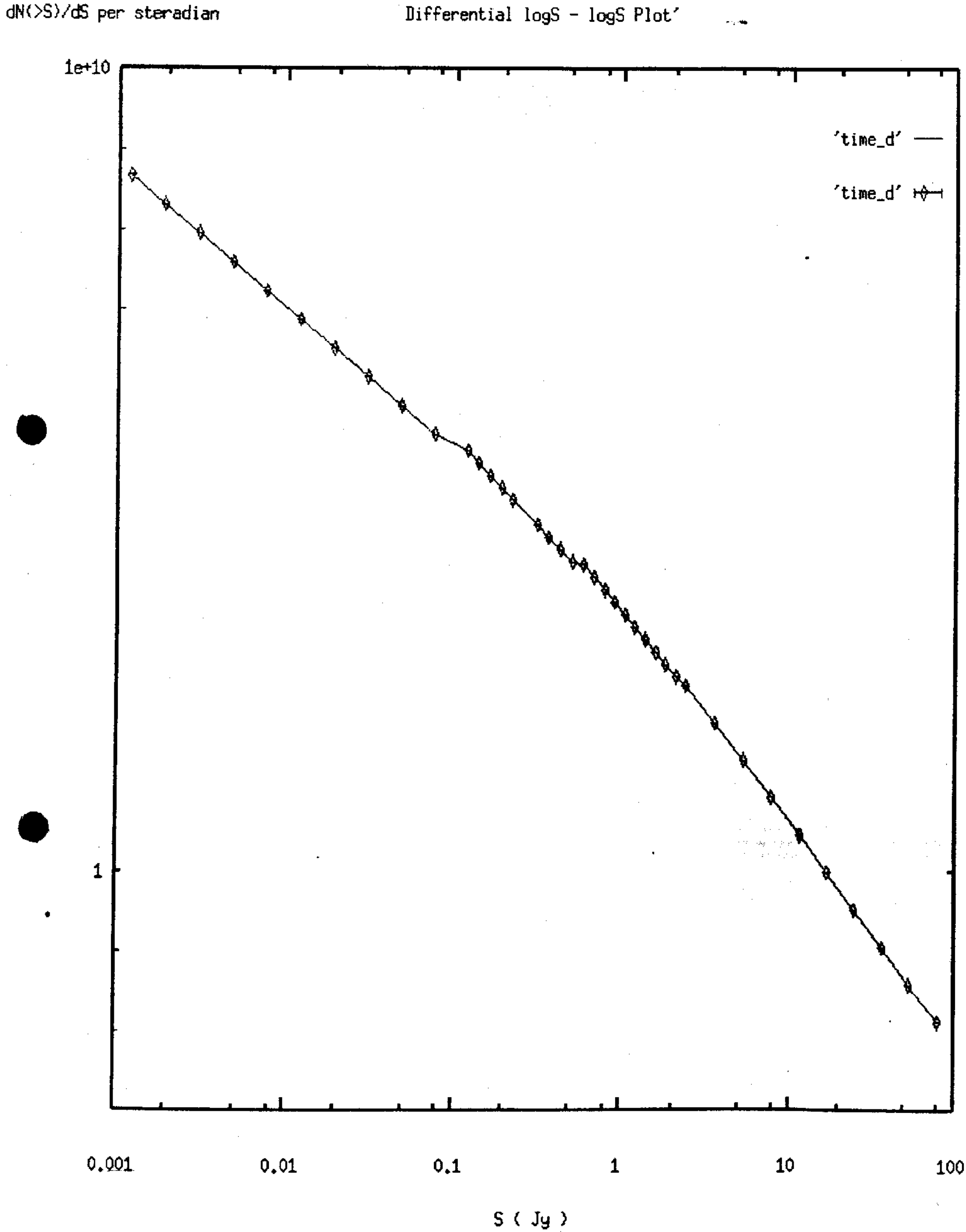
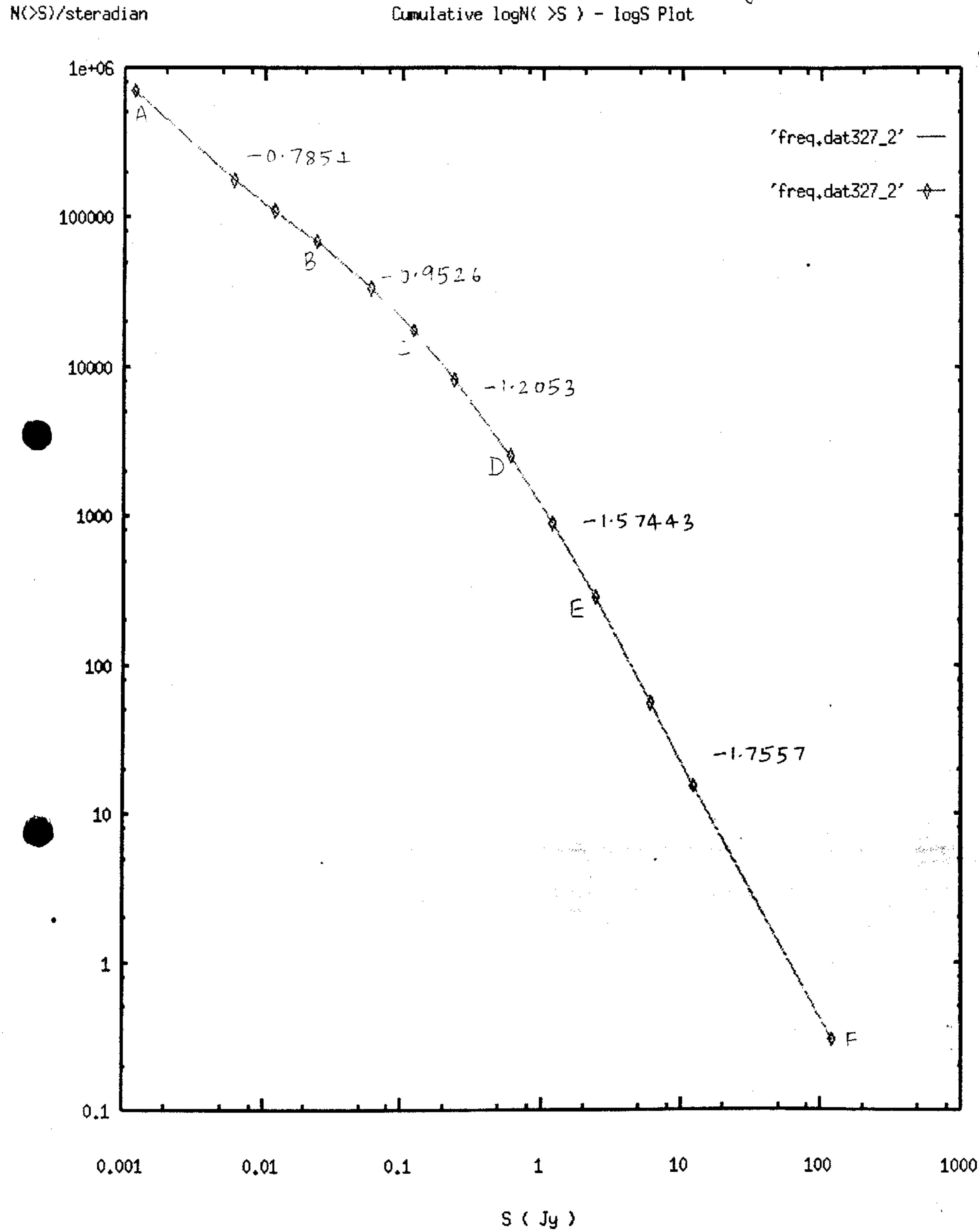

dN(>S)/dS per steradian
Differential logS - logS Plot'
1e+10
1
'time_d'
'time_d'
0.001
0.01
0.1
1
10
100
S (Jy)

Fig 0


N(>S)/steradian
Cumulative logN(>S) - logS Plot
'freq.dat327_2'
'freq.dat327_2'
1e+06
100000
10000
1000
100
10
1
0.1
0.001
0.01
0.1
1
10
100
1000
S (Jy)
A
-0.7851
B
-0.9526
D
-1.2053
-1.57443
E
-1.7557
F

Extrapolation of the logS - logN(>S) curve for S<1mJy was done in the following way-

We had assumed the same slope, which was at the lowest given flux density limit.

Step4

After fitting lines to logS - logN(>S) curve in different regions of flux densities (the equation of each of them is then known). Writing it in the form-

$$logN = mlogS + c \tag{18}$$

$$\Rightarrow N = 10^c + (S)^m \tag{19}$$

$$\Rightarrow \frac{dN}{dS} = m(S)^{m-1}10^c \tag{20}$$

Step5

We write the second integral in equation(6) as

$$\int_0^{S_{lim}} S^2\frac{dN}{dS}dS = \int_0^A S^2\frac{dN}{dS}dS + \int_A^B S^2\frac{dN}{dS}dS + ...\int_{..}^{S_{lim}} S^2\frac{dN}{dS}dS \tag{21}$$

Consider the equation(21) as

$$I = I_{ED} + I_{DC} + I_{CB} + I_{BA} \tag{22}$$

Determination of S_lim in practice

Now to evaluate equation(21), we do it numerically. We first assume some higher value of S_{lim} and determine the $ConfusionLimit$. Next assume this $ConfusionLimit$ as S_{lim} and once again determine $ConfusionLimit$. This way one iterates the $ConfusionLimit$ value until it converges to a contant value.

The above said method show good results as long as S_{lim} is above few times mJY. So this is suitable for 'Single Dish' and 'Central - Square' case. This method cannot be applied to 'GMRT Full Synthesis Array' because the evaluation of $ConfusionLimit$ also depends on first integral in equation(6), this integrand is of 10^{-8} order or less. So, evaluated $ConfusionLimit$ (found by this method) and hence S_{lim} becomes of the order of 10^{-3} or less even before the $ConfusionLimit$ value saturates. This is due to fact that we are no longer in the given logS - logN regime.

Thus, we've assumed S_{lim} as the 1mJy for all cases and have found the $ConfusionLimit$.

Procedure (for 327 MHz)

Note: Refer fig2

Equations of various lines fitted (values in brackets indicate lower limits and upper limits of the integrands of the RHS of equation(21)):

Equation of ED [0.0:$10^{-0.928}$]

$$logN = -0.7854logS + 3.55161 \tag{23}$$

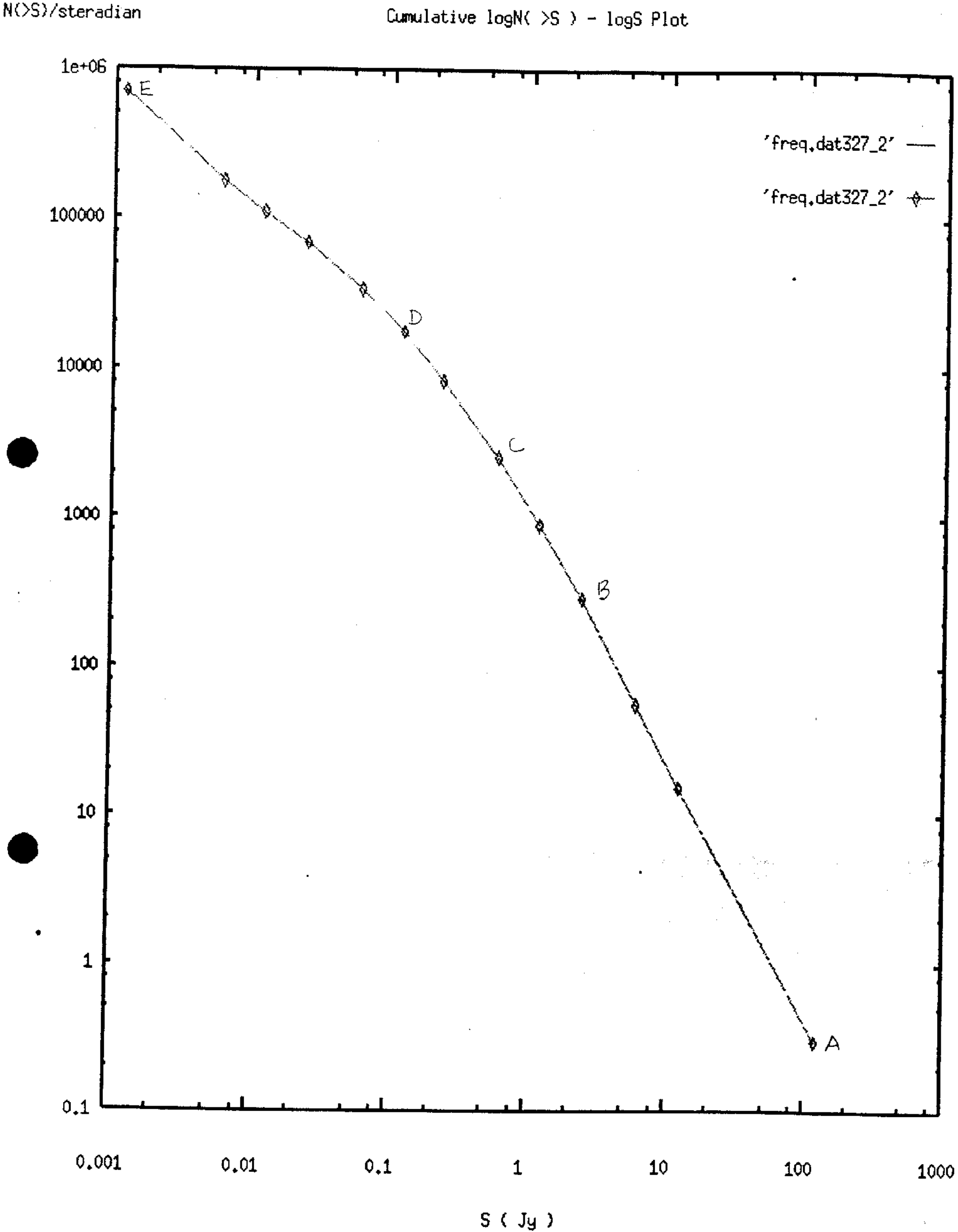
Equation of DC [$10^{-0.928}$:$10^{-0.229}$]

$$logN = -1.2053logS + 3.132 \tag{24}$$

Equation of CB [$10^{-0.229}$:$10^{0.373}$]

$$logN = -1.57443logS + 3.04744 \tag{25}$$

Fig 2



N(>S)/steradian
Cumulative logN(>S) - logS Plot
1e+06
100000
10000
1000
100
10
1
0.1
'freq.dat327_2'
'freq.dat327_2'
E
D
C
B
A
0.001
0.01
0.1
1
10
100
1000
S (Jy)

Equation of BA $[10^{0.373}:S_{lim}]$

$$logN = -1.7557 logS + 3.1151 \tag{26}$$

The values of various integrands are shown in TableB

TableB

	Upper Limit	Lower Limit	$I_{..}$
1	$10^{-0.928}$	0.0	$171.86 = I_{ED}$
2	$10^{-0.229}$	$10^{-0.928}$	$975.556 = I_{DC}$
3	$10^{0.373}$	$10^{-0.229}$	$2650.8 = I_{CB}$
4	S_{lim}	$10^{0.373}$	$18493.1 = I_{BA}$

Similarly we've found the similar values at various other frequencies. Using the all above values of integrands and $\tilde{\Omega}$, we then have found the following results.

Results

Table1: Confusion Limit for the Single Dish Gmrt Antenna

Freq.	$\int_0^{N_{lim}} S^2 dN$	S_{lim}	Conf.Lim. $=5\sigma=$CL in Jy per beam	$\tilde{\Omega}$ in Steradians	$Noise$ $=\sigma$ in Jy per beam
38 MHz	4.9311×10^5	476.9	476.3022	1.84026×10^{-2}	95.26044
150 MHz	3.9909×10^4	45.3	34.327	1.18104×10^{-3}	6.8654
233 MHz	2.0212×10^4	25.0	15.727	4.8948054×10^{-4}	3.1454
327 MHz	7.1343×10^3	7.0	6.687	2.48515×10^{-4}	1.3375
408 MHz	5.891×10^3	7.89	4.9238	1.596346×10^{-4}	0.985
610 MHz	2.6585×10^3	3.903	2.1786	7.14147×10^{-5}	0.436
1420 MHz	4.669×10^2	0.9	0.3922	1.317864×10^{-5}	0.07844

Table2: Confusion Limit for the Central Square GMRT Array

Freq.	$\int_0^{N_{lim}} S^2 dN$	S_{lim}	Conf.Lim. =5σ=CL in Jy per beam	$\tilde{\Omega}$ in Steradians	*Noise* =σ in Jy per beam
38 MHz	2.428015×10^{4}	3.126	2.4745	1.008762×10^{-5}	0.495
150 MHz	1.15×10^{2}	0.06141	0.04313	6.47401×10^{-7}	8.6256×10^{-3}
233 MHz	31.897322	0.02741	0.014627	2.6831438×10^{-7}	2.9255×10^{-3}
327 MHz	25.062854	0.01	9.23893×10^{-3}	1.3623×10^{-7}	1.85×10^{-3}
408 MHz	7.1217	0.01025	4.925×10^{-3}	8.7505524×10^{-8}	9.85×10^{-4}
610 MHz	0.575	1.34×10^{-4}	7.5×10^{-4}	3.91468×10^{-8}	1.5×10^{-4}
1420 MHz	0.061643	3.043×10^{-4}	1.056612×10^{-4}	7.224023×10^{-9}	2.11023×10^{-5}

Table3: Confusion Limit for the Gmrt Full Synthesis Array

Freq.	$\int_0^{N_{lim}} S^2 dN$	S_{lim}	Conf.Lim. $=5\sigma=$CL in Jy per beam	$\tilde{\Omega}$ in Steradians	$Noise$ $=\sigma$ in Jy per beam
38 MHz	1.9754	1.0×10^{-3}	1.6×10^{-3}	5.158×10^{-8}	3.192×10^{-4}
150 MHz	1.0153	1.0×10^{-3}	3.0×10^{-4}	3.31032×10^{-9}	5.8×10^{-5}
233 MHz	0.765	1.0×10^{-3}	1.62×10^{-4}	1.37196×10^{-9}	3.23905×10^{-5}
327 MHz	0.523	1.0×10^{-3}	9.542×10^{-5}	6.9656×10^{-10}	1.91×10^{-5}
408 MHz	0.533	1.0×10^{-3}	7.7233×10^{-5}	4.47437×10^{-10}	1.5447×10^{-5}
610 MHz	0.412	1.0×10^{-3}	4.539×10^{-5}	2.00167×10^{-10}	9.0775×10^{-6}
1420 MHz	0.239	1.0×10^{-3}	1.4856×10^{-5}	3.69382×10^{-11}	2.97×10^{-6}

Discussion

1. Single Dish: At 327 MHz the Confusion Limit is 6.687 Jy per beam, this corresponds to 86.8 beams per sources.

Central Square Array: At 327 MHz the Confusion Limit is 9.239 mJy per beam, this corresponds to 52 beams per sources.

Full Synthesis GMRT Array: At 327 MHz the Confusion Limit is 95.42μJy, this corresponds to 280.5 beams per sources.

2. The logS - logN curve assumes all sources to be point sources, which is not the case. In Synthesised beam case, the flux recieved from the point of the source must be considered and not as recieved from the complete source. So, it is likely that the Confusion Limit should go lower than the predicted values.

3. The Source - Count - Data at lower flux densities can give better results than the presented above.

4. AT 327 MHz, the Sensitivity as predicted in GMRT REPORT is 61μJy (5 RMS noise of image and 10 Hrs of integration). Since Confusion Limit is also near to this value, so observations may be Sensitivity limited or Confusion Limited. As said the better results can be prediced by better Source - Count - Data.

References

1. John D Kraus, *Radio Astronomy*

2. Kirsten Rohlfs, *Tools of Radio Astronomy*

3. J J Condon, *The Astronomical Journal*, 188, 279-286, 1974

4. M Ryle, *Annual Review of Astronomy and Astrophysics*, Vol. 6, 1968

5. John Dickell, *Methods of Experimental Physics*, Vol. 12, part C

6. W R Burns, *Astronomy and Astrophysics*, 19, 41-44, 1972

Data Aquisition Procedure

The data was collected with a single dish in each case kept pointing towards the sky with an elevation close to 90°. The sky was allowed to drift across the beam over a stretch of few hours depending upon the available time. Three data sets were collected on 14^{th} of June,17^{th} of June,and 25^{th} July with the antennae C12,C12, and C9 respectively, over the repspective time periods of 8 hours,12 hours, and 13 hours roughly. The parameters of the data aquiaition are listed below.

1. Signal was collected in total power mode on the PC in the control room.

2. Signal was given directly to the detector without passing it over to the correlator.

3. detector time constant =10 ms

4. sampling period =20 ms

5. ALC was off.

6. IF bandwidt = 16 *MHz* (12*MHz* was available acually)

7. The program used for aquisition was

$$c:\backslash ips \backslash ips2 \backslash ipsacq$$

The IST and LST were noted while starting the data aquisition. A plot of the raw data aquired is attached.

Gain Callibrations

Since the gain settings are not standard in the electronics, the scale of the data sets in counts is not defined. In order to convert the flux densities from the observed counts in the data set into Jy, it is important to calculate the conversion factor from counts to Jy for a particular antenna for the particular observations. To measure the conversion factor we observe a standard source, say Her−A (flux density = 136 Jy), and *on source* counts are noted. The antenna is then moved *off source* through about 5°, to measure the counts corresponding to the background. The conversion factor from counts to Jy was now calculated as,

$$conversion\ factor = \frac{on\ source\ counts - off\ source\ counts}{flux\ density\ of\ the\ source}\ counts/Jy \qquad (1)$$

In the case of the first data set. the source observed was Her-A (Flux density = 136 Jy). The conversion factor measurements using equation **1** were done twice, once beforė the data aquisition and then after the data aquisition was over. For the second data set only one such callibration measurement was done with the source Hydra (Flux density= 150 Jy),and no such callibration measurement was done for the third data set. A rough estimate of the conversion factor for the third data set was determined by comparing the peak in the counts due to the galaxy plane observed in the third data set with a simillar peak observed in the first data set. The results are listed below.

	date of aqusition	$\frac{counts}{Jy}$	
first data set	14^{th} June	1.0	(mean)
second data set	17^{th} June	1.6	
third data set	25^{th} July	~ 0.171	

Program Used to Average The Data

The raw data taken as it is contains interferrence as well as receiver noise over very short durations of time (20 ms - 30 sec). A program was written to clip and average the peaks which deviate from the mean by $\pm 2\ \sigma$. .The algorithm used to write the program is as follows:

1. A predecided length of data is read from the data file serially into an array, say data(i).

2. The mean and the standard deviation from this mean for this array of data points is calculated.

3. If any data point data(i) is found to be deviating from the above calculated mean by $\pm 2\sigma$ then the point is assigned a value equal to the earlier data point. i.e. data(i)=data(i-1).

4. Again the new values for the mean and σ are calculated.

5. Steps 3 and 4 are repeated 3 times.

6. Then the data array is averaged over the desired length, which is limited by the length of the data array and the average is written in the output file.

Integration over 4 minutes

At 327 *MHz*, the beam width at half maximum is 1° roughly. Since the sky was allowed to drift steadily across the beam, with the speed equal to the earth's rotation speed, one independent sample (patch) of the sky is acheived over 4 minute duration. Since for the r.m.s. fluctuation calculations around the mean (or confusion limit calculations) we need to consider independent patches of the sky from various directions, the data integrated over 4 minutes becomes an obvious choice to calculate the σ value of the fluctuations. A different program was written to average the data over 4 minute duration. The program performs box averaging over 4 minute durations.

The plots of 4 minute integration for the three data sets are attached.

Features Observed In The Data Set

There are very few sources above the flux density 15 Jy, in fact the exact number can be calculated from the cummulative log N - log S curve to be 11 in all over the sky. A typical point source will take 4 minutes to pass through the beam, and an extended source will show a slow rise in counts on a larger time scale. Such peakes are seen in the data,which we can identify to be extended sources. A sudden gain change as well as interferrence are also seen. In the third data set, the three peaks observed were identified with the Galactic plane,galactic anticentre, and the Sun. Such peaks were observed in the other data sets as well. These peaks are marked in the plots attached.

Though the strong sources could be identified, we couldn't characterise the slow gain changes since no actual measurements were done. Hence the data is prone to errors due to such slow gain variations. Though the interferrence is present in the third data, it can be seen that it's averaged out in the 4 minute integration plot. It's high frequency doesn't allow it to contribute much to the confusion noise.

Analysis Using A Two Point Difference Function

All the three data sets were integrated over 4 minute duration. The data set now contains observations from different parts of the sky, with a little overlap which can be taken as a random variable, say X(i). To calculate the r.m.s. fluctuations about the mean, we need to define the mean of the data set. But due to various features such as sources and drifts the mean is not well defined, or the local mean fluctuates with large values. To get around with this difficulty we construct a two point difference function Y(i), such that

$$Y(1)=X(2)-X(1),\ Y(2)=X(4)-X(3),\ \text{etc}$$

Any strong source showing a peak over a duration of 4 minutes in the plot of X(i) now appears as a double peak in the plot of Y(i), one positive peak followed by a negative peak or vice versa. Slower variations are cancelled out through differences, and the mean is around zero. The strong peaks above $\pm\, 5\sigma$ level, which corresponds to a source or a large variation in X(i) are deleted manually. They are not considered in the further analysis. Then the new value of σ was calculated. Iteratively any peak deviating from zero by $\pm\, 5\, \sigma$ is deleted. The σ value thus calculated for Y(i) is related to that of X(i) through the relation

$$\sigma_y = \sqrt{2}\, \sigma_x$$

Results

The confusion limit at 327 MHz from

1. The first data set for C12 is

$$5\,\sigma = 20.4\ Jy$$

2. The second data set for C12 is

$$5\,\sigma = 16.5\ Jy$$

3. The third data set for C09 is

$$5\,\sigma = 26.25\ Jy$$

Sources Of Errors

1. Gain changes over a long duration of time,e.g. for the first data conversion factor before and after the aquisition had values 0.8 and 1.28 respectively. We consider a mean value of 1 counts/Jy in the analysis.

2. The beam angle was considered to be 1°, when the more accurate value is larger than this.

3. Interferrence can spoil the game.

4. We only have selective patches of the sky. The analysis will be reliable if we have larger data to analyse.

Suggestions And Remarks For The Repeation Of the Expt.

1. The conversion factor variation is a major source of error in this whole procedure. So more callibrations should be done, especially in between the confusion measurements.

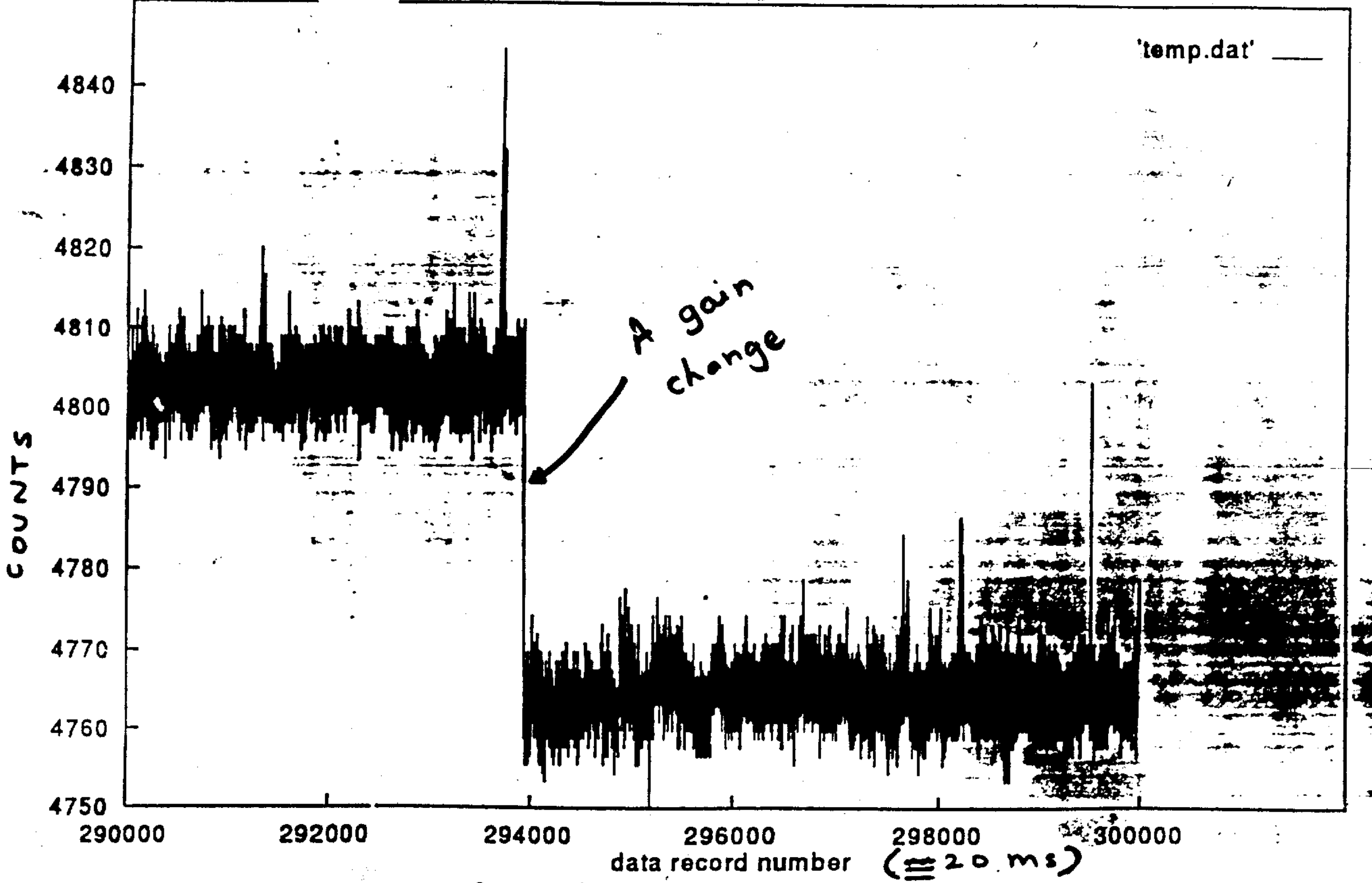
2. To locate the sources crossing the beam, we need to know the exact IST and LST while starting the observations. This enables to calculate IST and LST at any time during the observations.

3. We need to take care of the interferrence due to unwanted sources as that may affect the confusion calculations finally.

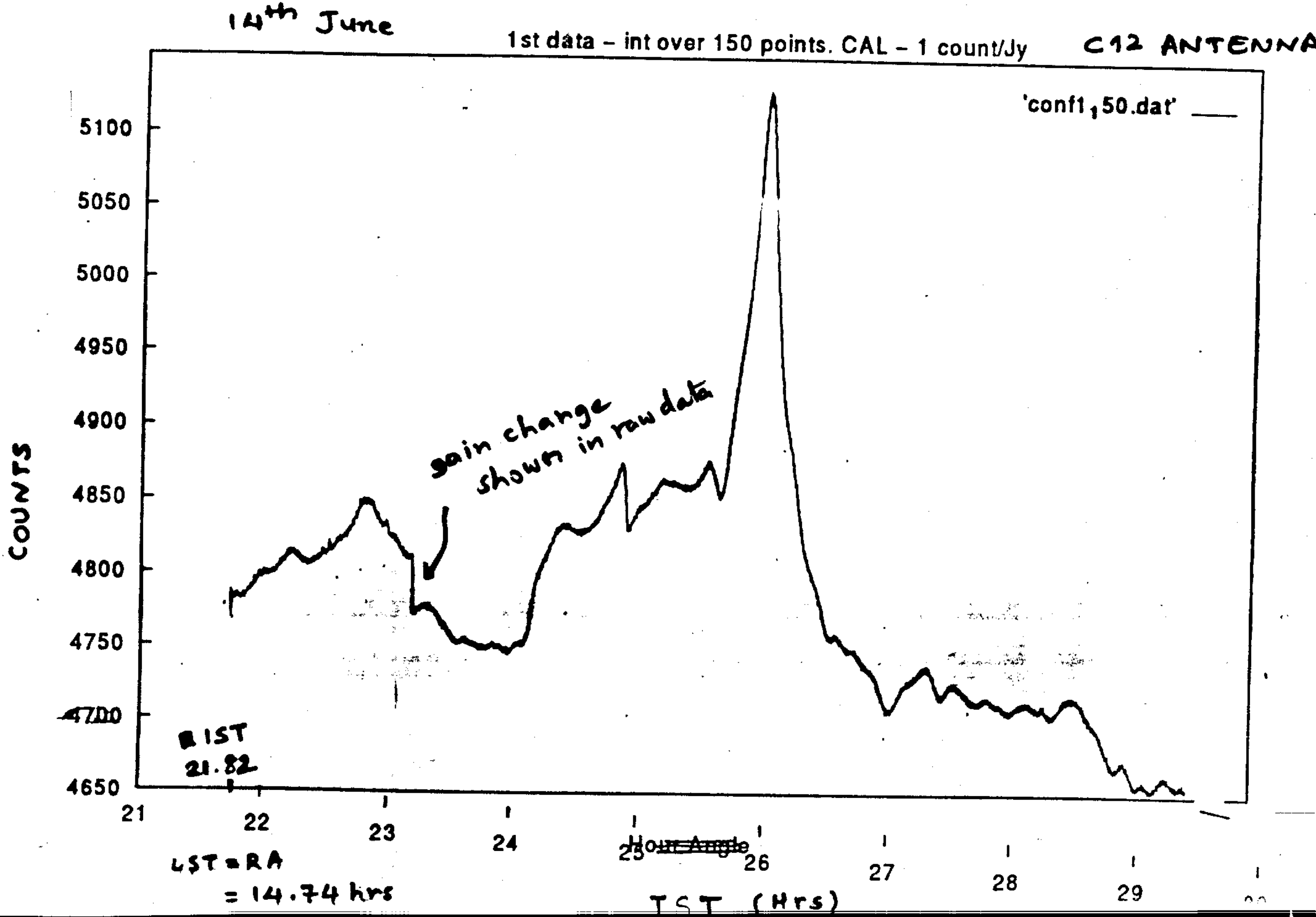
4. During the above analysis the beam size was considered to be 1 degree and this may result in coupling between two points in the two point difference function. This needs to be avoided in the next analysis.

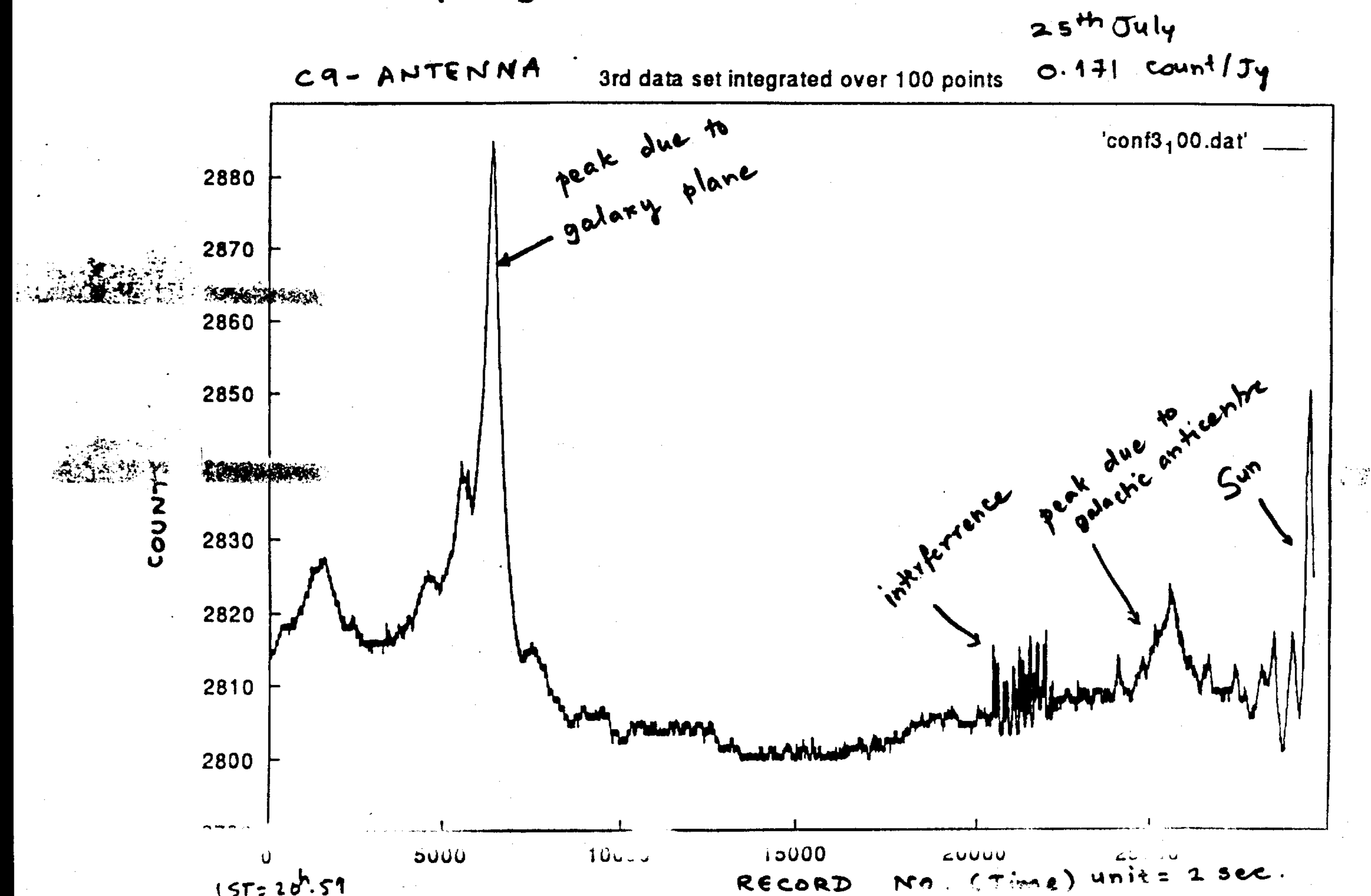
14th June

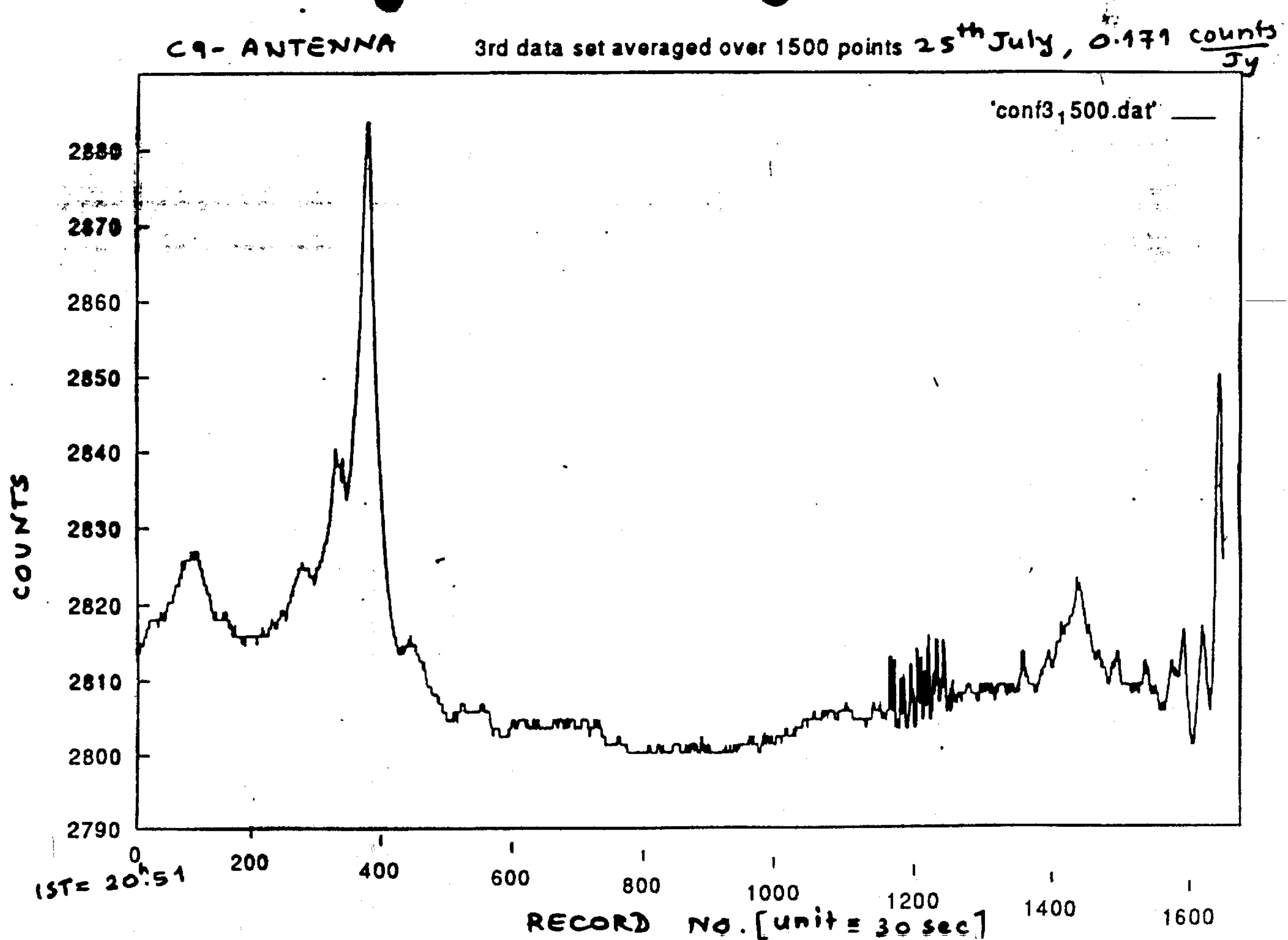
C12. RAW DATA. gain change isolated - first data set 1 COUNT/Jy



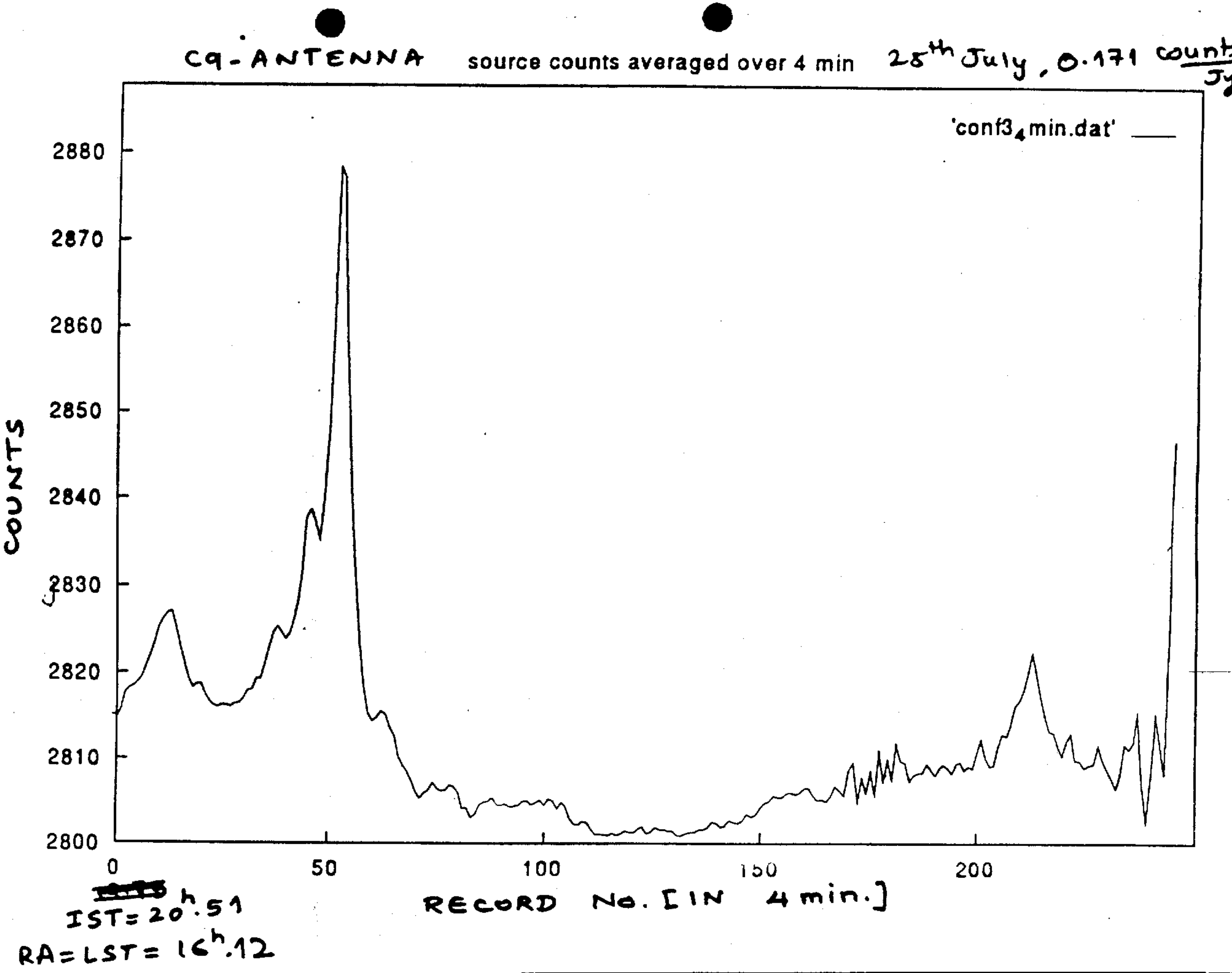
293924


14th June
1st data - int over 150 points. CAL - 1 count/Jy
C12 ANTENNA
'conf1,50.dat'
COUNTS
5100
5050
5000
4950
4900
4850
4800
4750
4700
4650
gain change shown in raw data
IST 21.82
21
22
23
24
25
Hour Angle
26
27
28
29
LST = RA = 14.74 hrs
IST (Hrs)

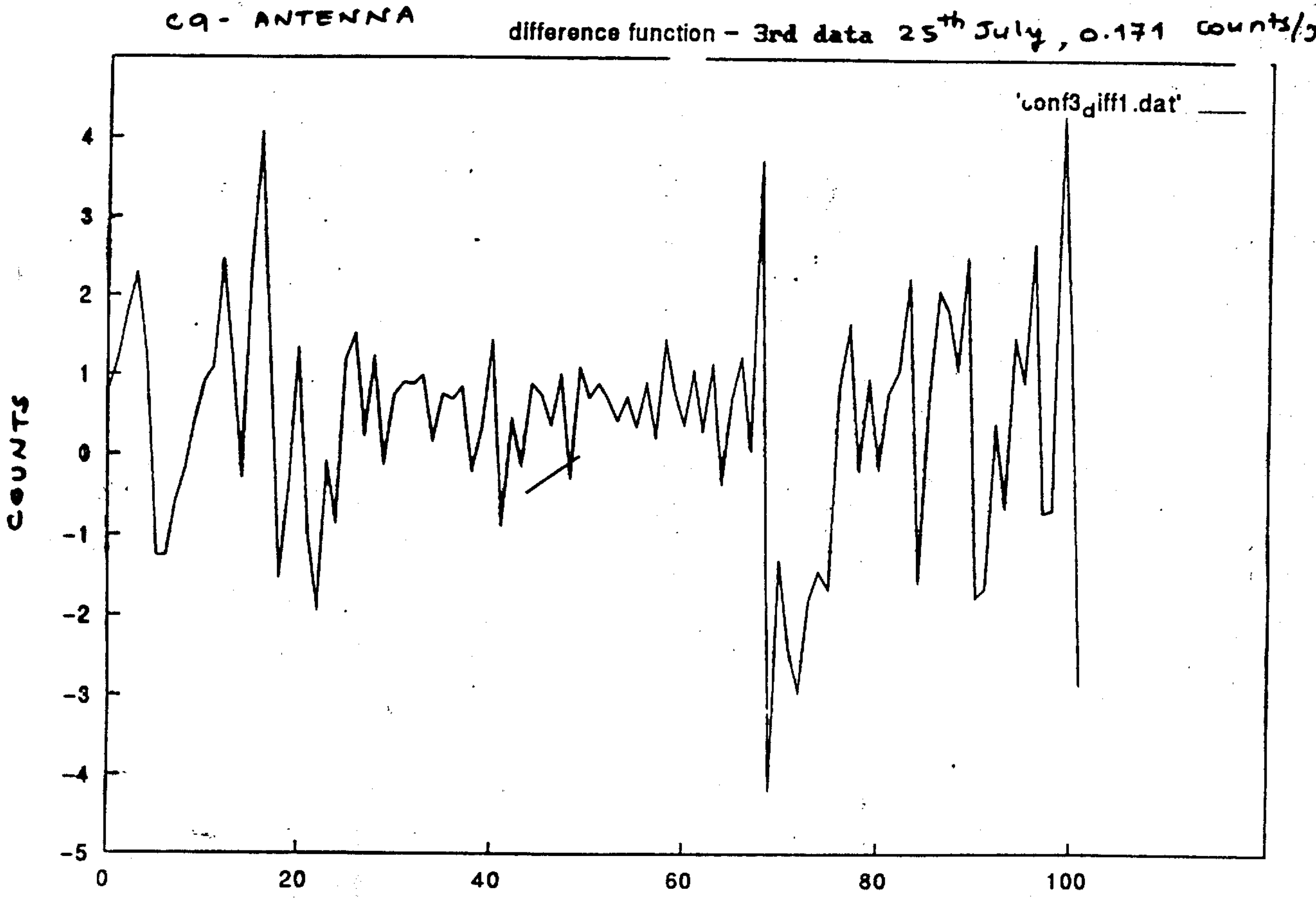

25th July
C9- ANTENNA
3rd data set integrated over 100 points
0.171 count/Jy
'conf3,00.dat'
peak due to galaxy plane
interferrence
peak due to galactic anticentre
Sun
COUNT
2880
2870
2860
2850
2830
2820
2810
2800
0
5000
15000
LST= 20h.59
RECORD No. (Time) unit = 2 sec.



C9- ANTENNA
3rd data set averaged over 1500 points 25th July, 0.171 counts/Jy
'conf3_1500.dat'
2880
2870
2860
2850
2840
2830
2820
2810
2800
2790
COUNTS
0
200
400
600
800
1000
1200
1400
1600
IST= 20h.51
RECORD NO. [unit = 30 sec]



Cq-ANTENNA source counts averaged over 4 min 28th July, 0.171 counts/Jy
'conf3₄min.dat'
COUNTS
2800
2810
2820
2830
2840
2850
2860
2870
2880
0
50
100
150
200
RECORD No. [IN 4 min.]
IST = 20h.51
RA = LST = 16h.12



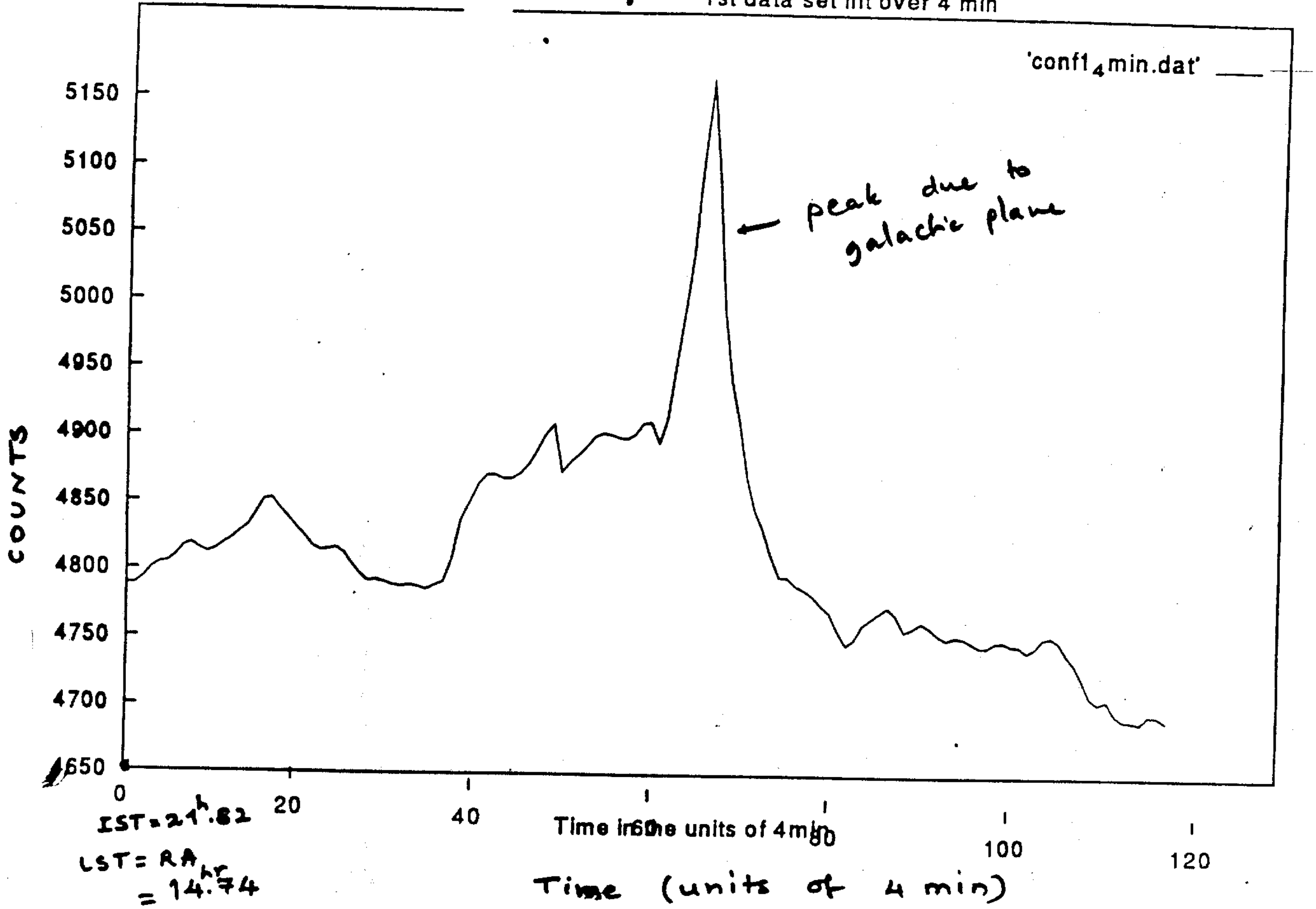
C9- ANTENNA
difference function - 3rd data 25th July, 0.171 counts/
'conf3_diff1.dat'
COUNTS
4
3
2
1
0
-1
-2
-3
-4
-5
0
20
40
60
80
100

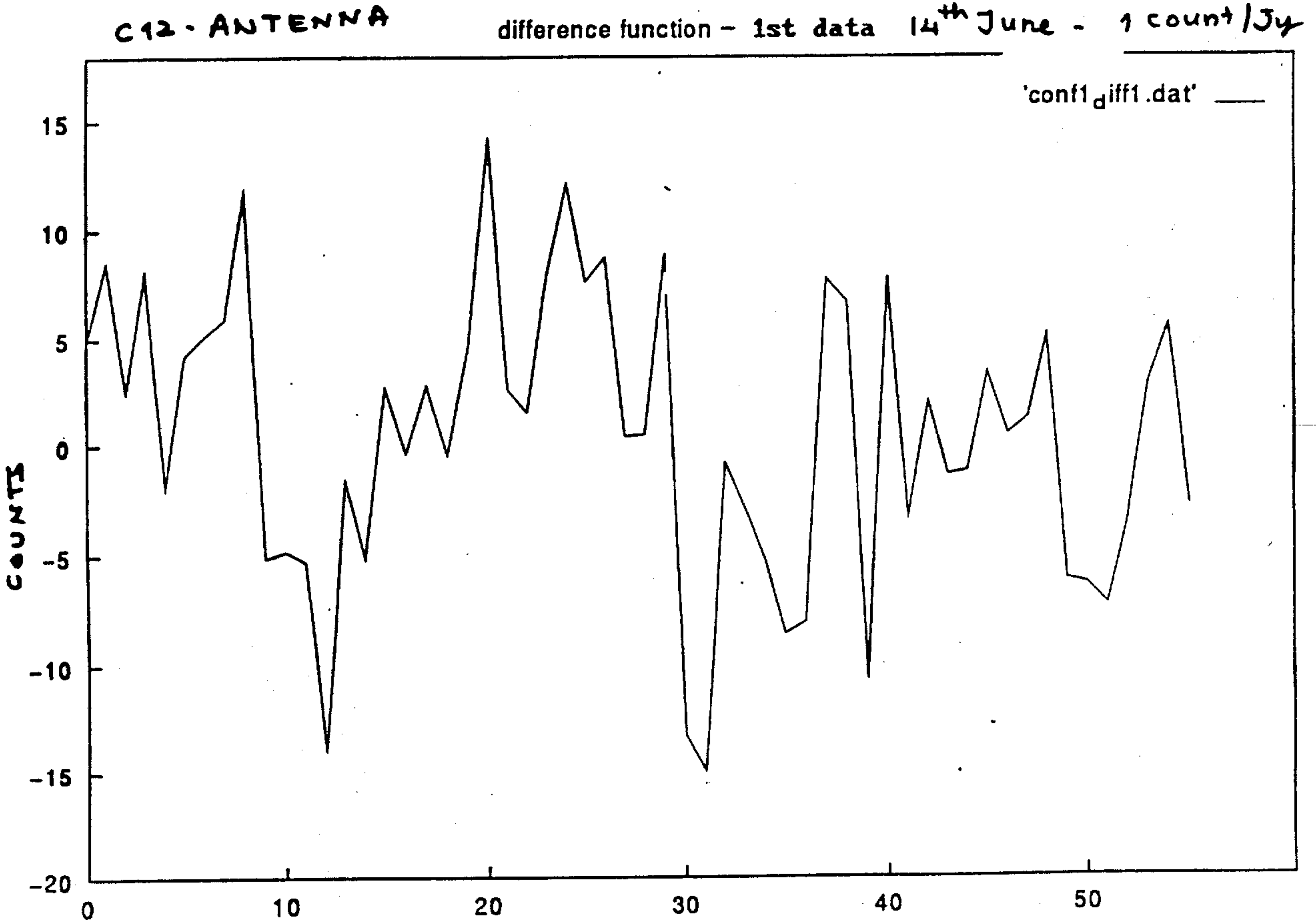
14th June

C12 - ANTENNA, 1 count/Jy

ERRORBARS = 0·0576 coun[ts]
RECEIVER NOISE

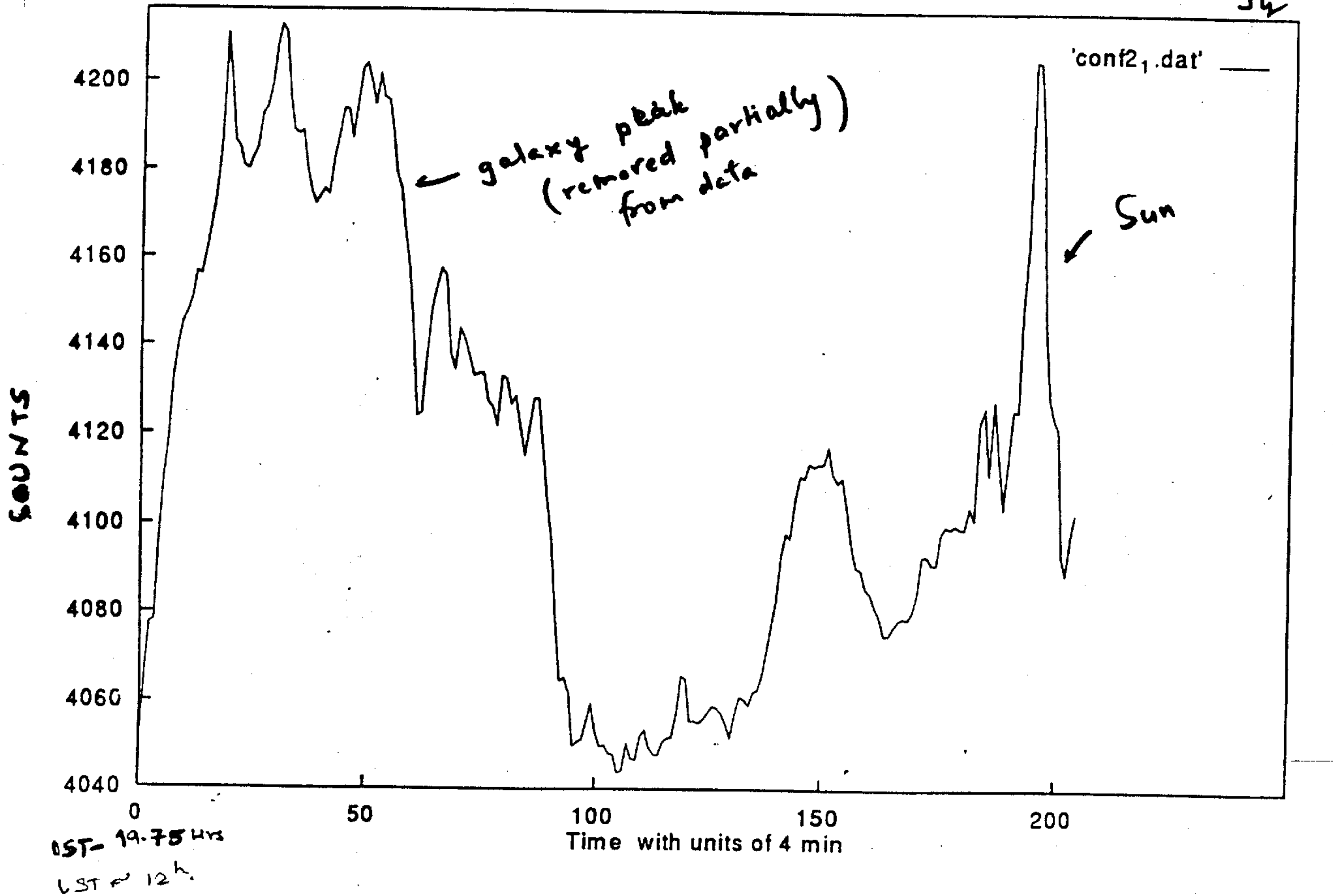
1st data set int over 4 min

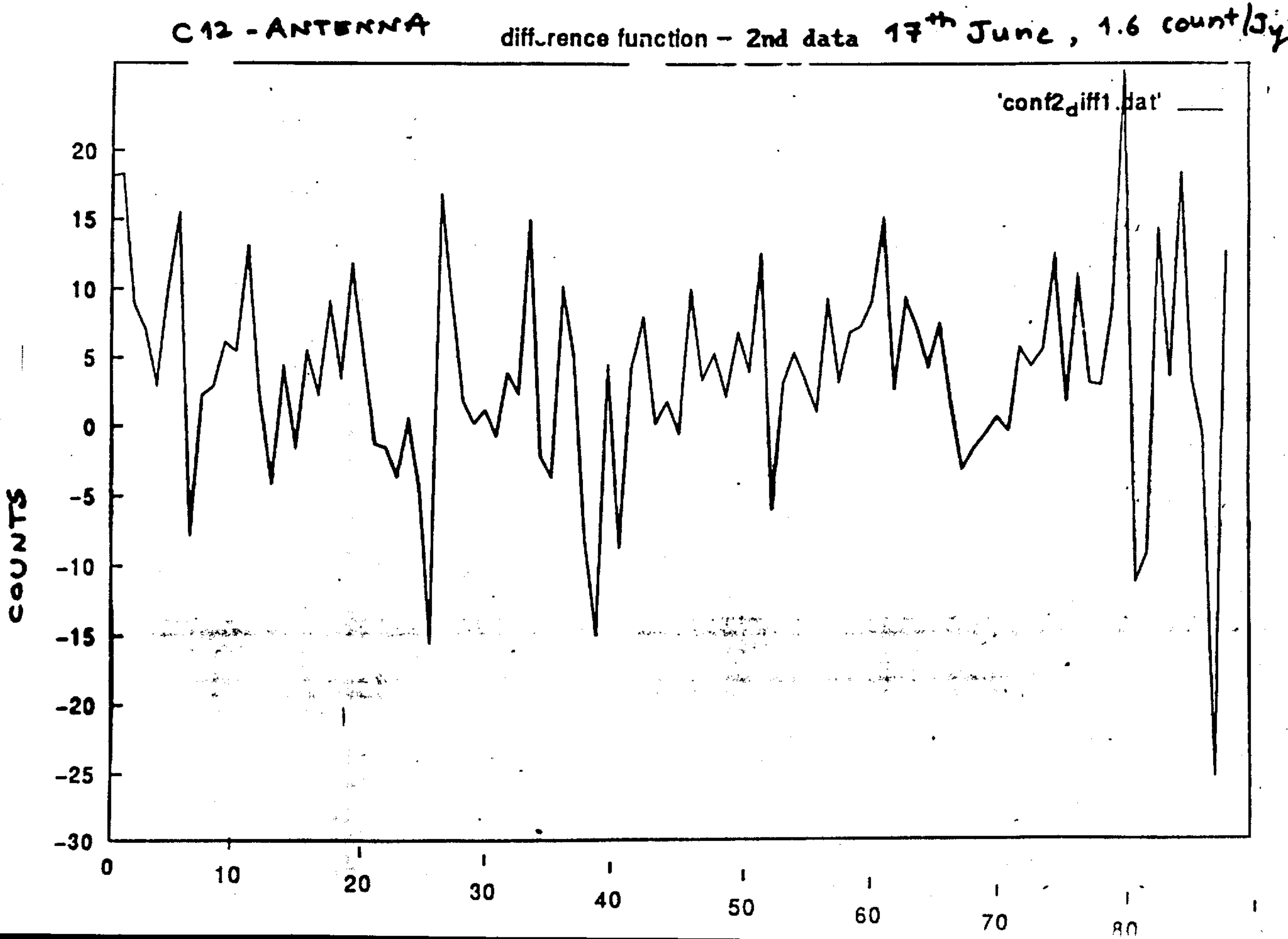




C12 - ANTENNA
difference function - 1st data 14th June - 1 count/Jy
'conf1_diff1.dat'
COUNTS
15
10
5
0
-5
-10
-15
-20
0
10
20
30
40
50

second data with 4 min integration C12 - ANTENNA. 17^{th} June, 1.6 counts/Jy





C12 - ANTENNA
diff.rence function - 2nd data 17th June, 1.6 count/Jy
'conf2diff1.dat'
COUNTS
20
15
10
5
0
-5
-10
-15
-20
-25
-30
0
10
20
30
40
50
60
70
80