NOTE ON TRACKING WITH GMRT

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The mounts for antennas in GMRT are alt-azimuth. For tracking a source, the coordinates(azimuth and elevation) (henceforth, coordinate means either the azimuth or the elevation), rate of change of coordinates and the time interval, during which the antenna moves uniformly at the specified rate are to be specified. Since the coordinate as a function of time is not linear, this linear approximation is not precise. The problem is to find out, for a given accuracy(i.e. maximum allowable error), the time interval in which the coordinate as a function of time can be represented by a straight line. This has implications on how frequently the antenna tracking software should update the coordinates.

The accuracy required is calculated as follows. The position of the antenna is represented by a 17-bit encoder. Therefore the coordinate of the antenna can be changed in steps of $360.0 \times 3600/2^{17} \approx 10$ arcseconds. So, the accuracy was chosen to be 5 arcseconds.

PROCEDURE

For a given source, coordinate is a function of hour angle and hour angle itself is a linear function of Universal time (see calculations for the relation between hour angle and time). Hence we consider the coordinate as a function of hour angle and find out the hour angle interval Δt in which the coordinate can be represented by a straight line. To get ΔT in terms of Universal time interval, we have to divide the hour angle interval by $\frac{d(h)}{dt} = 0.9972695663$.

The coordinate as a function of hour angle is expanded in Taylor series. If we approximate the function by a straight line, then the magnitude of the error is of the order of the second term. If n is the accuracy (i.e. maximum allowable error) required, then the hour angle interval Δt in which, the path is a straight line is given by $[\frac{2n}{d^2(el)/dh^2}]^{1/2}$. Δt is calculated for different hour angles.

CALCULATIONS

given:

ϕ	=	latitude of the place
δ	=	declination of the source

$sin(el) = \quad sin(\delta) \times sin(\phi) + cos(\delta) \times cos(\phi) \times cos(h)$

where

el = elevation of the source

h = hour angle

$sin(el) = \quad a + b \times cos(h)$

where

$a = sin(\delta) \times sin(\phi)$ and $b = cos(\delta) \times cos(\phi)$ are constants for a given source

$$\frac{d(el)}{dh} = \frac{-b \times sin(h)}{cos(el)}$$

$$\frac{d^2(el)}{dh^2} = \frac{sin(el) \times (\frac{d(el)}{dh})^2 - b \times cos(h)}{cos(el)}$$

$$\Delta t_e = [\frac{2 \times N_e}{abs(d^2(el)/dh^2)}]^{1/2}$$

where

N_e = accuracy required(maximum allowable error) = 5"

Δt_e = hour angle interval in which the elevation can be approximated by a straight line

$$h = lst - \alpha = gst + long - \alpha$$

$$= gst0 + 0.9972695663 \times t + long - \alpha$$

where

h = hour angle

lst = local sidereal time

α = right ascension of the source

gst = Greenwich sidereal time

long = Longitude of the place

gst0 = Greenwich sidereal time at 0h UT on the date of observation

$$\frac{d(el)}{dt} = \frac{d(el)}{dh} \times \frac{d(h)}{dt}$$

where

$$\frac{d(h)}{dt} = 0.9972695663$$

$$\frac{d^2(el)}{dt^2} = \frac{d^2(el)}{dh^2} \times (\frac{d(h)}{dt})^2$$

Therefore,

$$\Delta T_e = [\frac{2 \times N_e}{abs(d^2(el)/dt^2)}]^{1/2} = \frac{\Delta t_e}{(d(h)/dt)}$$

where

ΔT_e = Universal time interval in which the elevation can be approximated by a straight line

cos(az) = (c× sec(el)) - (d × tan(el))

where

az = azimuth of the source

$c = \frac{sin(\delta)}{cos(\phi)}$ and $d = tan(\phi)$ are constants for a given source

$$\frac{d(az)}{dh} = -\frac{[c \times sec(el) \times tan(el) - d \times sec^2(el)] \times d(el)/dh}{sin(az)}$$

$$\frac{d^2(az)}{dh^2} = -\frac{1}{sin(az)} \times [cos(az) \times (d(az)/dh)^2 + [c \times sec(el)(2 \times tan^2(el) + 1) - 2d \times sec^2(el) \times tan(el)](d(el)/dh)^2 + [c \times sec(el) \times tan(el) - d \times sec^2(el)] \times d^2(el)/dh^2]$$

$$\Delta t_a = \left(\frac{2 \times N_a}{abs(d^2(az)/dh^2)}\right)^{1/2}$$

where

N_a = accuracy required(maximum allowable error) = 5"

Δt_a = hour angle interval in which azimuth can be approximated by a straight line

$$\Delta T_a = \left[\frac{2 \times N_a}{abs(d^2(az)/dt^2)}\right]^{1/2} = \frac{\Delta t_a}{(d(h)/dt)}$$

where

ΔT_a = Universal time interval in which the azimuth can be approximated by a straight line

RESULTS

Let us consider the elevation case. Figs(1)-(8) gives the plot of el, $\frac{d(el)}{dh}$, $\frac{d^2(el)}{dh^2}$ as a function of hour angle for few declinations.Δt is calculated for elevation $> 15^0$, since the antenna does not track the source below this limit.

For a given source, elevation increases, with hour angle till hour angle = 0. Then it starts decreasing (see fig.1 and fig.2). We expect that the linear approximation to be poorer at hour angle = 0. For declination>latitude, the curvature at the hour angle=0 decreases as declination increases(fig.2). For declination < latitude, the curvature decreases as declination decreases at hour angle = 0 (fig.1). So, linear approximation is poorest for declination = latitude of the place and hour angle =0 (fig.9,10,11).

let us consider the azimuth case. Figs(12)-(22) gives the plot of az, $\frac{d(az)}{dh}$, $\frac{d^2(az)}{dh^2}$ as a function of hour angle for few declinations.

For a given source, azimuth is a monotonic function of hour angle. $\frac{d(az)}{dh}$ decreases(or increases) as hour angle increases upto hour angle = 0, then it starts increasing(or decreasing). Therefore we expect a turn-over at two points in $\frac{d^2(az)}{dh^2}$, one for hour angle < 0.0 and another for hour angle > 0.0, at which $\frac{d^2(az)}{dh^2}$ reaches minimum(or maximum). (fig.18 to fig.22). So, we expect the linear approximation to be poorest at this hour angle (fig.(23)-(26)). Hour angle = 0 is a point of inflection. Therefore we expect approximation to be exact (upto second order) at hour angle = 0 (fig.(23)-(26)).

If we choose $\Delta t = 10$ seconds, we can approximate the elevation by a straight line except for the range $18^0 < dec < 20^0$ and $-0.1h < hourangle < 0.1h$. For the same time interval, azimuth can be represented by straight line except for the range $15^0 < dec < 24^0$ and $-0.2h < hourangle < +0.2h$. For the time inte rval = 5 seconds, the azimuth can be represented by a straight line except for the range $18^0 < dec < 21^0$ and $-0.1h < hourangle < 0.1h$.

LIST OF FIGURES

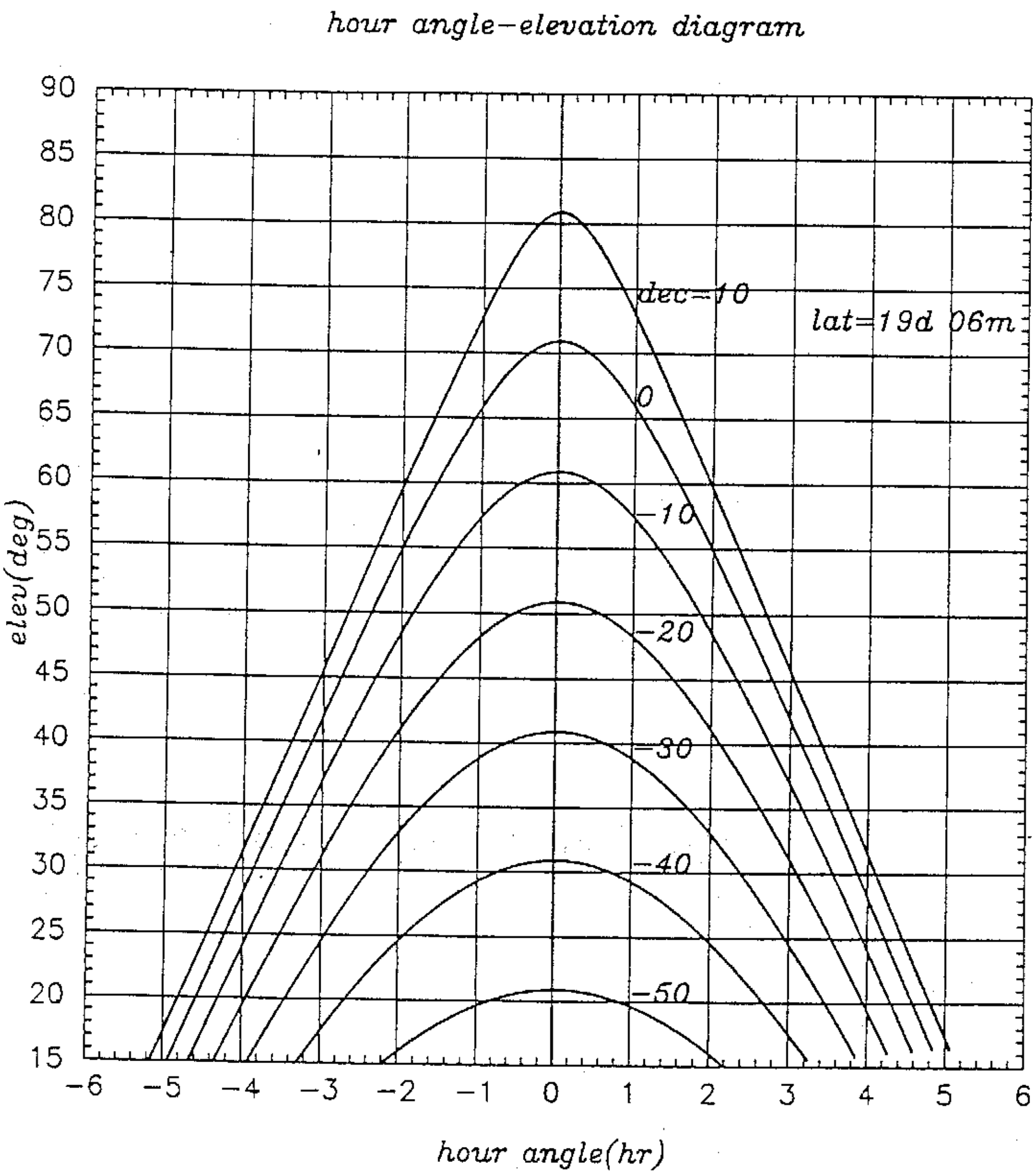

hour angle−elevation diagram
dec=10
lat=19d 06m
0
−10
−20
−30
−40
−50
elev(deg)
hour angle(hr)
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5
6

Fig. 1

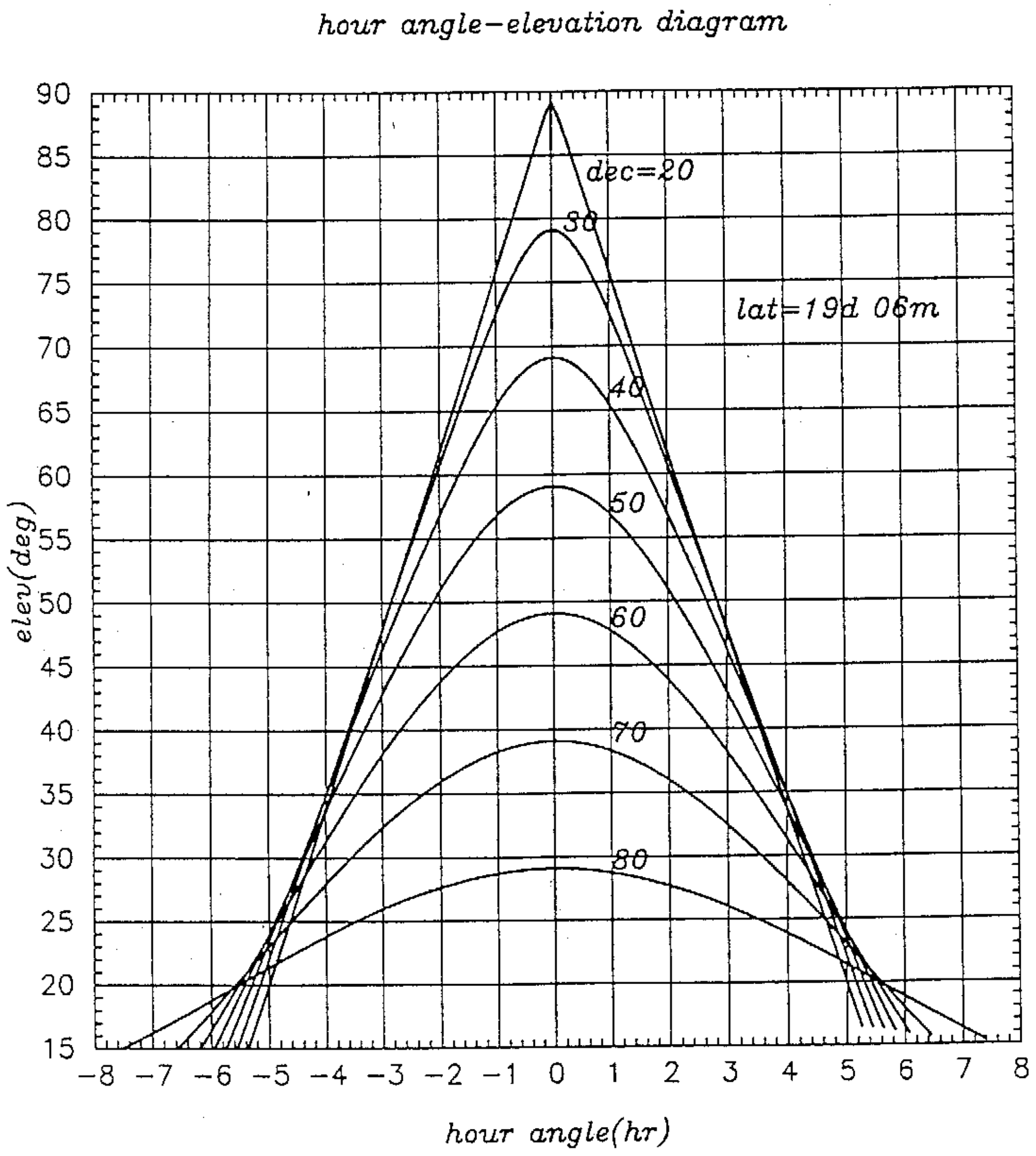
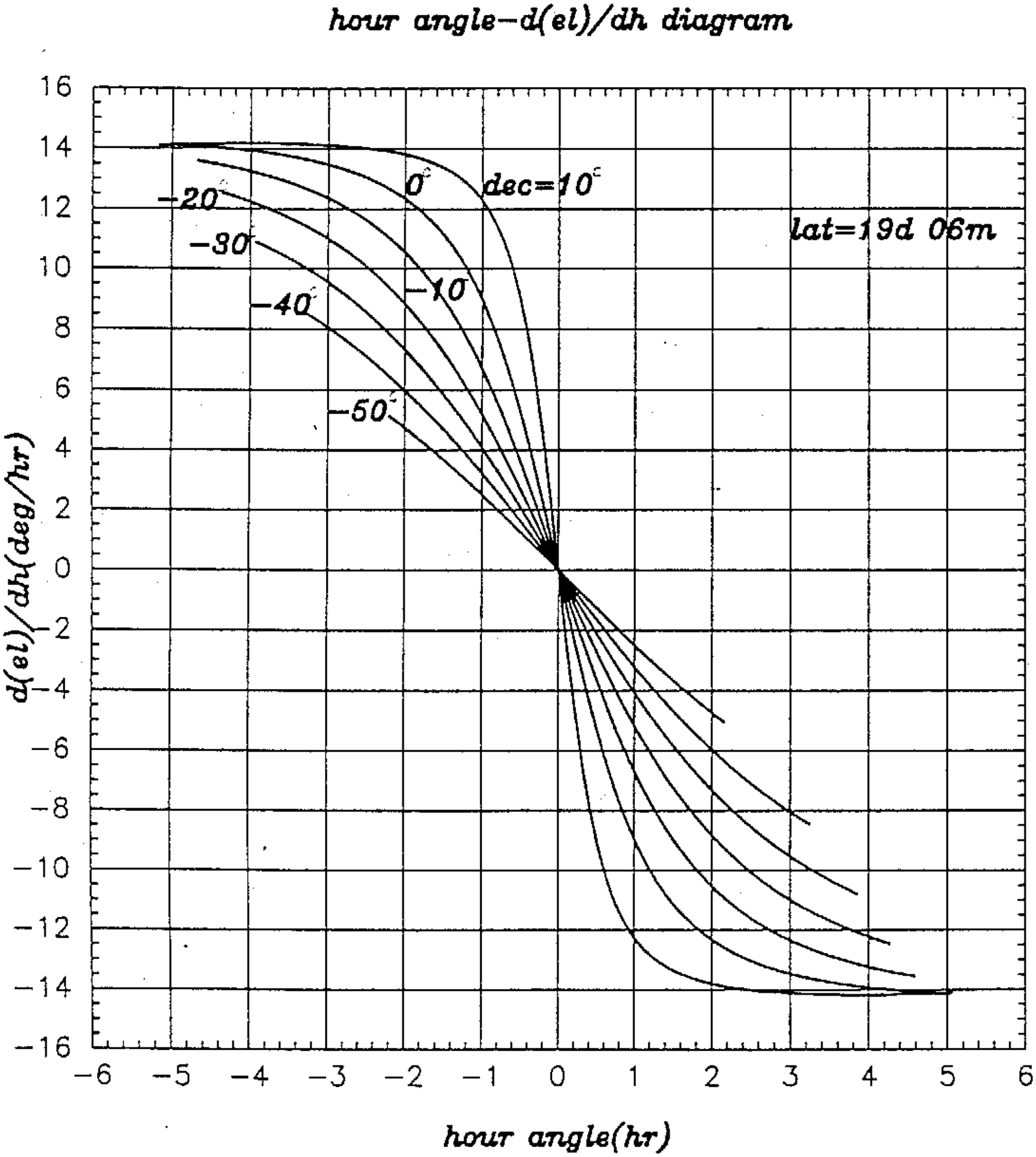
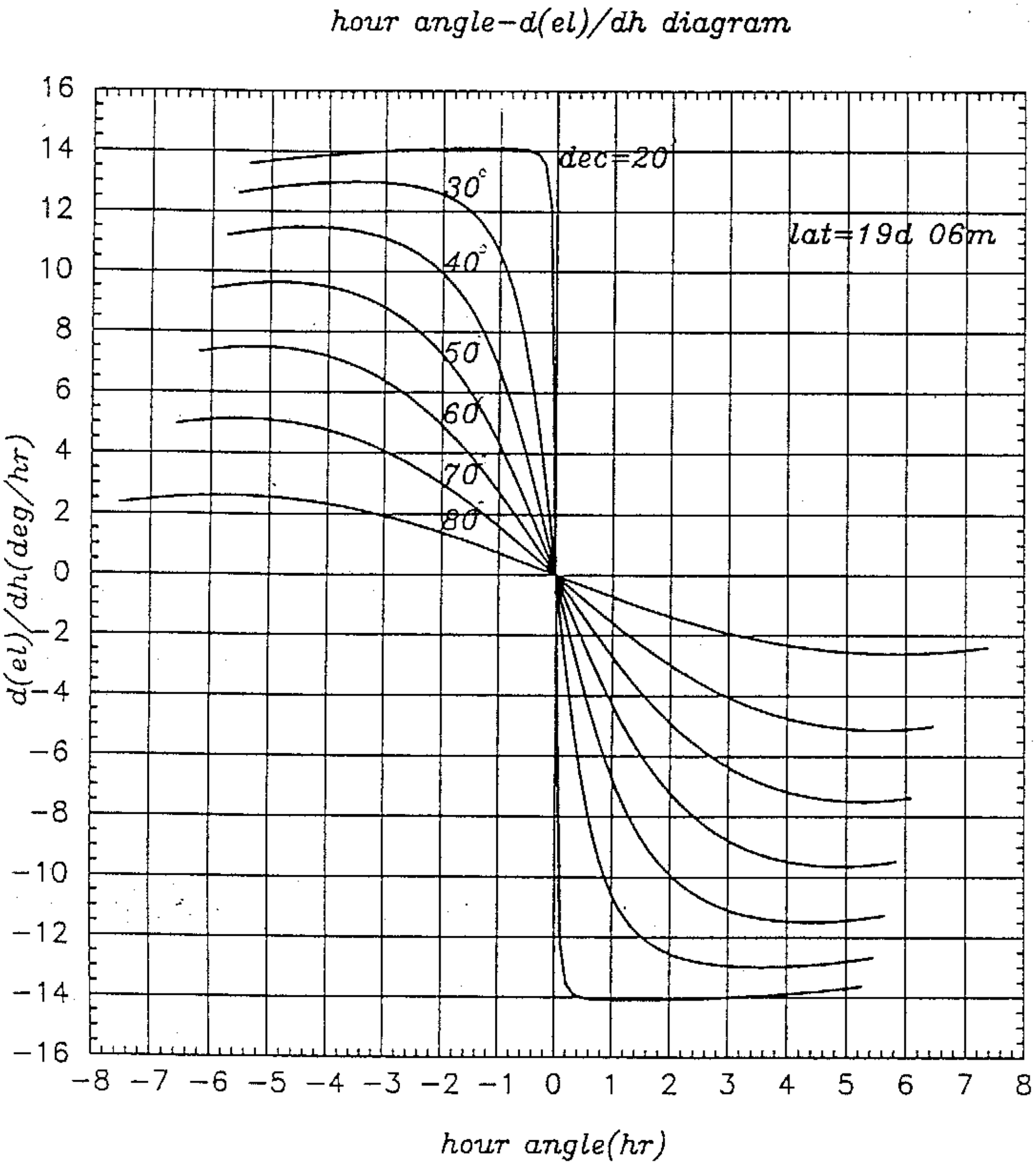

hour angle-elevation diagram
dec=20
30
40
50
60
70
80
lat=19d 06m
elev(deg)
hour angle(hr)
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
−8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8

Fig. 2



hour angle–d(el)/dh diagram
dec=10°
0°
−10°
−20°
−30°
−40°
−50°
lat=19d 06m
d(el)/dh(deg/hr)
hour angle(hr)
16
14
12
10
8
6
4
2
0
−2
−4
−6
−8
−10
−12
−14
−16
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5
6

Fig. 3



hour angle-d(el)/dh diagram
dec=20°
30°
40°
50°
60°
70°
80°
lat=19d 06m
d(el)/dh(deg/hr)
hour angle(hr)
16
14
12
10
8
6
4
2
0
−2
−4
−6
−8
−10
−12
−14
−16
−8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8

Fig. 4

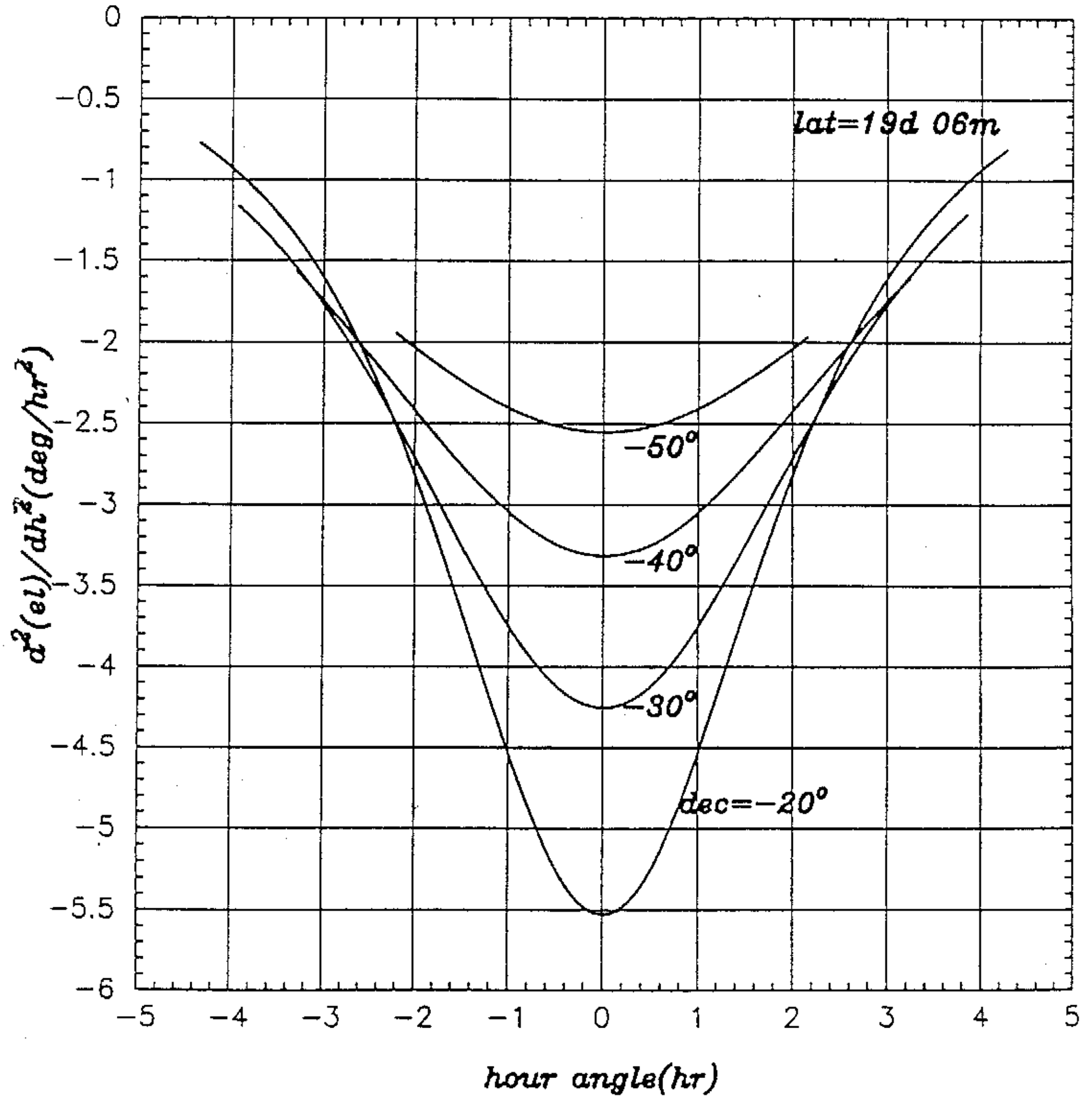


Fig.5

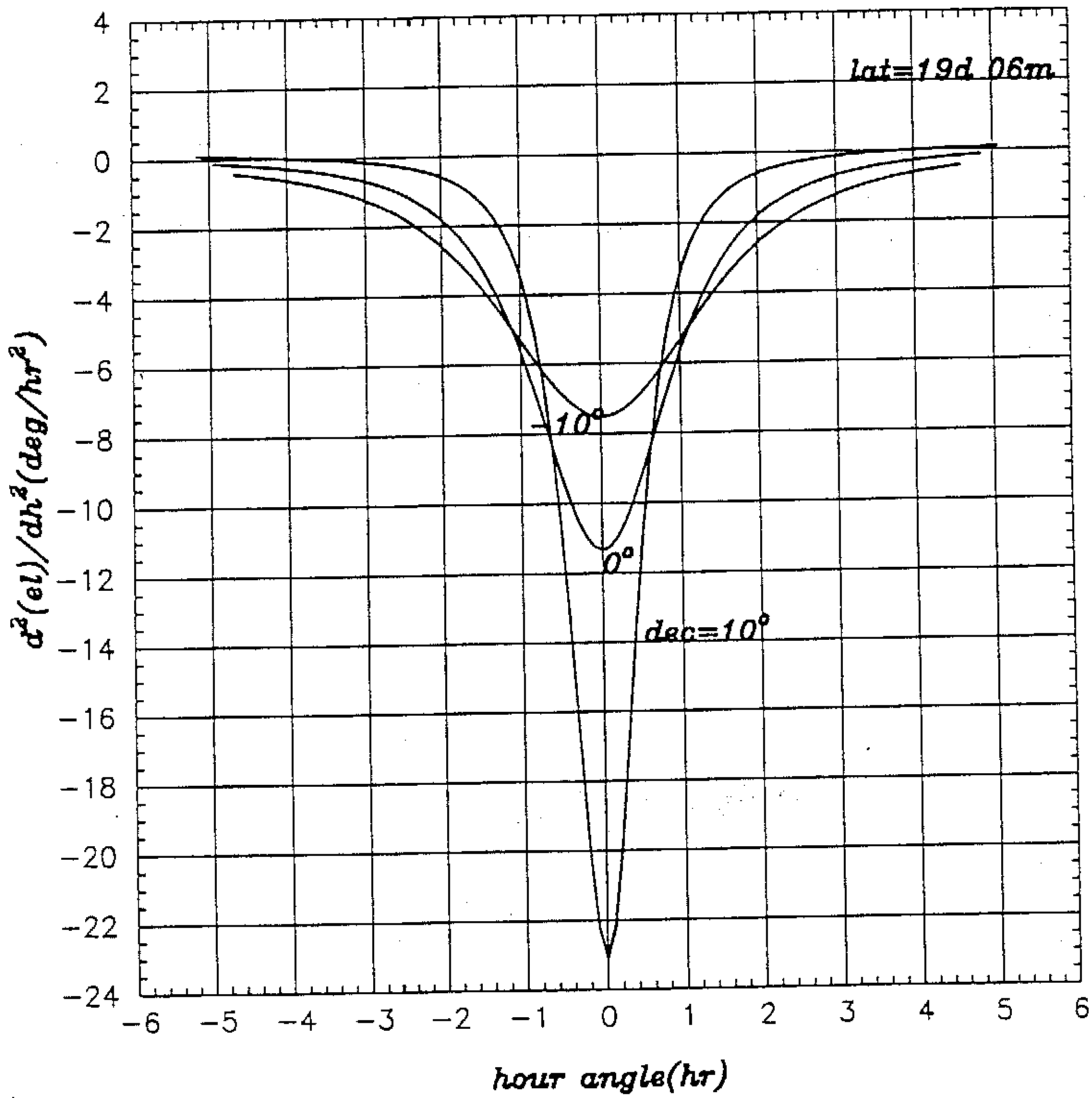

lat=19d 06m
-10°
0°
dec=10°
4
2
0
−2
−4
−6
−8
−10
−12
−14
−16
−18
−20
−22
−24
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
hour angle(hr)
$d^2(el)/dh^2(deg/hr^2)$

Fig. 6

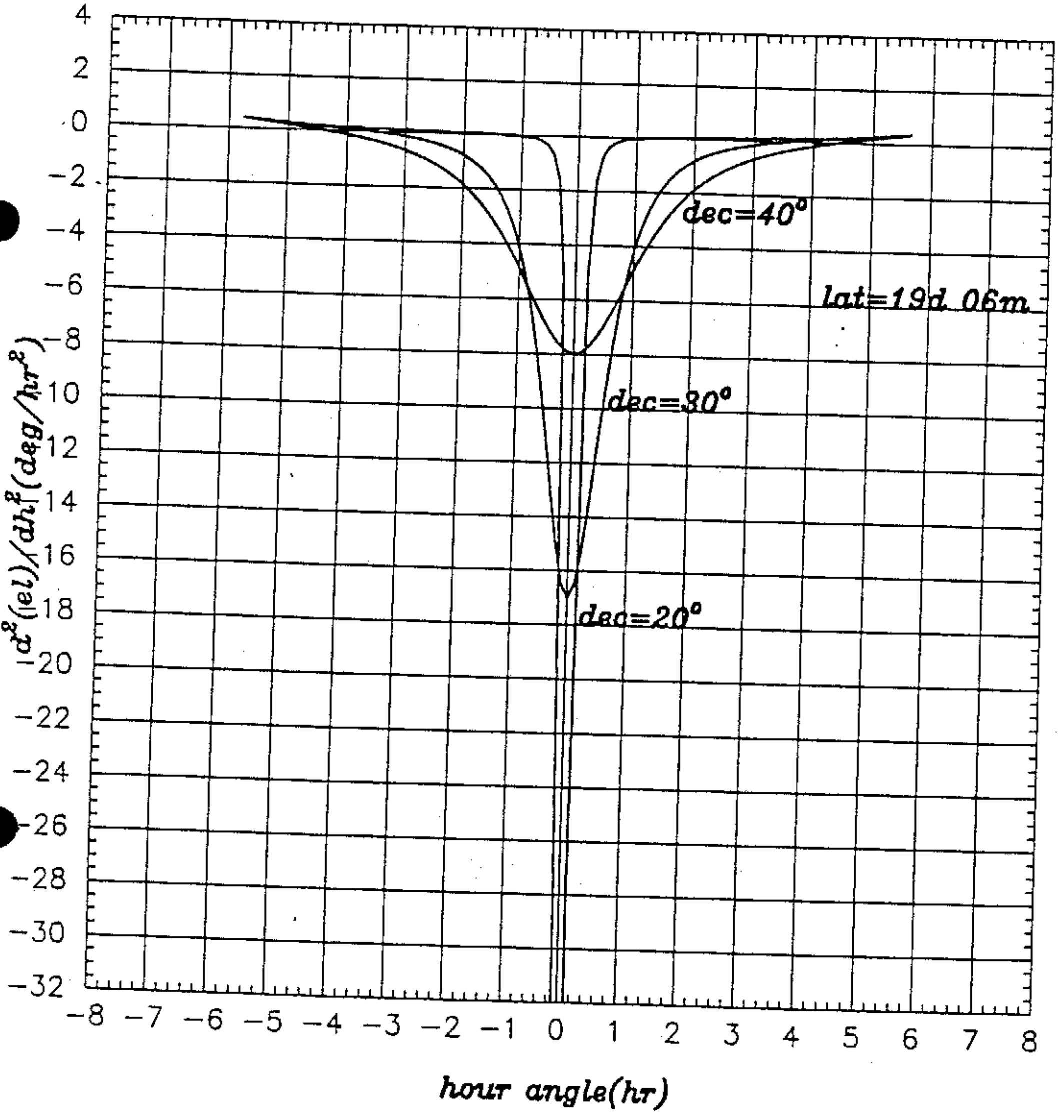
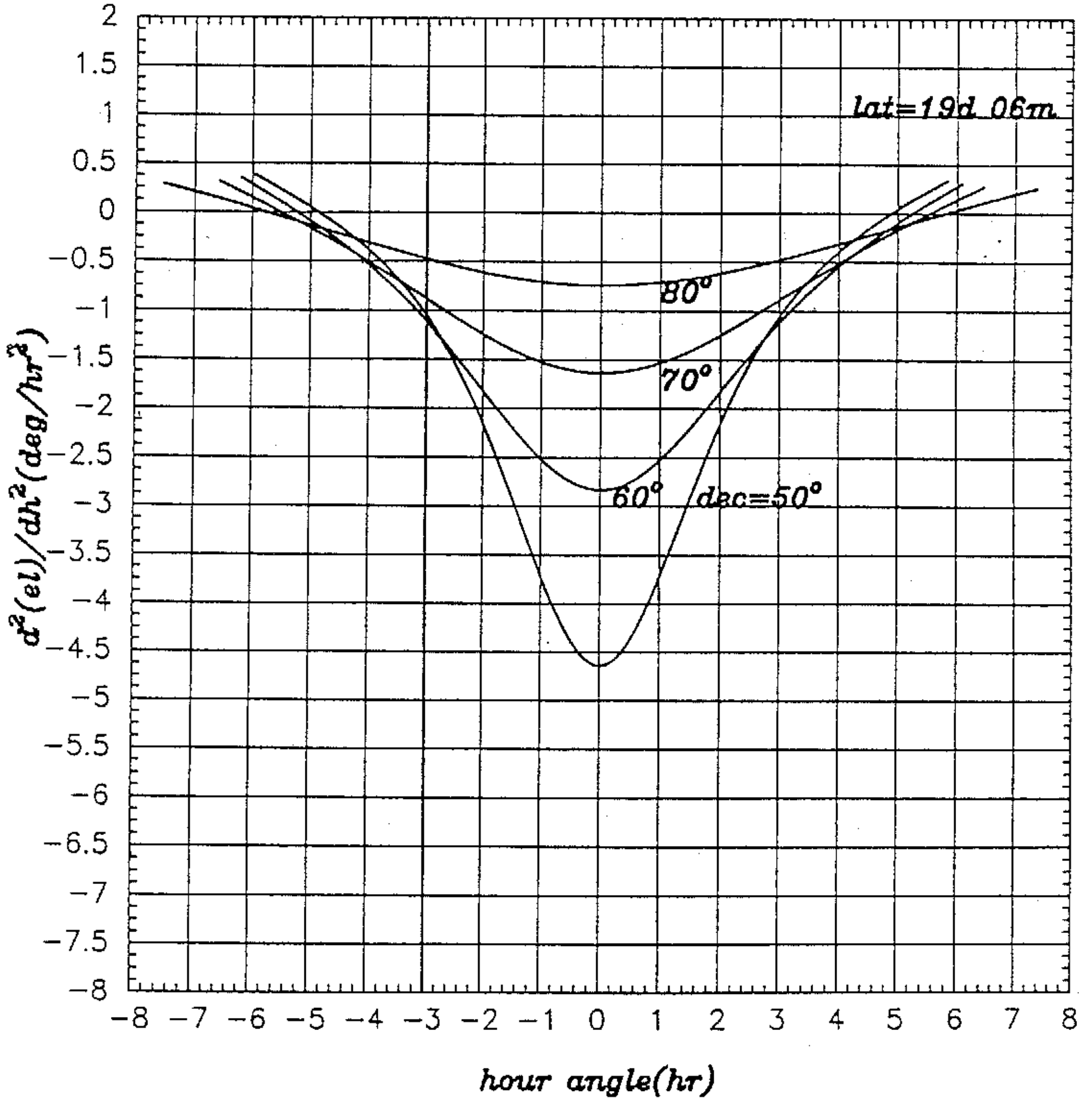


Fig. 7



lat=19d 06m
80°
70°
60°
dec=50°
$d^2(el)/dh^2(deg/hr^2)$
hour angle(hr)
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
−3
−3.5
−4
−4.5
−5
−5.5
−6
−6.5
−7
−7.5
−8
−8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8

Fig. 8

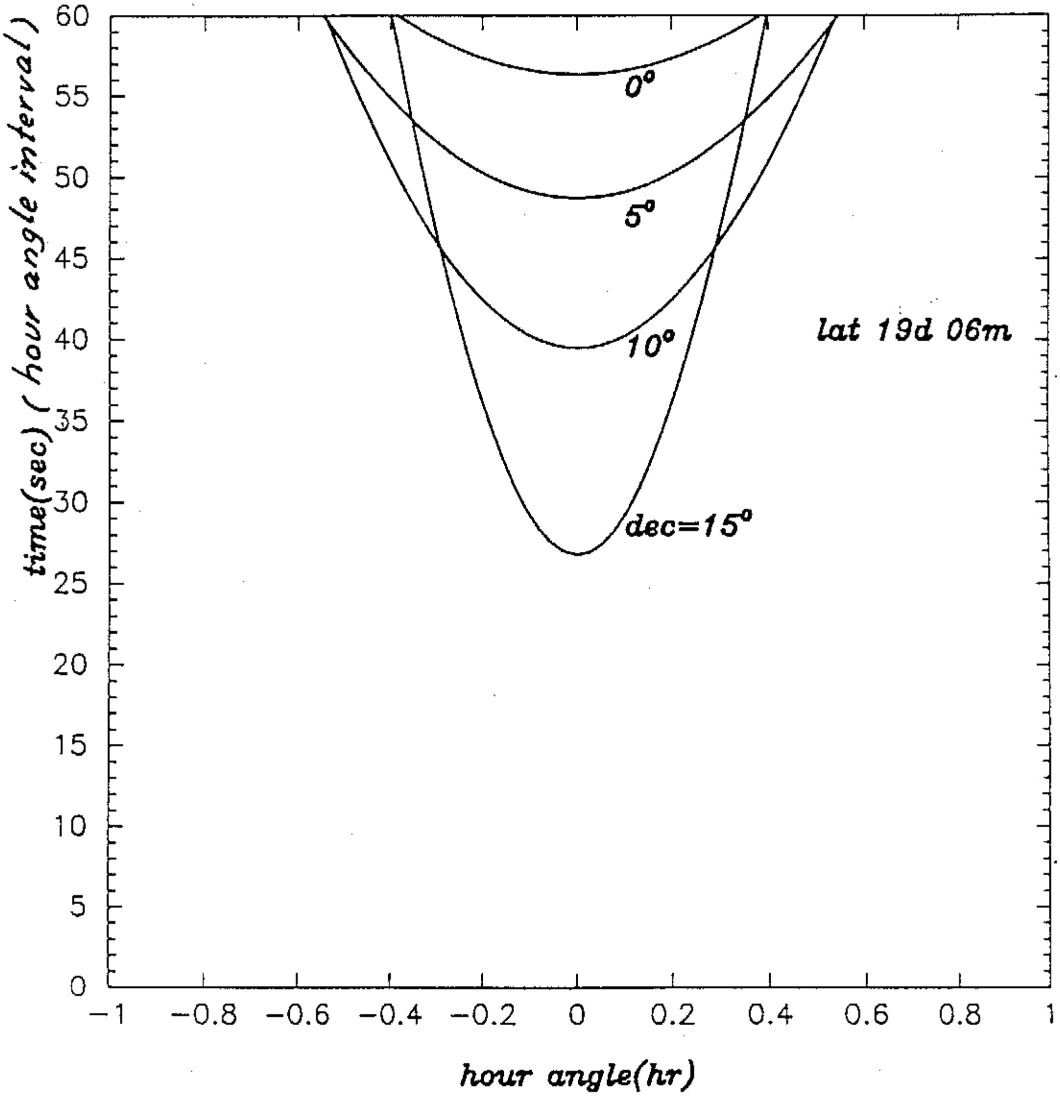

time(sec) (hour angle interval)
hour angle(hr)
0°
5°
10°
dec=15°
lat 19d 06m
60
55
50
45
40
35
30
25
20
15
10
5
0
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1

Fig. 9

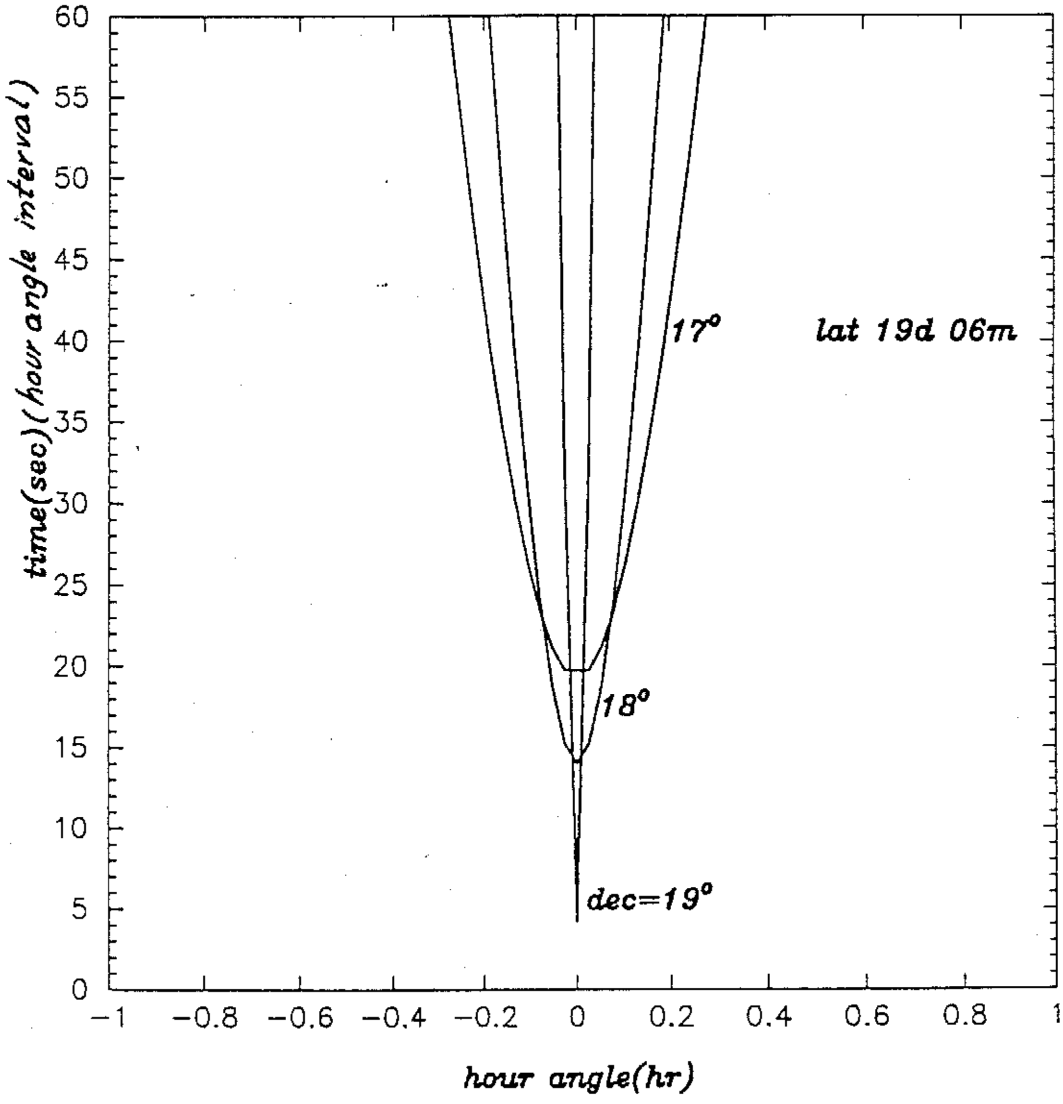

60
55
50
45
40
35
30
25
20
15
10
5
0
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
time(sec)(hour angle interval)
hour angle(hr)
17°
lat 19d 06m
18°
dec=19°

Fig. 10

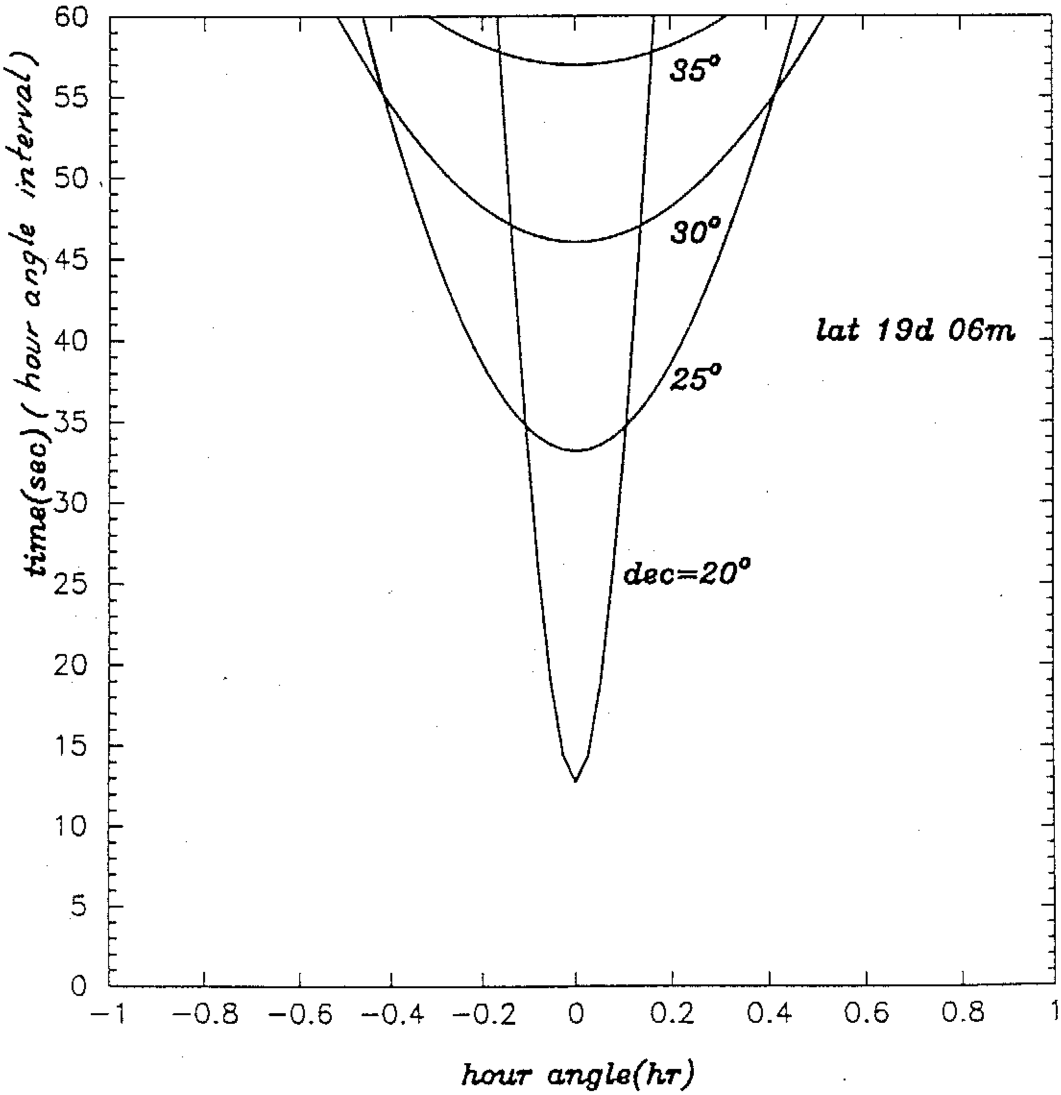

35°
30°
25°
dec=20°
lat 19d 06m
time(sec) (hour angle interval)
hour angle(hr)
60
55
50
45
40
35
30
25
20
15
10
5
0
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1

Fig. 11

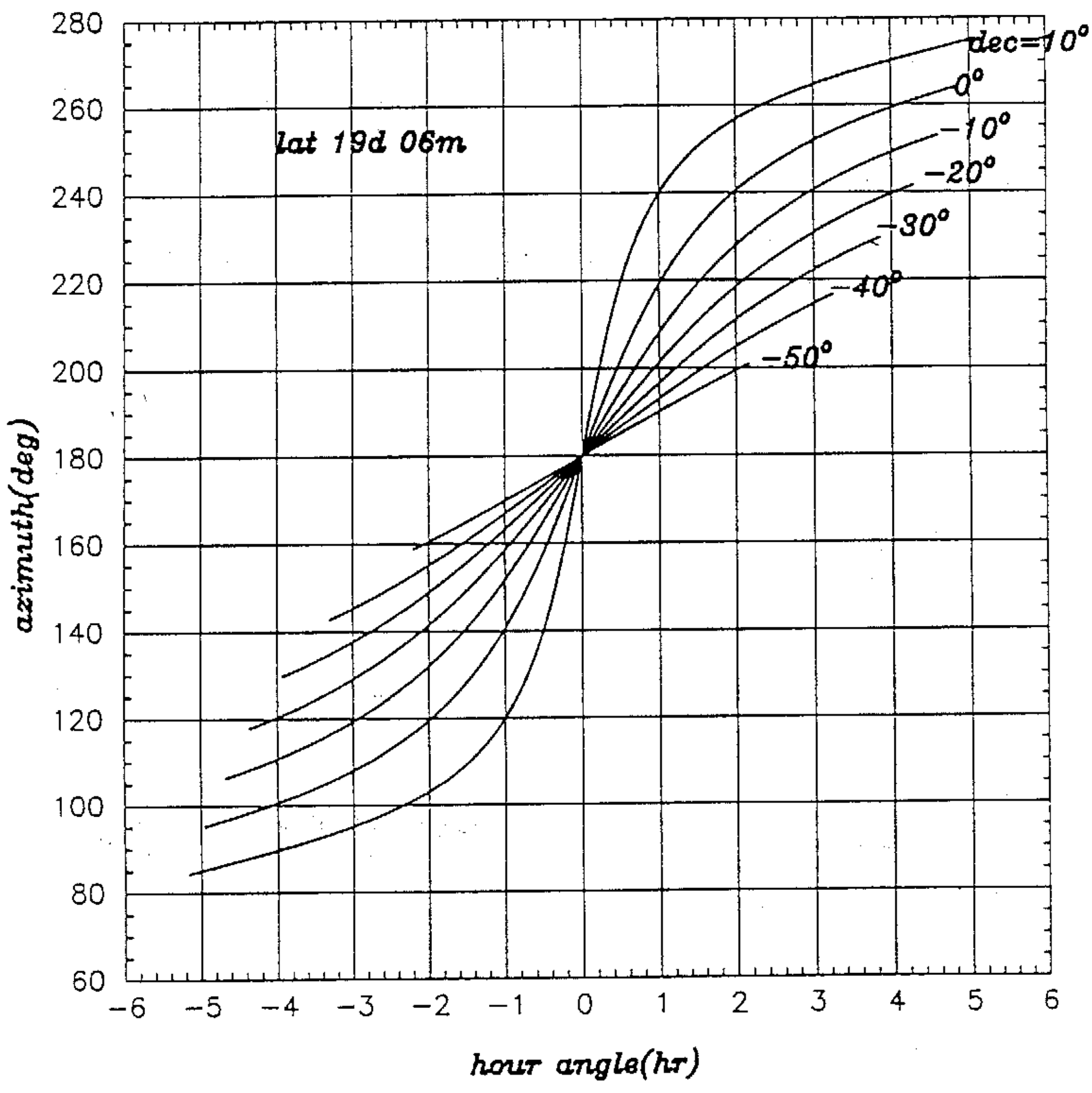

280
260
240
220
200
180
160
140
120
100
80
60
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
dec=10°
0°
−10°
−20°
−30°
−40°
−50°
lat 19d 06m
azimuth(deg)
hour angle(hr)

Fig. 12

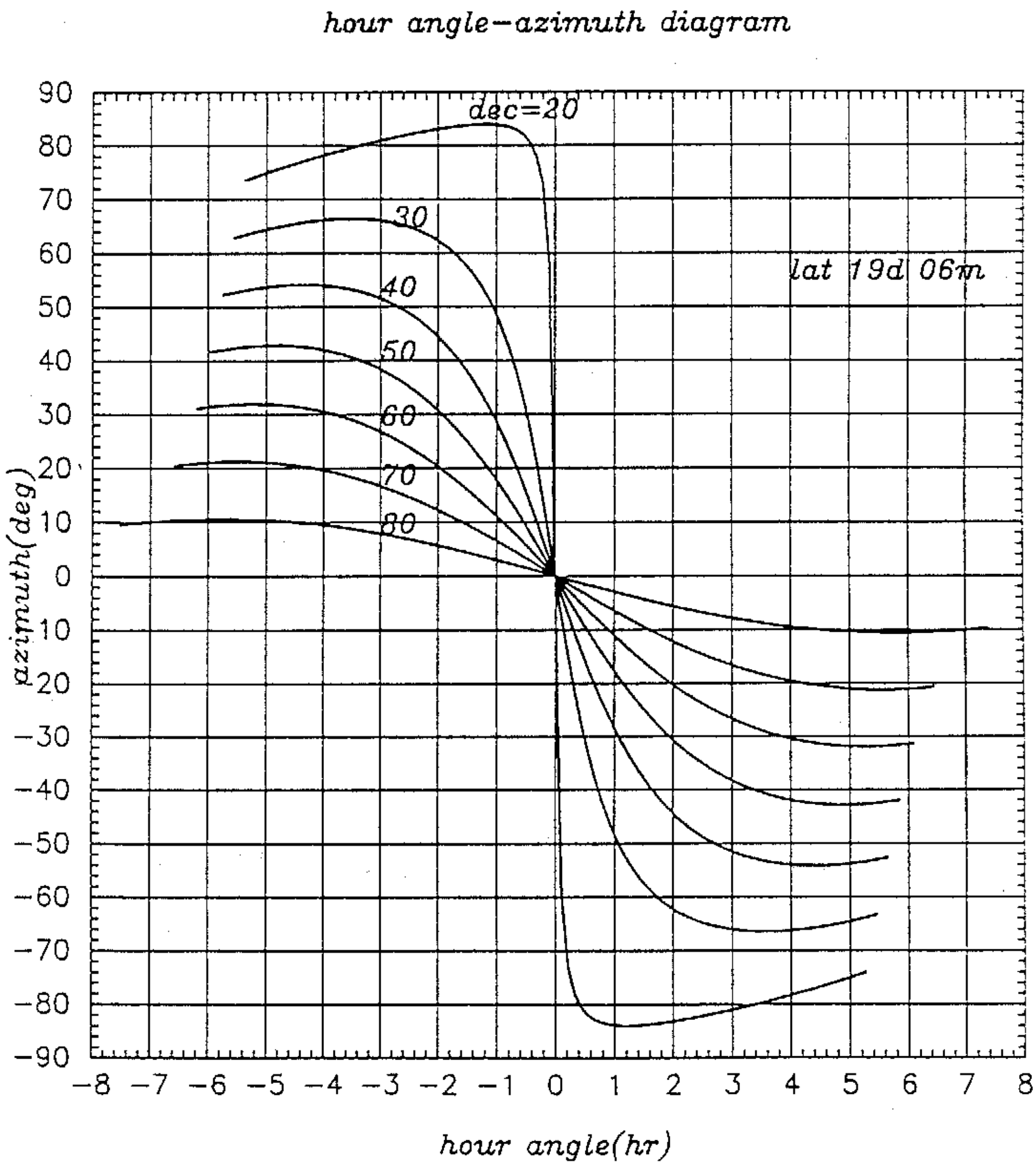

hour angle-azimuth diagram
dec=20
30
40
50
60
70
80
lat 19d 06m
azimuth(deg)
90
80
70
60
50
40
30
20
10
0
−10
−20
−30
−40
−50
−60
−70
−80
−90
−8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8
hour angle(hr)

Fig. 13

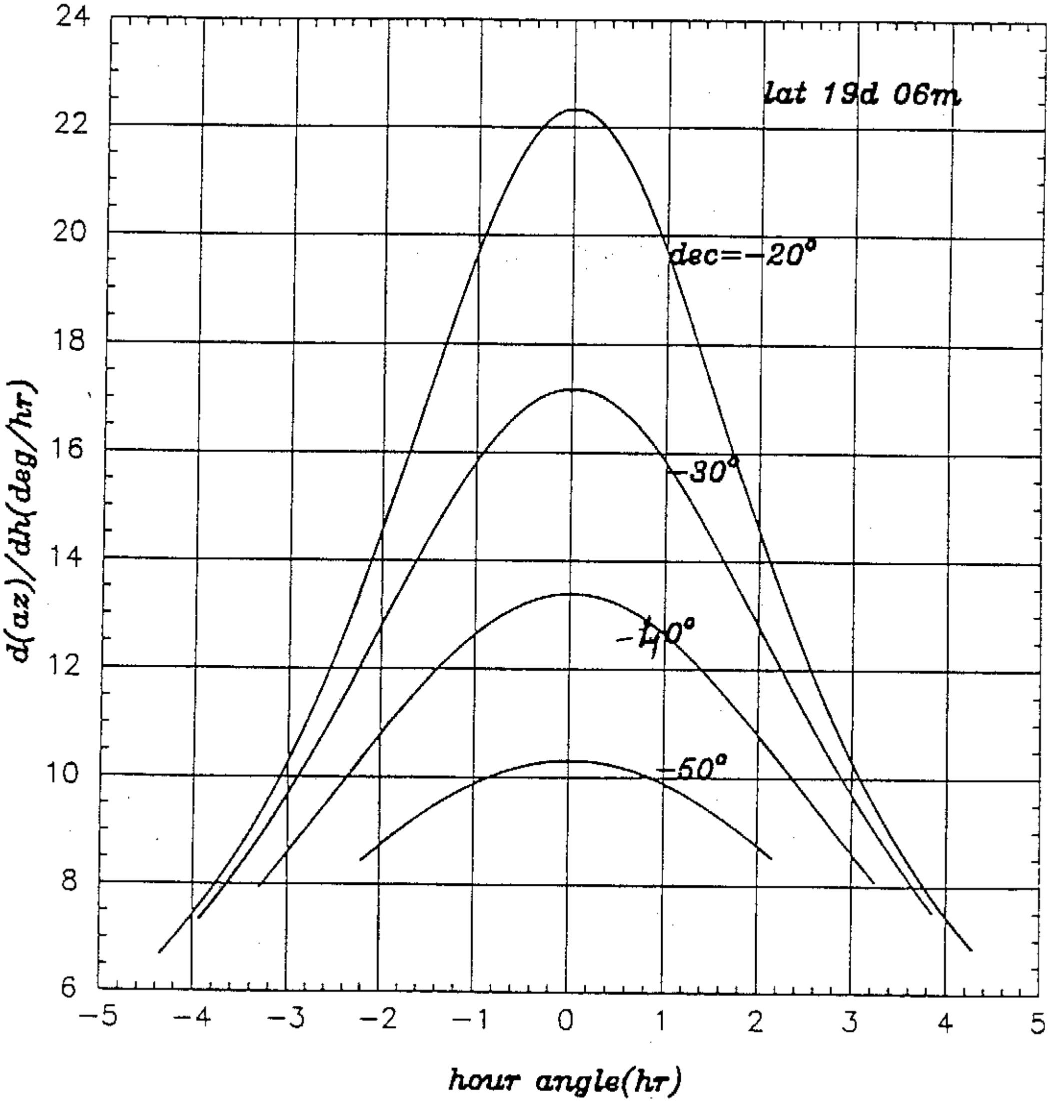
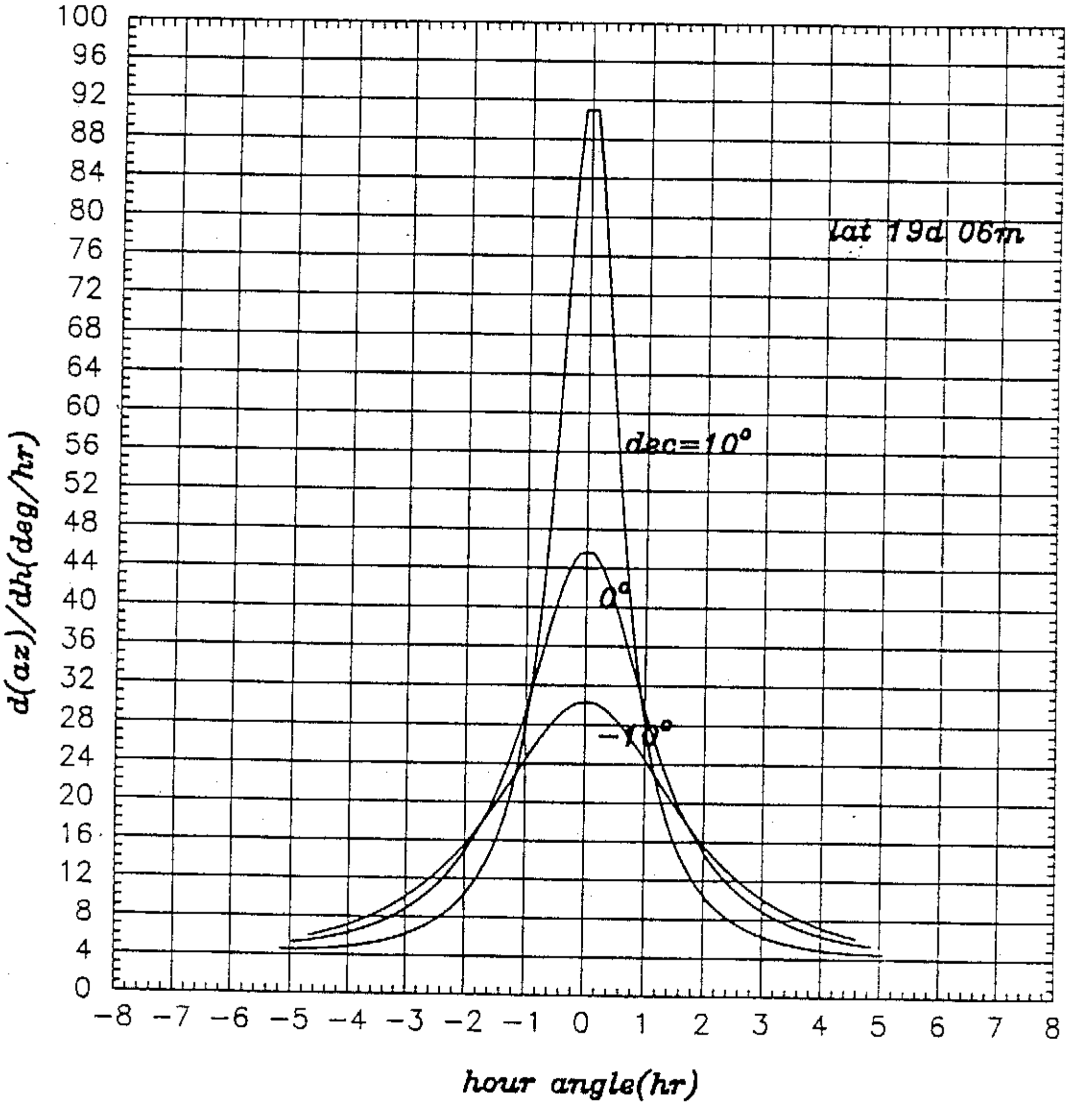

lat 19d 06m
dec=−20°
−30°
−40°
−50°
d(az)/dh(deg/hr)
hour angle(hr)
24
22
20
18
16
14
12
10
8
6
−5
−4
−3
−2
−1
0
1
2
3
4
5

Fig. 14.



100
96
92
88
84
80
76
72
68
64
60
56
52
48
44
40
36
32
28
24
20
16
12
8
4
0
d(az)/dh(deg/hr)
lat 19d 06m
dec=10°
0°
-10°
−8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8
hour angle(hr)

Fig. 15

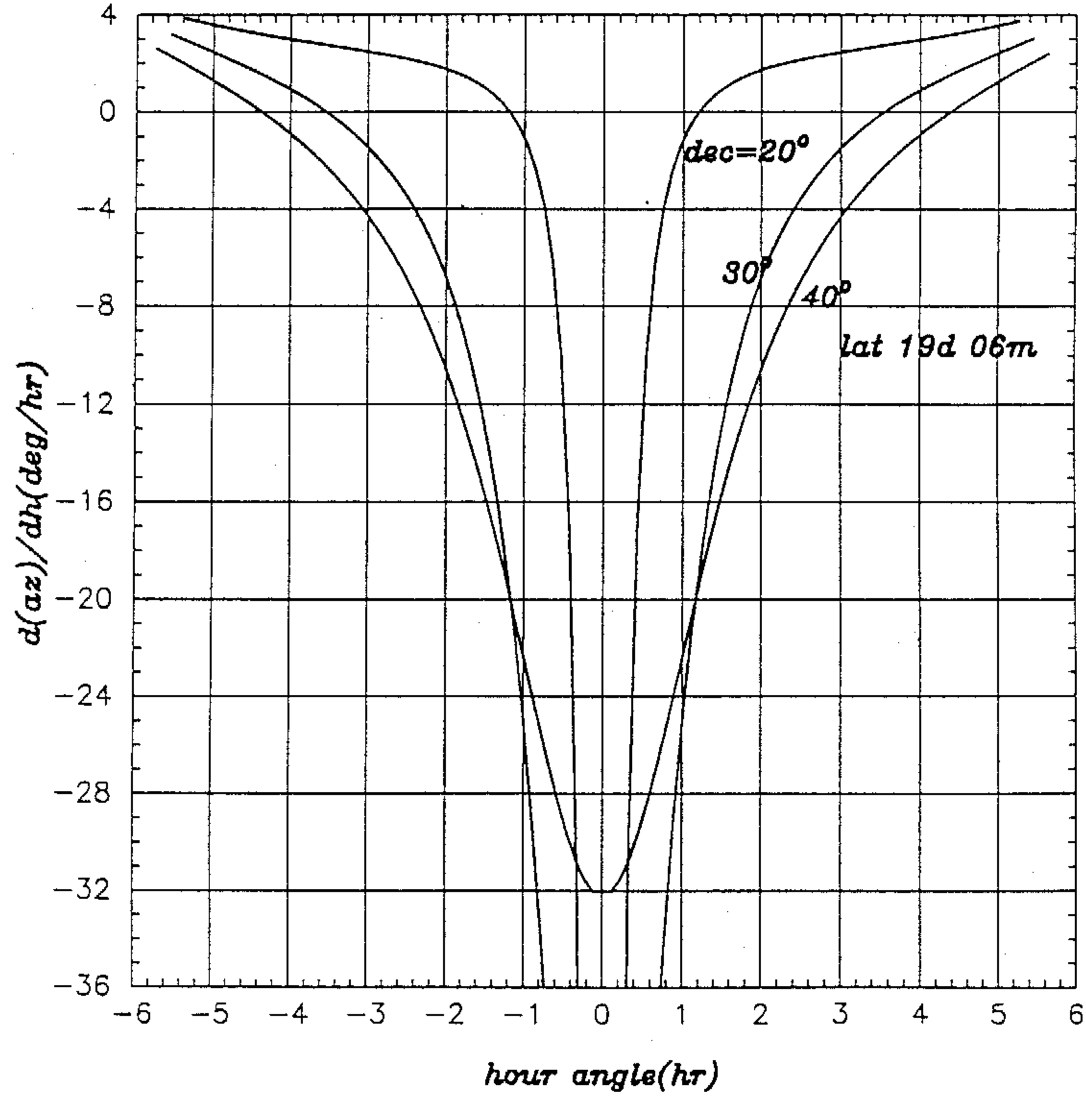
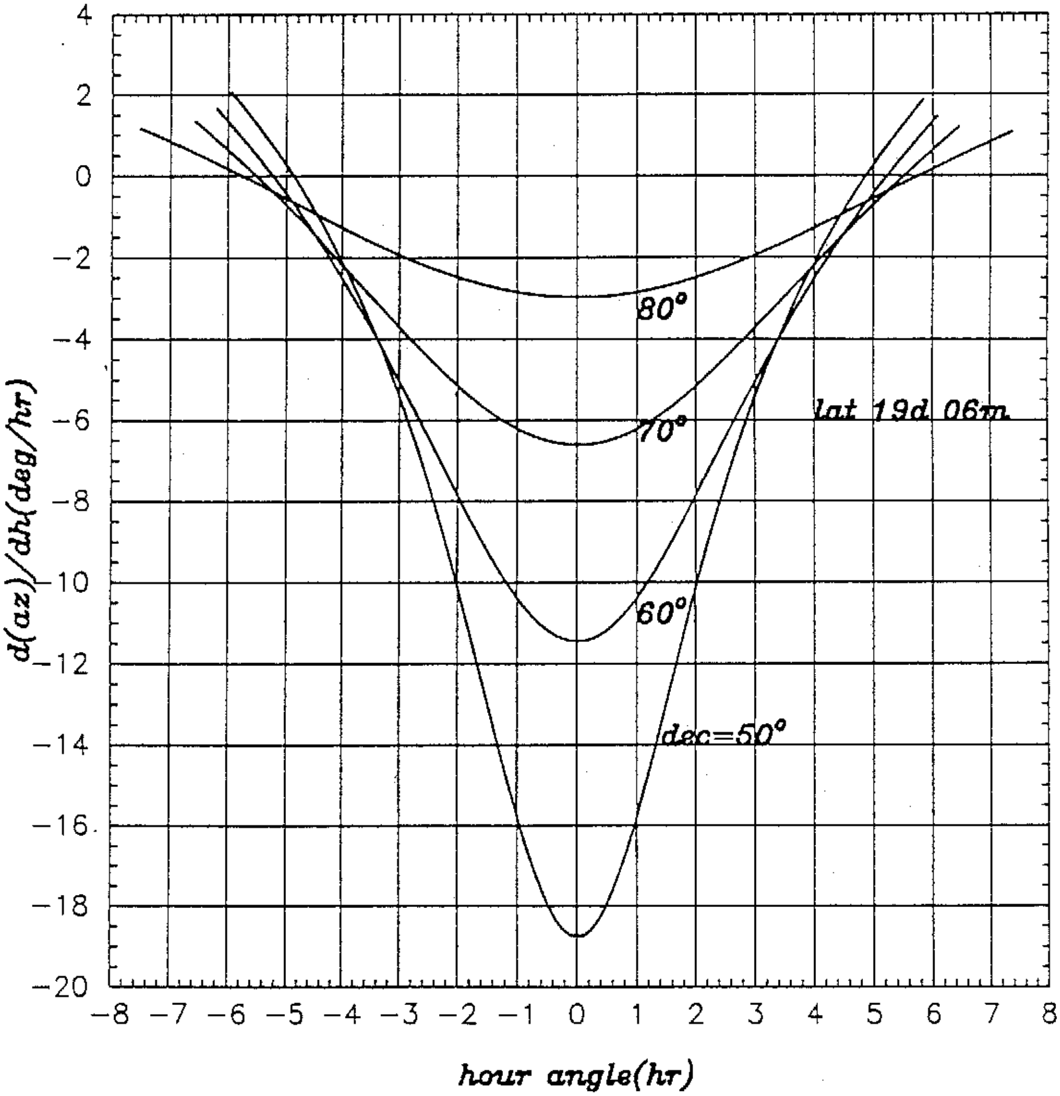
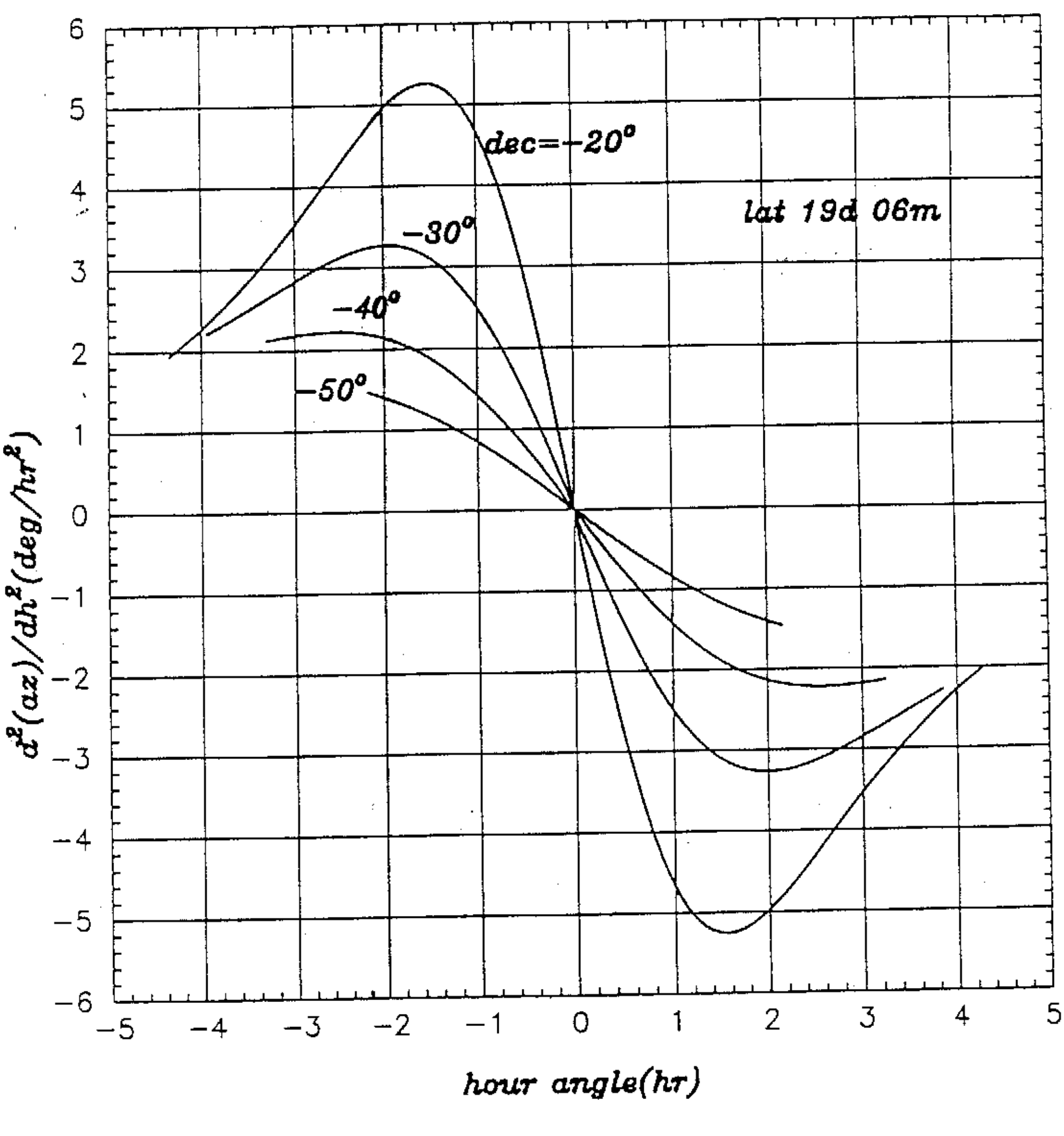

dec=20°
30°
40°
lat 19d 06m
d(az)/dh(deg/hr)
hour angle(hr)
4
0
−4
−8
−12
−16
−20
−24
−28
−32
−36
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5
6

Fig. 16



4
2
0
−2
−4
−6
−8
−10
−12
−14
−16
−18
−20
d(az)/dh(deg/hr)
80°
70°
60°
dec=50°
lat 19d 06m
−8
−7
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
7
8
hour angle(hr)

Fig. 17



dec=−20°
−30°
−40°
−50°
lat 19d 06m
$d^2(az)/dh^2(deg/hr^2)$
hour angle(hr)
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5

Fig. 18

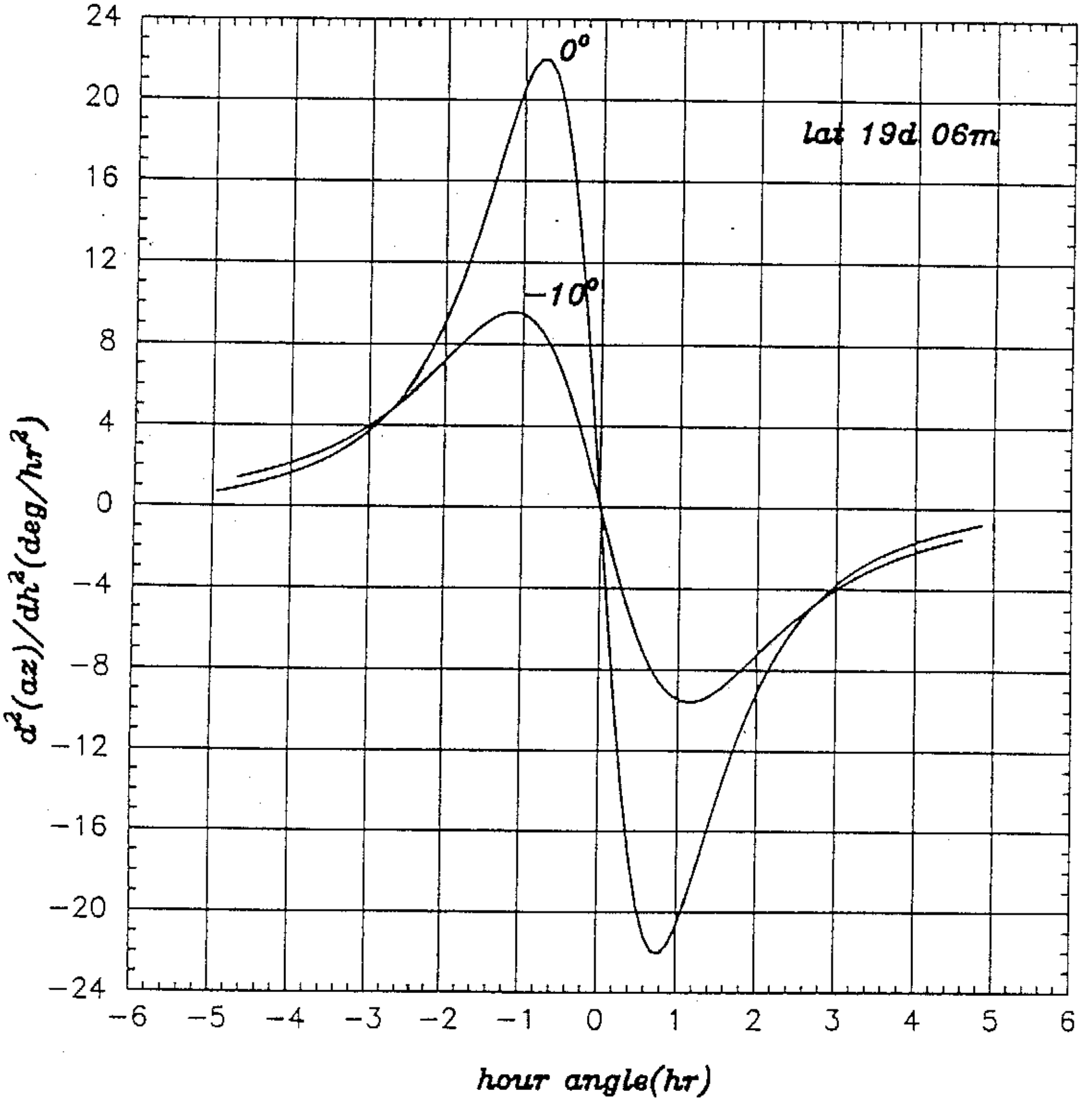
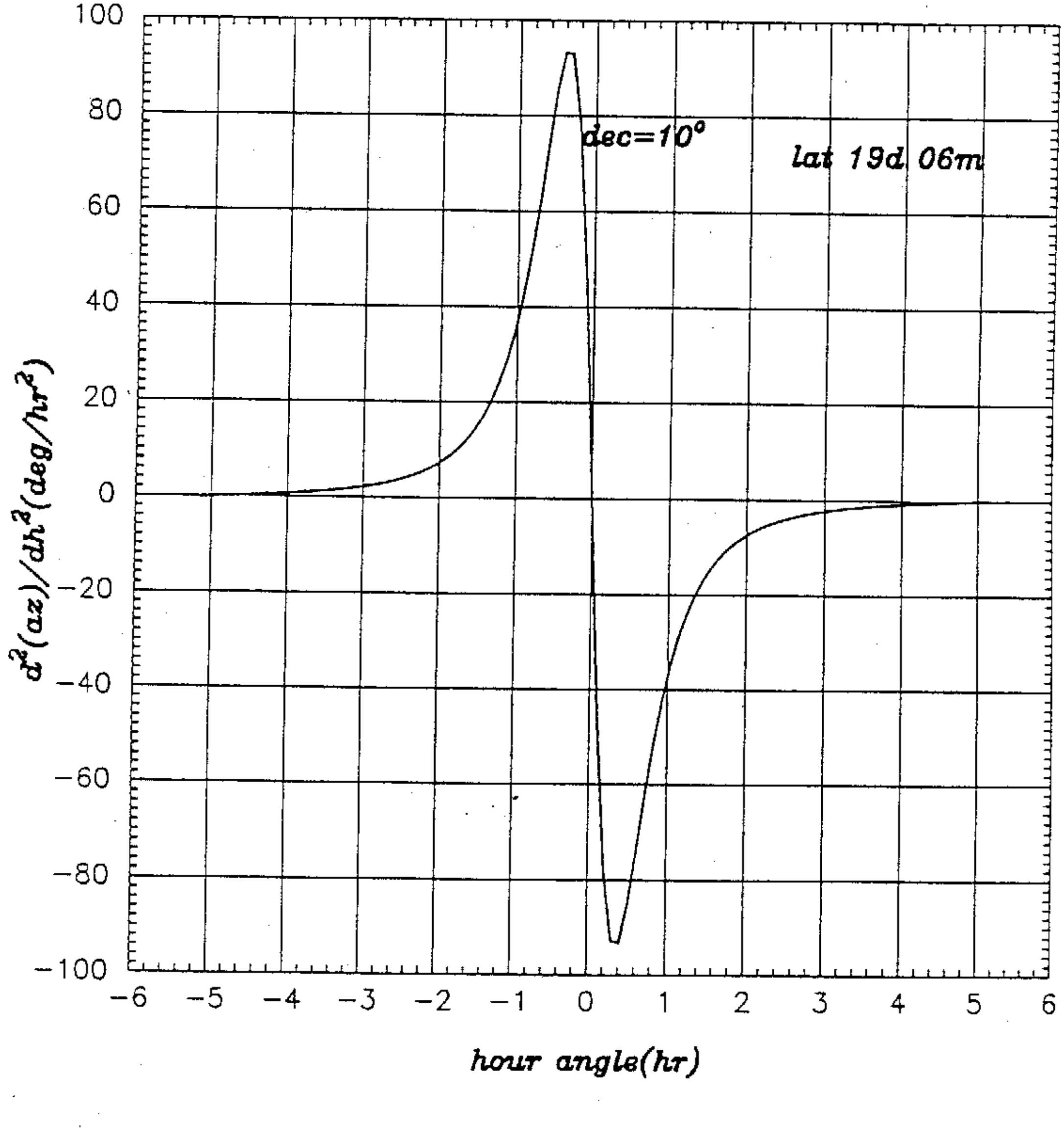
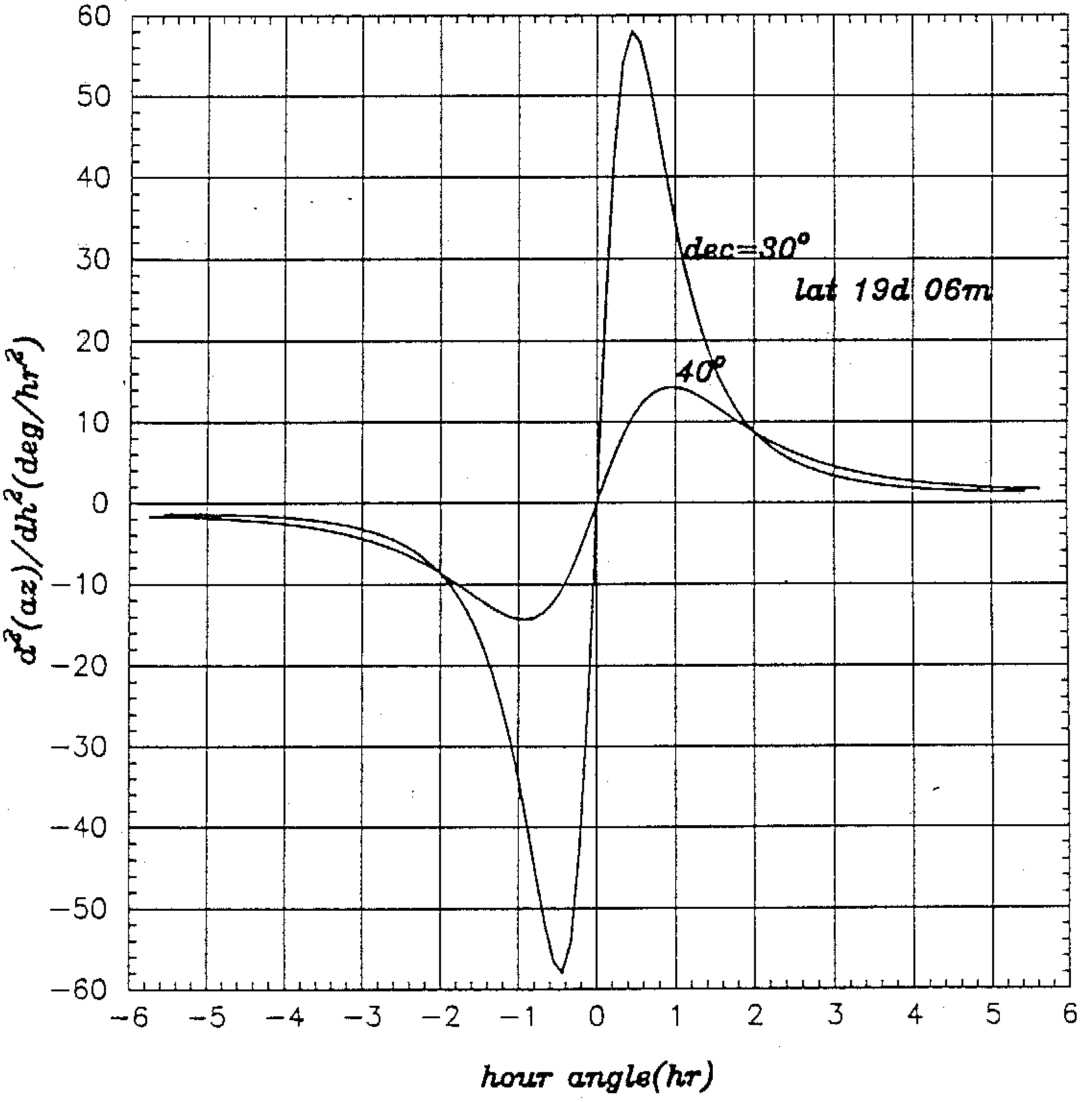

0°
-10°
lat 19d 06m
24
20
16
12
8
4
0
−4
−8
−12
−16
−20
−24
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
$d^2(az)/dh^2(deg/hr^2)$
hour angle(hr)

Fig. 19



100
80
60
40
20
0
−20
−40
−60
−80
−100
−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6
dec=10°
lat 19d 06m
d²(az)/dh²(deg/hr²)
hour angle(hr)

Fig. 20



60
50
40
30
20
10
0
−10
−20
−30
−40
−50
−60
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
dec=30°
lat 19d 06m
40°
d²(az)/dh²(deg/hr²)
hour angle(hr)

Fig. 21

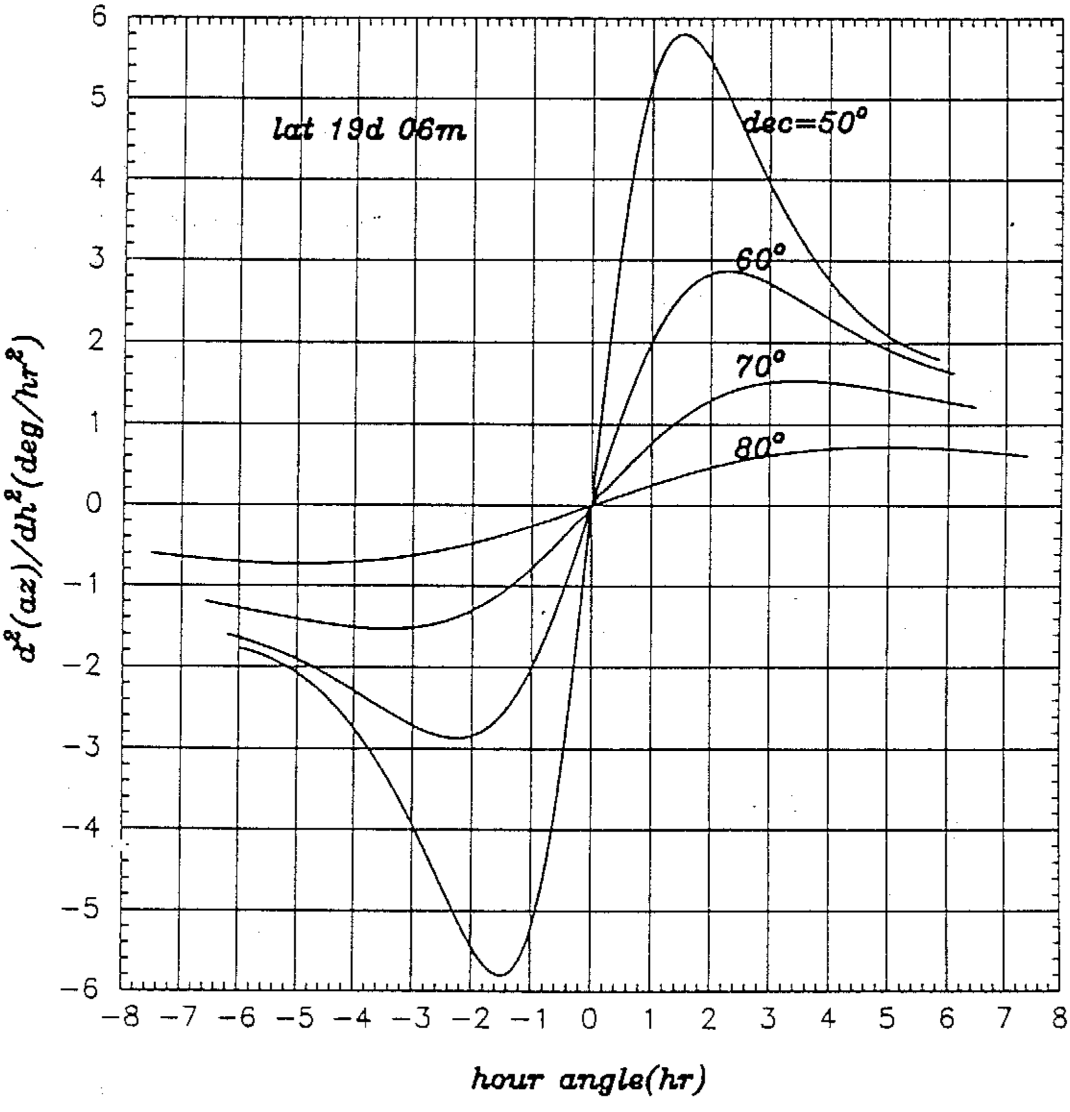
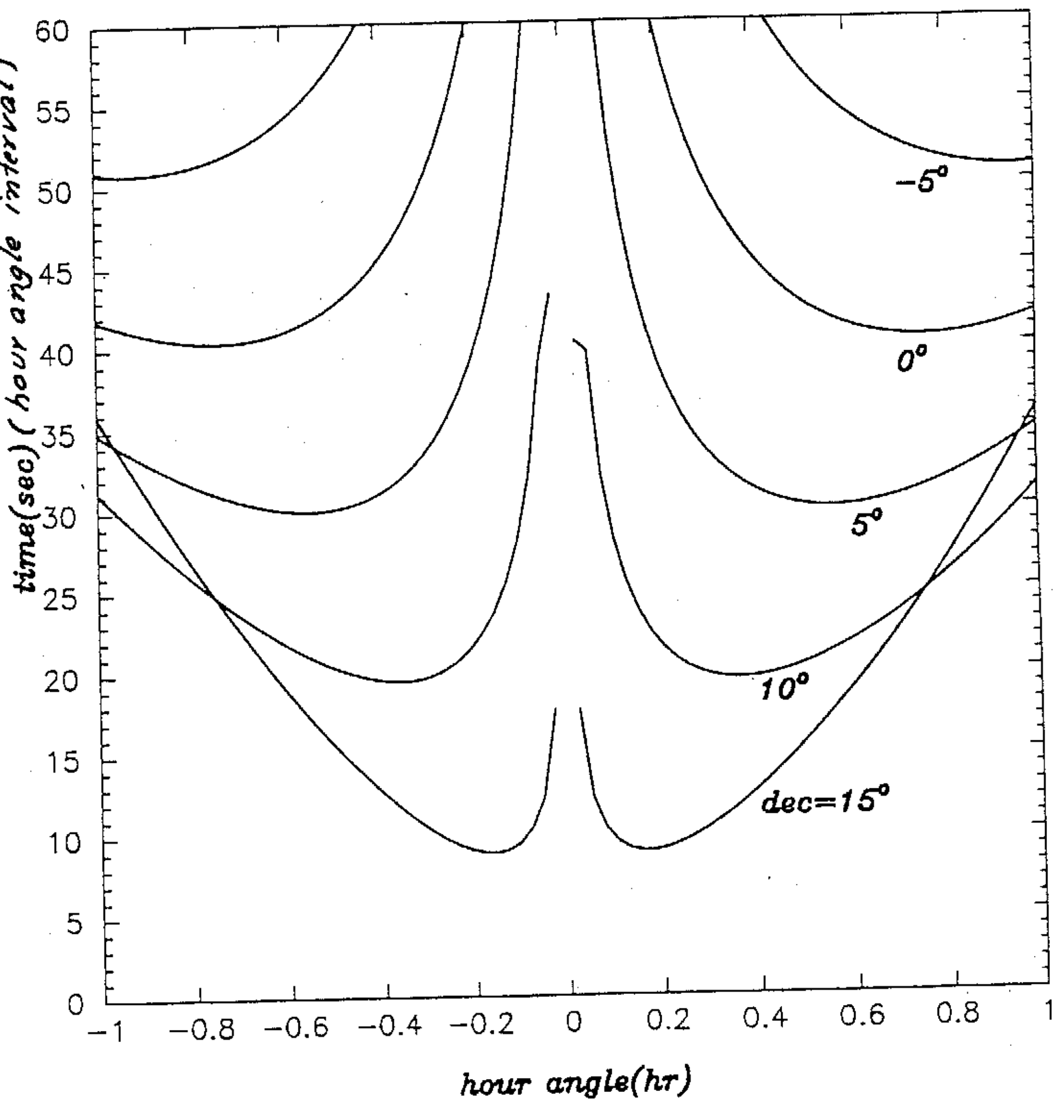
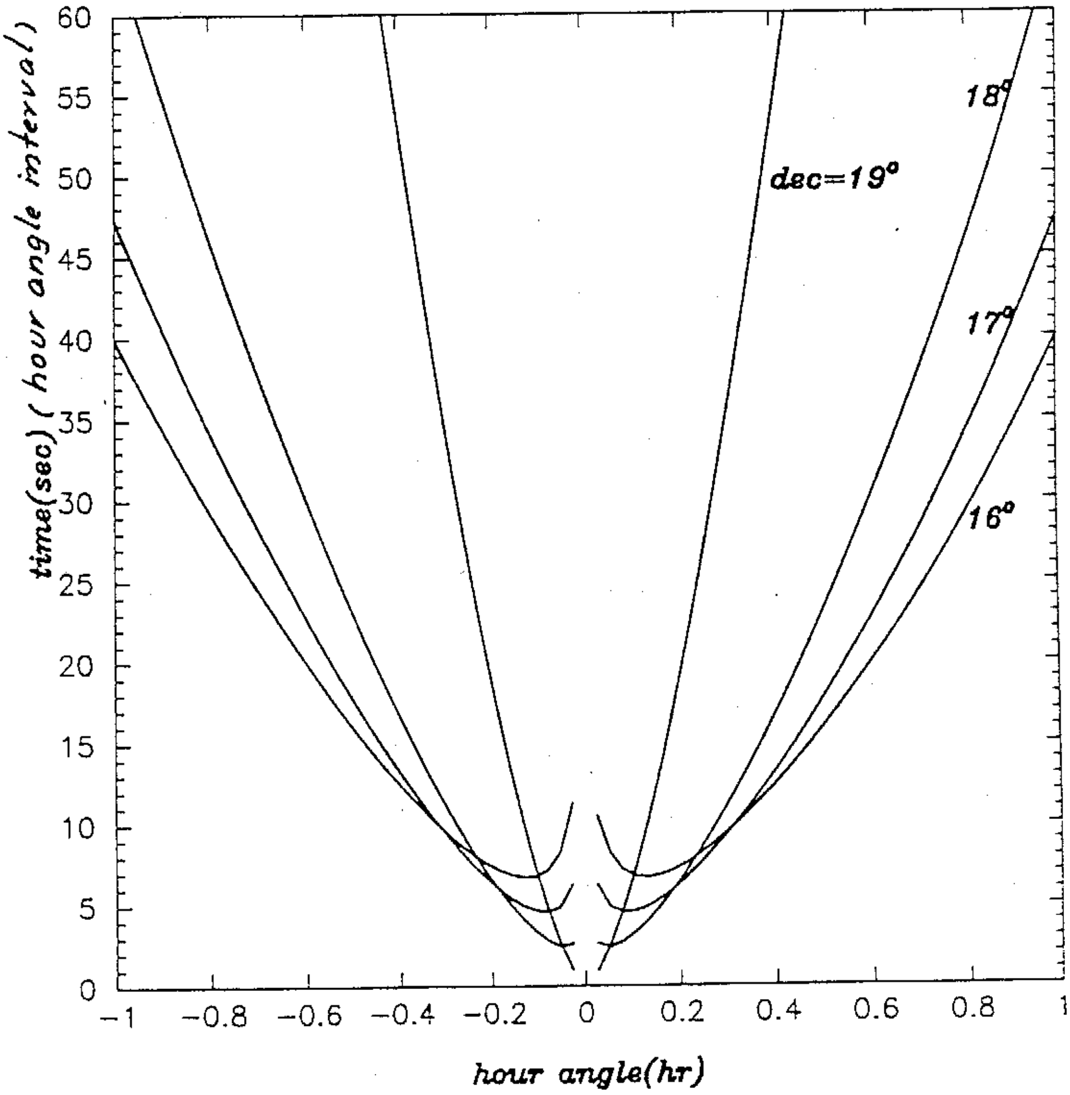

lat 19d 06m
dec=50°
60°
70°
80°
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
−8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8
$d^2(az)/dh^2(deg/hr^2)$
hour angle(hr)

Fig. 22



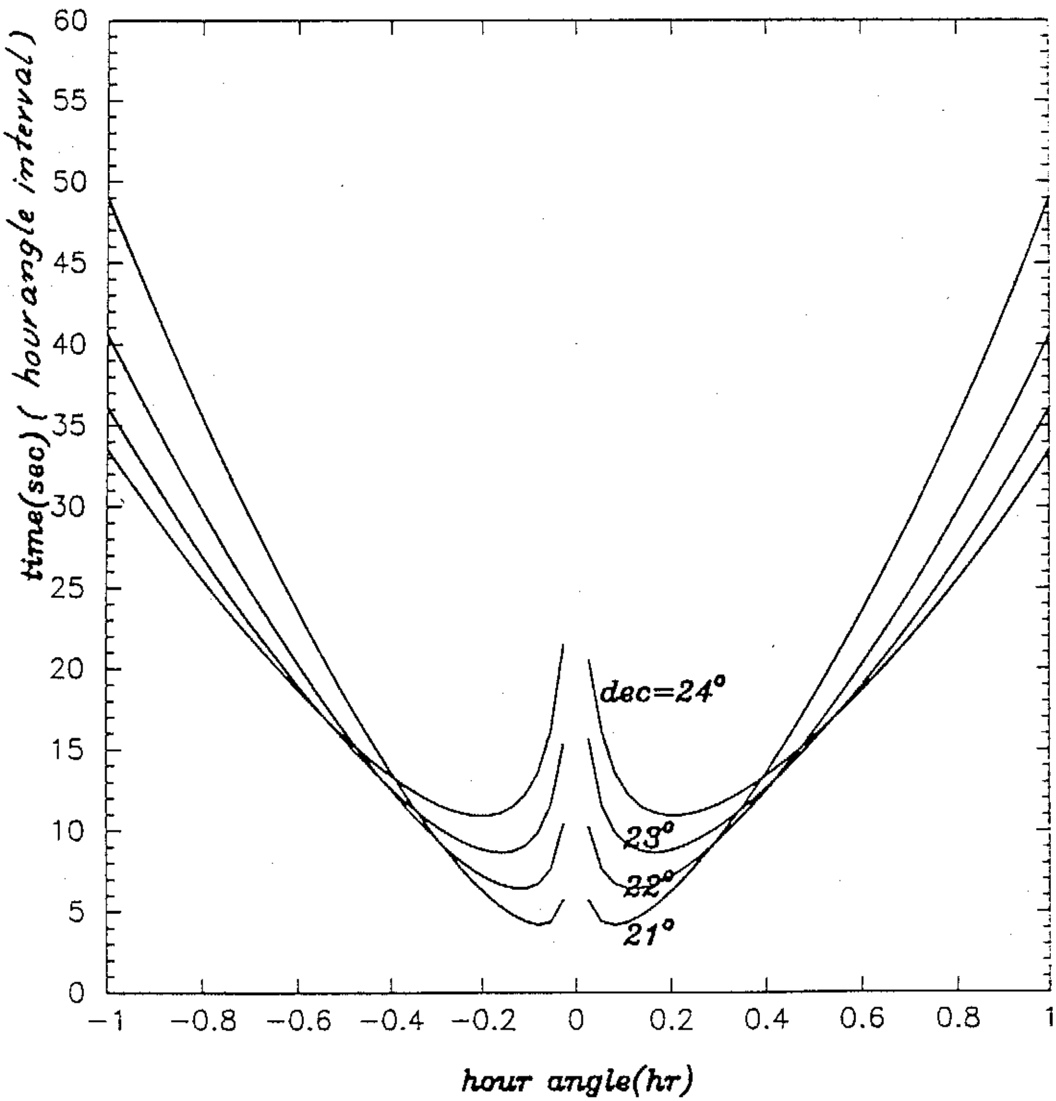
time(sec) (hour angle interval)
60
55
50
45
40
35
30
25
20
15
10
5
0
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
hour angle(hr)
−5°
0°
5°
10°
dec=15°

Fig. 23



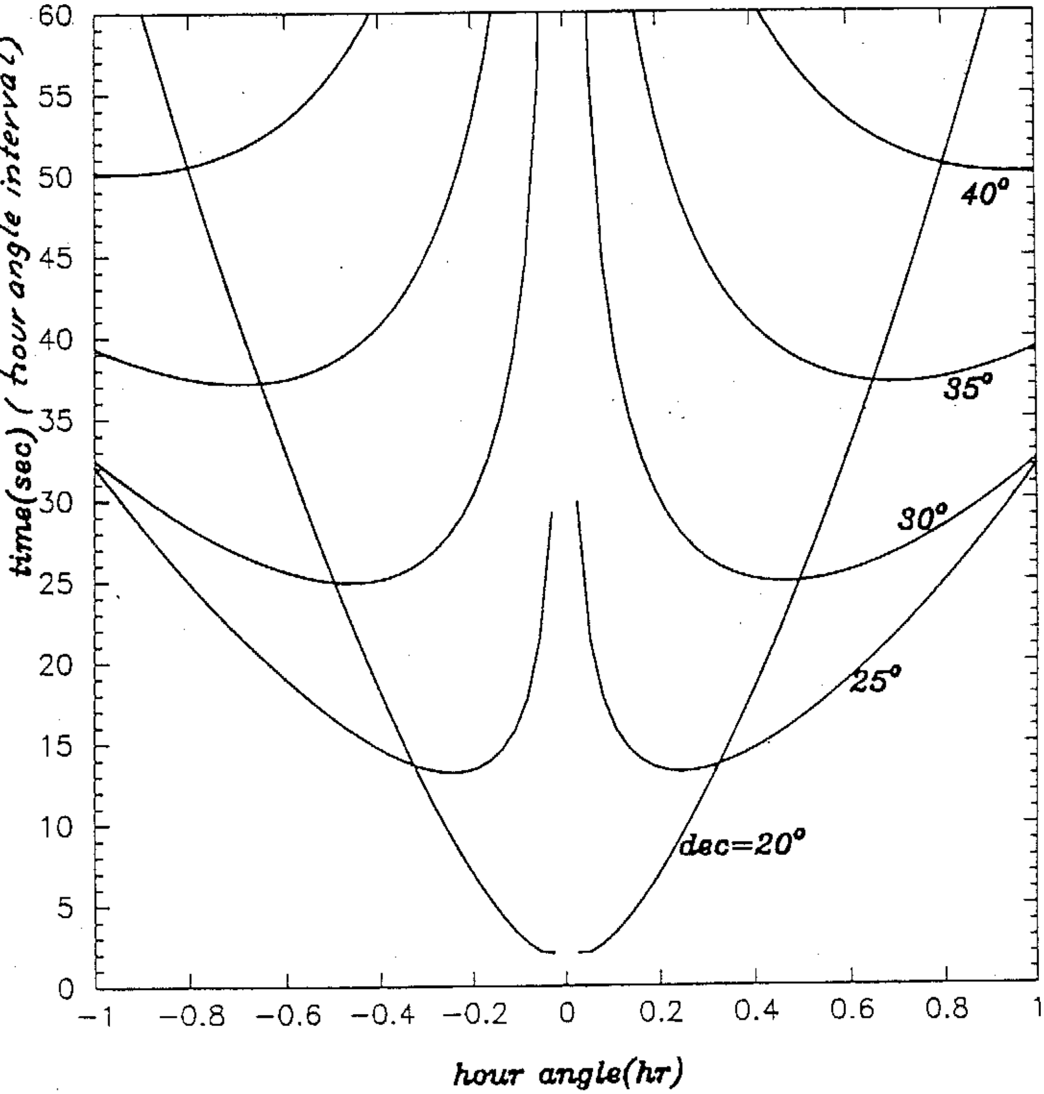
60
55
50
45
40
35
30
25
20
15
10
5
0
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
dec=19°
18°
17°
16°
time(sec) (hour angle interval)
hour angle(hr)

Fig. 24



60
55
50
45
40
35
30
25
20
15
10
5
0
time(sec) (hour angle interval)
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
hour angle(hr)
dec=24°
23°
22°
21°

Fig. 25



60
55
50
45
40
35
30
25
20
15
10
5
0
time(sec) (hour angle interval)
40°
35°
30°
25°
dec=20°
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
hour angle(hr)

Fig. 26