Reduction in directivity of a GMRT dish due to displacement of feed centre from the focus point

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April 1992

1 Introduction

In this note we estimate the degradation in efficiency of a GMRT dish due to a small displacement of the feed phase centre from the true focus point of the paraboloid.

A small displacement of the feed centre from the focus can be considered to have two components, one along the axial direction which has a defocussing effect and the other along the lateral direction which is like a squint whose principal effect is a slight shift in the beam direction.

2 Axial Error

The geometry is shown in Fig. 1. Let the physical displacement be d. This causes a phase error that varies with angle θ roughly as $\cos\theta$. The phase error has a minimum value at the edge of the aperture that is $\delta_{min} = (2\pi d/\lambda)\cos 62.^\circ 5$ radians and a maximum value at the centre of the dish of $\delta_{max} = (2\pi d/\lambda)$ radians ($62.^\circ 5$ is the half angle of the GMRT dish from the focus point).

The axial gain of a circular aperture with an arbitrary phase error $\delta(\rho,\phi)$ can be written as (Ruze 1966),

$$G = \frac{4\pi}{\lambda^2}\frac{|\int_0^{2\pi}\int_0^{\rho_0} f(\rho,\phi)\, e^{j\delta(\rho,\phi)}\,\rho\, d\rho\, d\phi|^2}{\int_0^{2\pi}\int_0^{\rho_0} f^2(\rho,\phi)\,\rho\, d\rho}$$

Here ρ and ϕ are the radial and angular coordinates of a point in the aperture plane and $f(\rho,\phi)$ in the normalized illumination field pattern in the aperture. Assuming circular symmetry in the illumination,

$$G = \frac{4\pi}{\lambda^2}\frac{|\int_0^{\rho_0} f(\rho)\, e^{j\delta(\rho)}\,\rho\, d\rho|^2}{\int_0^{\rho_0} f^2(\rho)\,\rho\, d\rho}$$

For small phase errors, the exponential can be expanded as a power series and one can show that the ratio of the gain(G) to the no-error gain($G_\circ$) is

$$\eta = (G/G_\circ) = 1 - \bar{\delta^2} + \bar{\delta}^2$$

where

$$\bar{\delta^2} = \frac{\int_0^{\rho_\circ} f(\rho)\, \delta^2(\rho)\, \rho\, d\rho}{\int_\circ^{\rho_\circ} f(\rho)\, \rho\, d\rho}$$

and

$$\bar{\delta} = \frac{\int_\circ^{\rho_\circ} f(\rho)\, \delta(\rho)\, \rho\, d\rho}{\int_\circ^{\rho_\circ} f(\rho)\, \rho\, d\rho}$$

In general, the phase reference plane can be chosen so that $\bar{\delta}$, the illumination weighted mean phase error is zero. The efficiency is then

$$\eta = 1 - \bar{\delta_\circ^2}$$

where $\bar{\delta_\circ^2}$ is now the illumination weighted variance of $\delta_\circ$, calculated from the mean phase plane i.e. $\delta_\circ = \delta - \bar{\delta}$.

We have estimated the loss in directivity for the GMRT dish for a typical illumination (taken from the coaxial feed power pattern as given in Fig. 3 of Kapahi's technical report on 'Surface Errors and Aperture Efficiency for GMRT dishes' dated 5.2.1991) that has an edge taper of 10 db. The illumination weighted mean phase in radians works out to

$$\bar{\delta} = 0.766\,(2\pi d/\lambda) = 4.813\,(d/\lambda)$$

and the weighted variance,

$$\bar{\delta_\circ^2} = 0.0234\,(2\pi d/\lambda)^2$$

i.e.

$$\bar{\delta_\circ^2} = 0.924\,(d/\lambda)^2$$

so that

$$\eta = 1 - 0.924\,(d/\lambda)^2$$

It is seen from the above result that for a feed centre displacement of $d = 3$ cm (corresponding to $\sim \lambda/7$ at 21 cm wavelength and $\sim \lambda/16$ at 49 cm) in the axial direction, the loss in Directivity of the GMRT dish is only about 1.9% and 0.35% at 1.4 GHz and 610 MHz respectively. The loss in directivity for some axial displacements is given in Table 1, and a plot of η against d/λ is shown in Figure 2.

Axial Displacement	Loss in Directivity
$\lambda/20$	0.23%
$\lambda/16$	0.36%
$\lambda/12$	0.64%
$\lambda/8$	1.44%
$\lambda/6$	2.57%
$\lambda/4$	5.78%
$\lambda/3$	10.3%
$\lambda/2$	23.1%

Table 1: Loss in Directivity for different axial displacements

3 Lateral Displacement (Squint)

The principal effect of a small lateral displacement of the feed centre is to introduce a phase error that varies with θ as $\delta{\sim}(2\pi d/\lambda)\sin\theta$ radians. In the case of the GMRT dish this varies almost linearly from a value of $0.89(2\pi d/\lambda)$ radians at one edge of the aperture to zero at the aperture centre and a value of $-0.89(2\pi d/\lambda$ at the opposite end of the aperture (see Figure 3). This has the effect of tilting the beam in a direction opposite to the feed displacement by an angle of about $\Delta\beta$ radians given by

$$\Delta\beta = d/\rho_o$$

Note that the squint is independent of wavelength of operation. As an example, a lateral displacement of the feed centre by 3 cm shifts the beam by about 4.6 arcmin. For small displacements the beam shift is about 1.5 arcmin for every 1 cm of lateral error. The shift can in principle be determined by calibration of sources in the sky and an appropriate pointing correction made.

As the maximum deviation of the actual phase error over the linear approximation (see Fig. 3) is always smaller than about $0.1(2\pi d/\lambda)$ radians, the loss in directivity is likely to be $\ll 1\%$ even for reasonably large values of d and at the highest frequency of 1.4 GHz. It can therefore be ignored.

References

Ruze, J. "Antenna Tolerance Theory - A Review", Proc. IEEE, **54**, 633(1966).

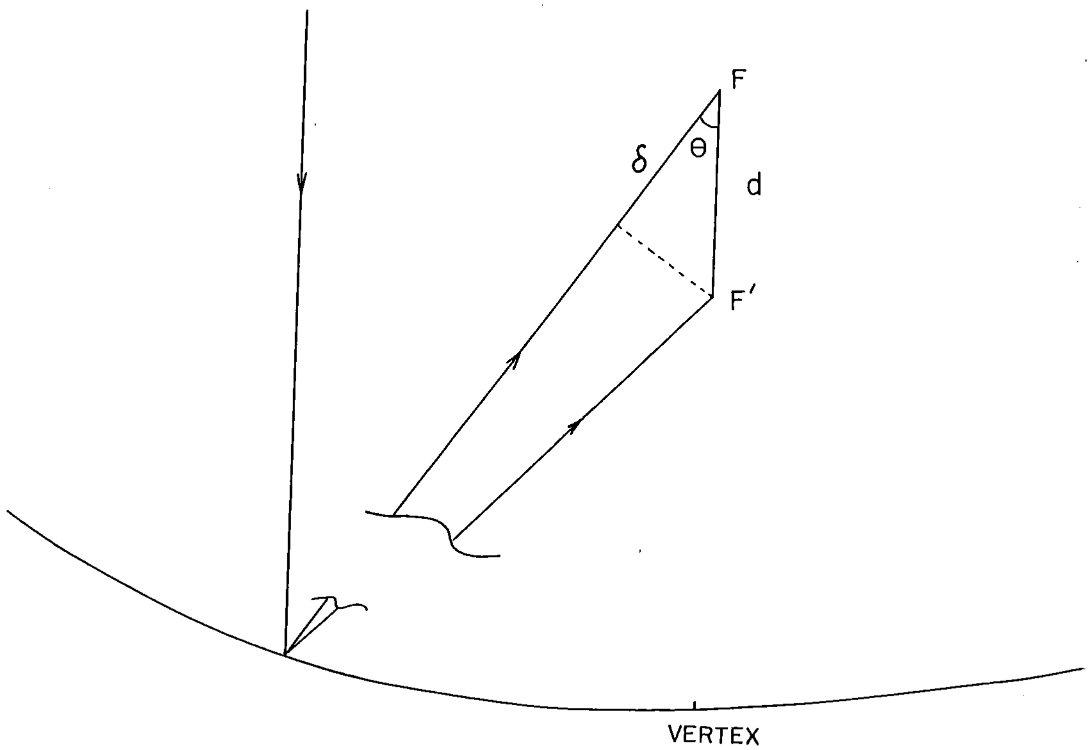

FIGURE - I

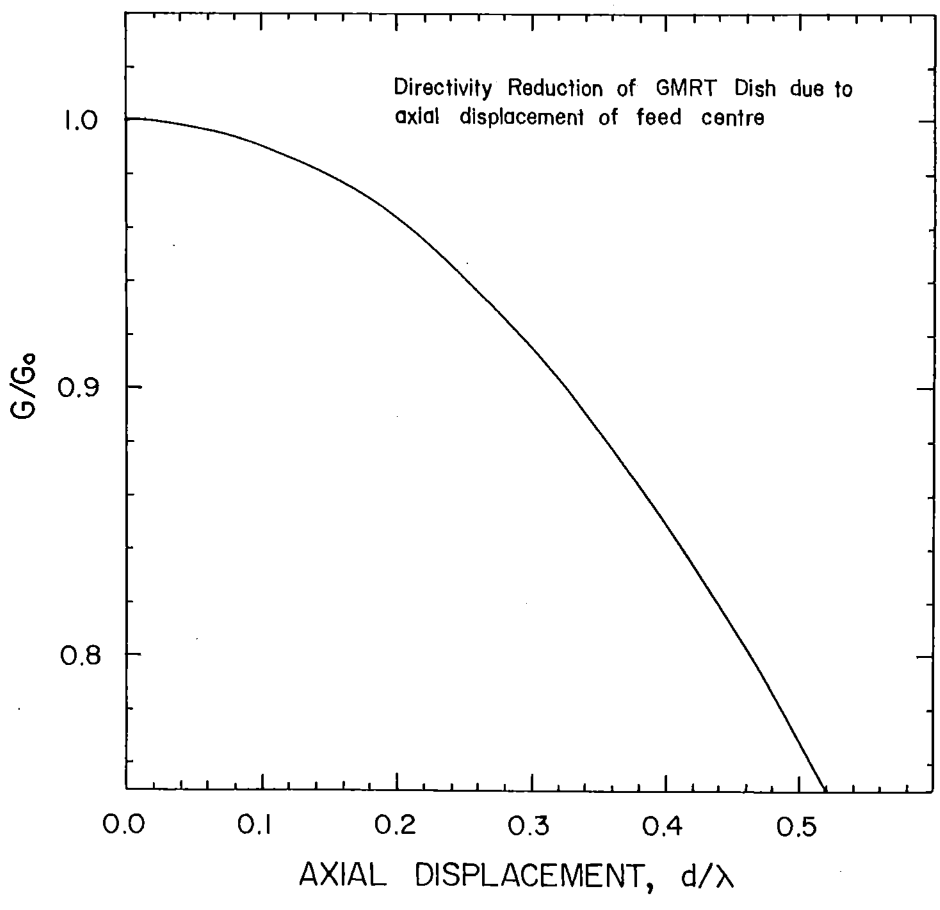

FIGURE - 2

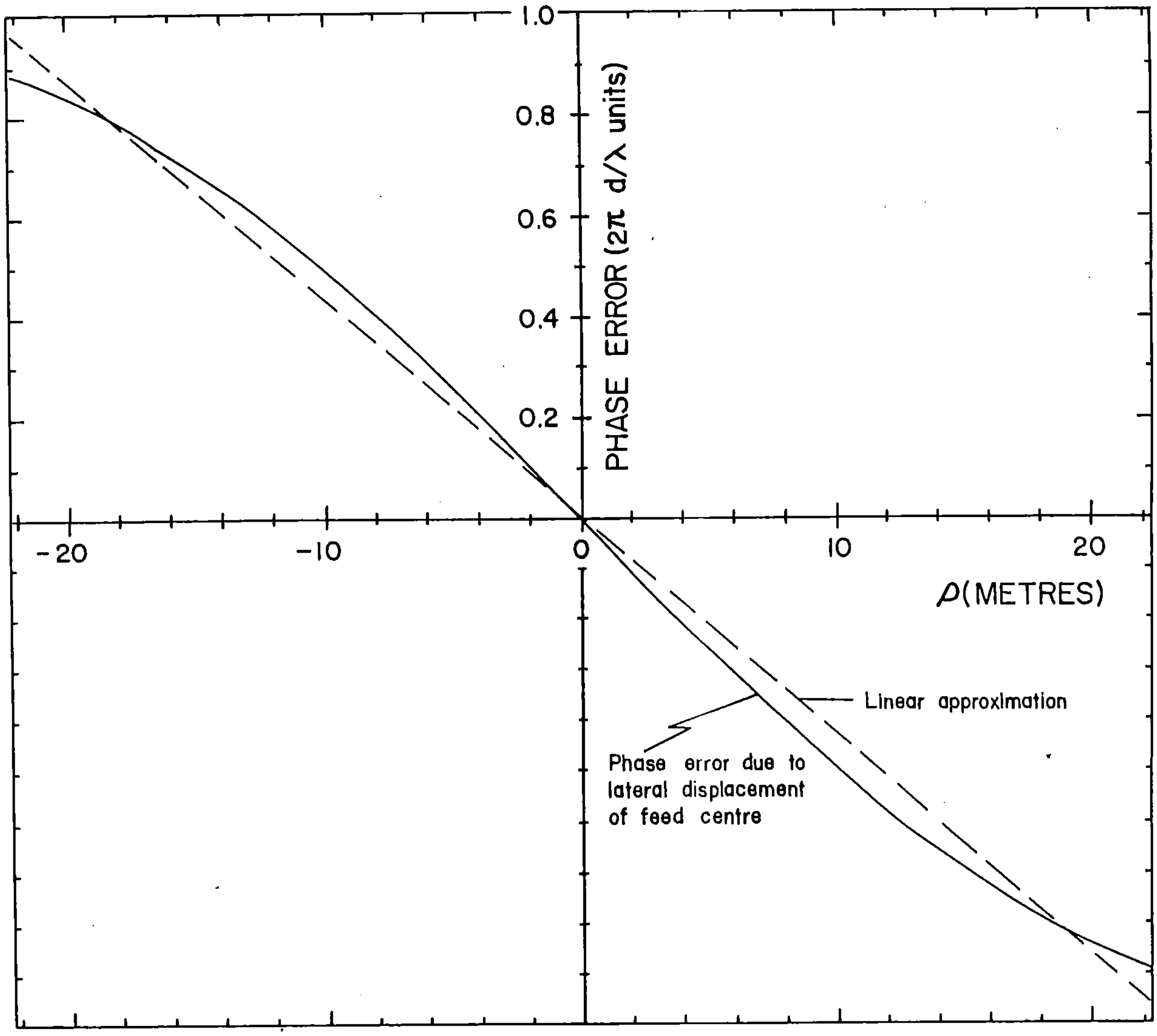
1.0
0.8
0.6
0.4
0.2
PHASE ERROR (2π d/λ units)
-20
-10
0
10
20
ρ(METRES)
Linear approximation
Phase error due to
lateral displacement
of feed centre

FIGURE - 3