GMRT:PSR:pol_cal:02

GMRT Polarisation Calibration and Pulsars

an internal report prepared May 25, 1991 by Y. Gupta and S. Upreti.

1. Aims of the report :

i. To discuss instrumental effects in polarisation studies (those relevant to GMRT in general and pulsar observations in particular).

ii. To explore the possibility of using pulsars as calibration sources for GMRT.

This report concentrates on the 327 and 610 MHz bands of GMRT.

2. Intorduction :

Pulsars are some of the most highly polarised radio sources known to us, with some pulsars having virtually 100% polarisation, mostly linear. Accurate polarimetry of pulsars is very important. Also, several extra-galactic sources have very small levels of polarisation and here again it is critical to be able to measure this as accurately as possible. In order to achieve these polarimetry goals, it is essential to measure instrumental polarisation effects accurately and be able to correct for them. For a variety of reasons, this is not a simple problem at metre wavelengths, especially for a multi-element system like GMRT.

Several instrumental effects can contribute to errors in polarimetry for a radio source at the centre of field of view of an antenna. These include non-orthogonality of the feed elements of a single dish (e.g. crossed dipoles for GMRT at 327 MHz), irregularities of reflecting surface and those due to feed support structure of a dish, non-alignment of the feeds from one dish to another, different path lengths and gains for the two polarisation channels for a dish, cross coupling of the voltages from the two channels via a switch or other electronic circuitry etc. For the analysis that follows we will assume the net result is that the two orthogonal polarisation channels being received by the system are corrupted and so are the estimates of the Stokes parameters made from these components. In the case of GMRT, the orthogonal channels received are the two circular polarisations. Linear polarisations from the feed are converted into circular using a polariser.

3. Calibration of instrumental effects (single dish case):

The net result of the instrumental effects for a single dish can be represented as a leakage (or cross coupling) of the response from one polarisation channel to the other (Conway and Kronberg, Stinebring et.al.). The measured responses for the left and right circularly polarised components, E_L' and E_R', are then given in terms of the true responses, E_L and E_R, by

$$E_L' = G_L \left[\frac{E_L}{(1+\varepsilon_1^2)^{1/2}} + \frac{E_R\, \varepsilon_1 e^{i\psi_1}}{(1+\varepsilon_1^2)^{1/2}} \right] \tag{3.1a}$$

$$E_R' = G_R \left[\frac{E_R}{(1+\varepsilon_2^2)^{1/2}} + \frac{E_L\, \varepsilon_2 e^{-i\psi_2}}{(1+\varepsilon_2^2)^{1/2}} \right] \tag{3.1b}$$

where ε_1, ε_2, ψ_1, ψ_2 are real constants specifying the cross coupling and G_L, G_r are the complex gains for the left and right channels. The cross coupling results in the circular polarisations being corrupted to elliptical polarisations. For typical antenna systems, the values for ε_1, ε_2 are of the order of 5-10% (requirements for minimisation?). The Stokes parameters (I', Q', U', V') calculated from these measured responses can then be related to their true values (I,Q,U,V) by the antenna polarisation matrix which depends solely on the cross coupling coeffecients and the gains. The exact description of this matrix in terms of these coeffecients is attached as a separate sheet (here it is assumed that the gains are known and have been corrected for). The antenna polarisation matrix is inverted to yield the true Stokes parameters in terms of the measured values. This is called the correction matrix (Stinebring et.al.). The accuracy of the polarimetric observations is thus determined not so much by the magnitude of the cross coupling parameters (or their variations, provided calibration is done often enough), but by the accuracy to which these parameters can be determined.

The channel gains and the cross coupling parameters can be determined by appropriate calibration observations using sources with known polarisation properties. The quantities monitored are the measured self and cross products of E_L' and E_R. These are related to the true Stokes parameters of the source and the cross coupling coeffecients as follows (Conway and Kronberg):

$$|E_R'|^2 = |G_R|^2[\ (I-V)/2 + mI\ \varepsilon_2\cos(\theta-\psi_2)\ ] \tag{3.2a}$$

$$|E_L'|^2 = |G_L|^2[\ (I+V)/2 + mI\ \varepsilon_1\cos(\theta-\psi_1)\ ] \tag{3.2b}$$

$$E_L' E_R'^{*} = \frac{1}{2}\, G_L\, G_R^{*}[\ mI\, e^{i\theta} + I\, \sigma\, e^{i\phi_1} - V\, \delta\, e^{i\phi_2}\ ] \tag{3.2c}$$

Here σ, δ, ϕ_1 and ϕ_2 are given by

$$\varepsilon_1\, e^{i\psi_1} + \varepsilon_2\, e^{i\psi_2} = \sigma\, e^{i\phi_1} \tag{3.3a}$$

$$\varepsilon_1\, e^{i\psi_1} - \varepsilon_2\, e^{i\psi_2} = \delta\, e^{i\phi_2} \tag{3.3b}$$

and we have made use of the relations :

$$Q + iV = mI\, e^{i\theta} = 2\, E_L^{*}\, E_R \tag{3.4a}$$

$$I = |E_R|^2 + |E_L|^2 \tag{3.4b}$$

$$V = |E_L|^2 - |E_R|^2 \tag{3.4c}$$

where m is the fraction of linearly polarised power from the source with θ ($= 2\chi - 2\eta$) being the effective position angle (χ is the true position angle and η is the parallactic angle). In eqns.(3.2) we have neglected all second order products of instrumental polarisation parameters.

One way of doing the calibration is to first observe a strong unpolarised source (Q=U=V=0) and measure the parallel hand correlations. Eqns.(3.2a & b) then yield estimates for the amplitude of the gains of the two channels. Next observe a source with known linear polarisation (m and χ) and $V \approx 0$, for a range of parallactic angle η. Then the sinusoidal fluctuations of $|E_L'|$ and $|E_R'|$ directly yield estimates for the cross coupling parameters. The relative phase difference between the two channels is then got from the cross-correlation product (eqn.(3.2c)).

Note that because of the nature of eqns.(3.2), any error in θ will show up as a change in the estimated phase difference between the channels and a corresponding change in the phase of the estimated cross coupling parameters. This could cause problems at 327 MHz (and lower frequencies) where variations in the ionospheric Faraday rotation can cause large uncertainties in the estimate of θ. For example, at 327 MHz the Faraday rotation for nominal values of ionospheric parameters ($N = 10^{12} e\ cm^{-3}$; $B = 1G$; $d = 300$ km) is ~ 6.5 radians. If we take a 10% error in this estimate, the phases ψ_1 and ψ_2 and the phase difference between the two channels will be off by ~ 30 degrees.

Note, however, that if we use these erroneous calibration values for polarimetry of an unknown source which "sees" the same ionospheric conditions, then there should be no effect of the errors on the results. In practice, an "unpolarised" source can have some degree of polarisation, in which case there will be an error in the estimate of the gains. For example, if the source has 10% linear polarisation and ε_1, ε_2 are ~ 5%, then the worst case error in the gains due to this will be ~ 1%. The effect of this error in the gain estimate on the estimate of ε_1, ε_2 will be insignificant. The errors in these will be dominated by the errors in the estimate for m - the fractional linear polarisation.

4. Calibration of instrumental effects (multiple dish case):

For multiple dish systems, the instrumental effects need to be treated differently, depending on how the system is configured. For GMRT, the primary configuration is a synthesis instrument where polarisation channels from every two dish pair are correlated to yield the standard RR, LL, RL and LR products. In addition, for pulsar observations, GMRT will be used in a phased array configuration where the corresponding polarisation channels from each dish will be added up separately to produce E_{Ls} and E_{Rs} as the summed left and right circular polarisations. Here we will concentrate mainly on the problem of polarisation calibration in the phased array mode.

From eqn.(3.1), for a phased array, the measured E_{Ls} and E_{Rs} can be written as :

$$E'_{Ls} = \alpha_1 E_L + \beta_1 e^{i\gamma_1} E_R \quad (4.1a)$$

$$E'_{Rs} = \alpha_2 E_R + \beta_2 e^{-i\gamma_2} E_L \quad (4.1b)$$

where α_1 and β_1 are sums of the cross coupling parameters from the N dishes and are given by

$$\alpha_1 = \sum_{i=1}^{N} \frac{G_{L_i}}{(1+\varepsilon_{1_i}^2)^{1/2}} \quad (4.2a)$$

$$\beta_1 e^{i\gamma_1} = \sum_{i=1}^{N} \frac{G_{L_i}\, \varepsilon_{1_i} e^{i\psi_{1_i}}}{(1+\varepsilon_{1_i}^2)^{1/2}} \quad (4.2b)$$

and similarly for α_2 and β_2. Both sums are vector sums. Note that if we assume the channel gains from one dish to the next to be similar and if the cross coupling factors for the dishes have similar amplitude but random phases, it is possible to get a reduced level of cross coupling in the summed output (under what conditions can we ensure such an effect ?). This can provide a significant inprovement for polarisation studies because, if the parameters can still be determined to the same fractional accuracy, the effect of errors in their estimate will reduce.

From eqn.(4.1) one can reconstruct the antenna polarisation matrix in a manner similar for the single dish case. The matrix elements have somewhat more complicated expressions because, unlike the single dish case, we do not have $|\alpha_i|^2 + |\beta_i|^2 = 1$. Nevertheless, given the values of α_1, β_1, α_2, β_2, we can compute the polarisation matrix and invert it to obtain the correction matrix.

One possibility is to calibrate the phased array system (for polarisation observations) in a manner similar to the single dish system. Note that the estimate of the effective cross coupling parameters includes the gain terms for the individual dishes and these need not be determined individually in this case. However, for the system to work correctly in the phased array mode, these invidual complex gains must be known while observing the source and on line corrections made for these. The geometrical phase delays between the dishes can be easily calculated. To a large extent, the instrumental gains (amplitude as well as phase) can also be calibrated out. It is the varying ionospheric phase for each dish that is not known. Therefore, under conditions of non-isoplaneticity, one will have to rely on a separate calibration to provide these phases.

Alternatively, one can evaluate the sums in eqns.(4.2) and their counterparts by estimating the cross coupling parameters and the complex gains for each individual dish. These values can be obtained from the synthesis mode measurements of RR, LL, RL, LR for the various paired dish combinations (Conway and Kronberg, NRAO-VLA workshop proceedings). Just considering any two dishes (i and j), it can be shown that these four product terms can be expressed as :

$$E'_{Ri}\, E'^{*}_{Rj} = \frac{1}{2}\, G_{R_i} G^{*}_{R_j} \left[\, (I-V) + mI \left(\varepsilon_{2_i}\, e^{-i(\psi_{2_i}+\theta)} + \varepsilon_{2_j}\, e^{i(\psi_{2_j}+\theta)} \right) \right] \tag{4.3a}$$

$$E'_{Li}\, E'^{*}_{Lj} = \frac{1}{2}\, G_{L_i} G^{*}_{L_j} \left[\, (I+V) + mI \left(\varepsilon_{1_i}\, e^{i(\psi_{1_i}+\theta)} + \varepsilon_{1_j}\, e^{-i(\psi_{1_j}+\theta)} \right) \right] \tag{4.3b}$$

$$E'_{Li}\, E'^{*}_{Rj} = \frac{1}{2}\, G_{L_i} G^{*}_{R_j} \left[\, mI\, e^{-i\theta} + I \left(\varepsilon_{1_i}\, e^{i\psi_{1_i}} + \varepsilon_{2_j}\, e^{i\psi_{2_j}} \right) - V \left(\varepsilon_{1_i}\, e^{i\psi_{1_i}} - \varepsilon_{2_j}\, e^{i\psi_{2_j}} \right) \right] \tag{4.3c}$$

$$E'_{Ri}\, E'^{*}_{Lj} = \frac{1}{2}\, G_{R_i} G^{*}_{L_j} \left[\, mI\, e^{i\theta} + I \left(\varepsilon_{2_i}\, e^{-i\psi_{2_i}} + \varepsilon_{1_j}\, e^{-i\psi_{1_j}} \right) + V \left(\varepsilon_{2_i}\, e^{-i\psi_{2_i}} - \varepsilon_{1_j}\, e^{-i\psi_{1_j}} \right) \right] \tag{4.3d}$$

From the above it can be seen that for a linearly polarised test source, the products RR and LL show sinusoidal type variations around a mean value (as a function of parallactic angle), whereas the cross products (RL and LR) show sinusoidal variations with a dc offset, analogous to the single dish case. Appropriate observations of point sources can be used to extract the unknown gains and cross coupling coeffecients from the above equations. Analogous to the single dish case, observations of an unpolarised calibration source can be used to estimate the gains of the two channels for each dish. The main advantage between the multiple dish and single dish case is that, for more than a total of 4 dishes, the set of eqns.(4.3a & b) are overdetermined and the method of least squares estimation can be used to reduce the variance of the errors in the gain estimates. For similar reasons, the effect of a small fraction of linearly polarised component in the "unpolarised" source on the gain estimates will be much reduced. The phase values calculated for any one channel of a dish are relative and are referenced to the phase for that channel of a reference dish, which is taken to have zero phase. Thus an unknown phase difference remains between the right and left channels. Unlike the single dish case, observations of the same unpolarised source can be used to estimate the cross coupling parameters, using the cross correlation terms, RL and LR. This is because, for each i,j combination, eqns.(4.3c & d) yield estimates for different combinations of the cross coupling parameters which can then be estimated using a least squares fit. These estimates should be more robust than the estimates using eqns.(4.3a & b) with a linearly polarised source, since the terms of interest will have, in general, larger values in eqns.(4.3c & d) - leading to a better "signal to noise" performance. Also, errors in ‘m’ for known polarised sources will not bother. The only remaining unknown quantity - the phase difference between the right and left channels - needs observation of a known polarisation angle source and the application of eqns.(4.3c & d) to obtain a solution.

The above procedure (or something similar to it) is used for calibration in typical synthesis instruments (like the VLA). The calibration sources used need to be unresolved by the largest baseline being used in the calibration. The computations and the resulting corrections to the observed visibility functions are done off line during the map making process. For pulsar observations, we would need to do at least the gain calibration part of the above calculations on line so that the results can be used to set the correct delays between the signals from the dishes. Special software will need to be written to achieve this.

One of the important issues is the time stability of the instrumental polarisation, since it decides how often polarisation calibrations need to be done during an observation. Part of the fluctuations is due to actual temporal variations of the parameters while part can be due to variation of the cross coupling with changing pointing direction of the antenna system. Studies at the VLA show 0.5–1.0 % stability of the cross coupling parameters over times (a few hours) and improved stability (0.2 %) over long times (one year).

Another important stability question is that of the relative phase difference between the two polarisation channels. For circular polarisation channels, changes in this phase difference correspond to an extra rotation of the effective position angle (a similar effect is produced by varying Faraday rotation in the ionosphere). Such effects can corrupt the calibration process. VLA results show ~10 degrees change in the phase difference over 8-12 hour periods. However, there are hardware related, sporadic, large phase jumps which degrade the long term stability of the phase difference. Such characteristics would have to be determined accurately for the GMRT to make the polarisation calibration meaningful. A scheme to monitor the phase stability of the electronics using noise generators as calibrators is being worked out (see internal report GMRT:PSR:pol_cal.03 for details). Effects of ionospheric Faraday rotation can be corrected to a fairly large extent at the higher frequencies. But at lower frequencies (<= 327 MHz), this could become a serious problem.

5. Linear vs. Circular polarisation :

For GMRT, the two orthogonal polarisation channels received at the RF front end from the dish are linear polarisations (E_x , E_y) from a crossed dipole feed (e.g. at 327 MHz). These are converted to left and right circular components (E_r , E_l) in the front end by using power combiners and the rest of the receiver system processes these. It has been suggested that it might be better to stay with linear components. This is because, when using circular components, the Stokes parameter V is estimated as the difference of two large numbers ($E_r^2 - E_l^2$), each of which is scaled down by the appopriate gain factor (see section 3 of this report; also NRAO-VLA workshop proceedings). This reduces the accuracy to which V can be determined. Using linear components, V is estimated from a correlation product of E_x and E_y and the numbers subtracted to obtain the result are an order of magnitude smaller and hence gain errors are not so important. However, this simply means that the gain needs to be known to a greater accuracy when using circular polarisations and, according to the VLA report, this is not such a big problem with modern telescopes. Furthermore, as far as pulsars is concerned, the above is not a major problem as typical percentage of circular polarisation one is looking for is ~ 5-10%. For extra-galactic sources, where one rarely expects more than ~ 0.5% circular polarisation, this might be a problem. We feel this aspect should be discussed more with others, especially extra-galactic people.

There is however, one major advantage with using circular components. Any difference in phase between the two channels simply serves to rotate one circular component with respect to the other. This simply rotates the estimated position angle of a linearly polarised source and does not affect the estimate of V. Using linear components, the effect is not so easy to untangle as it can alter the measured state of polarisation of a polarised source. According to the VLA experience, the relative phase difference between the two channels has a rather poor long term stability (see last paragraph in section 4) and this could affect polarisation studies.

The use of linear components does not simplify the antenna polarisation matrix which relates the measured Stokes parameters to the true values. Thus it does not make the job of polarisation calibration any easier. In fact, it couples the polarisation of the calibrator source more strongly, affecting the accuracy of the gain calibration (ref: NRAO-VLA workshop proceedings).

Hence we feel that there is no advantage to using linear components for pulsar polarimetry and polarisation calibration. An option to bypass the power combiner and use linear components could however be provided if the switch needed does not raise the system temperature significantly.

6. Use of pulsars as polarisation calibrators :

As has been discussed above, point sources with significant (and known) polarisation make for good polarisation calibrators. The task of finding such candidates among compact extragalactic sources becomes difficult at metre wavelengths. To overcome this handicap, pulsars can be used as calibration sources (Stinebring et.al.). Many pulsars possess significant fraction of linear and circular polarisation. The ideal candidates would be strong pulsars having large linear polarisation.

Since GMRT will not have the facility of rotating the feeds, we can resort to two other techniques for varying the effective position angle of the linear component of the pulsar. The first is to use changing parallactic angle as the pulsar is tracked across the sky. We have attached plots showing the total parallactic angle change that can be obtained at the GMRT latitude, over a -6 hr to +6 hr range of hour angle, for different values of source declination. The results show that sources in the declination range 10 deg to 30 deg would show the largest change in parallactic angle over relatively short ranges of hour angles. For example, tracking the highly polarised pulsar PSR1929+10 for 2 hours before and after transit would give us ~140 deg change in parallactic angle. We need to find more suitable pulsars in the appropriate declination range. Two main problems with this technique are that the pulsar intensity may vary over the length of observation (due to scintillations or otherwise) and that the instrumental polarisation and the ionosphere may not be stable over the interval. The first difficulty can be overcome by normalising all measurements by the quantity [$I'^2 - (Q'^2+U'^2+V'^2)$], which remains invariant even when all other parameters are fluctuating.

The other technique which can be used to rotate the effective position angle is the differential Faraday rotation across the band. Thus, if the band is broken into narrow sections and the measurements made at each sub-band, a different position angle would be sampled. The advantages of this scheme are that it is much quicker than the first technique and that one does not have to worry so much about the fluctuations in the ionosphere, the polarisation parameters or the pulsar intensity. For the GMRT bandwidth of 16 MHz, the minimum rotation measure required for a 180 deg. rotation across the band at 327 MHz is 38 rad m^{-2}. There are several pulsars in the GMRT declination range that meet this requirement. With a little bit of research, it should be possible to find some among these that are bright and strongly linearly polarised. The main drawback with this scheme is that the cross coupling parameters may be frequency dependent across the 16 MHz band. For example, VLA measurements show that for the 20 cm band the parameters vary from 1 to 10% across the band. Most of these variations are believed to be due to standing waves caused by reflections at mismatches.

From the hardware point of view, using pulsars for polarisation calibration requires that the GMRT correlator system be able to accumulate the various two dish RR,LL,LR,RL products during a small fraction of the pulsar period only. This can be achieved by making use of the special "pulsar gate" feature provided in the NRAO correlator system design wherein one edge of a pulse can be used to clear the accumulators at a desired time and the other edge of this pulse can be used to zero the FFT output. One would then need to adjust this pulse to repeat at the pulsar period, be triggered in synchronism with the on-pulse of the pulsar and have a width which is the appropriate fraction of the pulsar profile. The major drawback with using pulsars is the poorer signal to noise ratio, as compared to extragalactic sources. This will require longer periods of time for calibration observations.

List of references :

1. Conway and Kronberg, MNRAS, 142, 11 (1969).

2. Stinebring et.al., Ap. J. Suppl., 55, 247 (1984).

3. Rankin, GMRT lecture notes (1991).

4. NRAO-VLA Workshop Proceedings (Synthesis Mapping) eds. Thompson and D'Addario (1982).

5. Synthesis Imaging in Radio Astronomy eds. Perley, Schwab and Bridle (1989).

Antenna Polarisation Matrix for Multiple Dish (Phased array):

Let,

$$E'_{ls} = \alpha_1 E_l + \beta_1 e^{i\gamma_1} E_r \qquad (5.1a)$$

$$E'_{rs} = \alpha_2 E_r + \beta_2 e^{-i\gamma_2} E_l \qquad (5.1b)$$

Then,

$$I' = \frac{\alpha_1^2 + \alpha_2^2 + \beta_1^2 + \beta_2^2}{2} I + [\alpha_1\beta_1 \cos\gamma_1 + \alpha_2\beta_2 \cos\gamma_2] Q$$
$$+ [\alpha_1\beta_1 \sin\gamma_1 + \alpha_2\beta_2 \sin\gamma_2] U + \frac{\alpha_1^2 - \alpha_2^2 - \beta_1^2 + \beta_2^2}{2} V$$

$$Q' = \frac{\alpha_1\beta_2 \cos\gamma_2 + \alpha_2\beta_1 \cos\gamma_1}{2} I + [\alpha_1\alpha_2 + \beta_1\beta_2 \cos(\gamma_1+\gamma_2)] Q$$
$$+ [\beta_1\beta_2 \sin(\gamma_1+\gamma_2)] U + \frac{\alpha_1\beta_1 \cos\gamma_2 - \alpha_2\beta_1 \cos\gamma_1}{2} V$$

$$U' = \frac{-\alpha_1\beta_2 \sin\gamma_2 - \alpha_2\beta_1 \sin\gamma_1}{2} I - [\beta_1\beta_2 \sin(\gamma_1+\gamma_2)] Q$$
$$+ [\alpha_1\alpha_2 - \beta_1\beta_2 \cos(\gamma_1+\gamma_2)] U + \frac{\alpha_2\beta_1 \sin\gamma_1 - \alpha_1\beta_2 \sin\gamma_2}{2} V$$

$$V' = \frac{\alpha_1^2 - \alpha_2^2 + \beta_1^2 - \beta_2^2}{2} I + [\alpha_1\beta_1 \cos\gamma_1 - \alpha_2\beta_2 \cos\gamma_2] Q$$
$$+ [\alpha_1\beta_1 \sin\gamma_1 - \alpha_2\beta_2 \sin\gamma_2] U + \frac{\alpha_1^2 + \alpha_2^2 - \beta_1^2 - \beta_2^2}{2} V$$

$$
\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix}
=
\begin{bmatrix}
1 & \left[\frac{\varepsilon_1\cos\psi_1}{1+\varepsilon_1^2}+\frac{\varepsilon_2\cos\psi_2}{1+\varepsilon_2^2}\right] & \left[\frac{\varepsilon_1\sin\psi_1}{1+\varepsilon_1^2}+\frac{\varepsilon_2\sin\psi_2}{1+\varepsilon_2^2}\right] & \frac{-\varepsilon_1^2+\varepsilon_2^2}{\varepsilon_0^2} \\
\left[\frac{\varepsilon_1\cos\psi_1+\varepsilon_2\cos\psi_2}{\varepsilon_0}\right] & \left[\frac{1+\varepsilon_1\varepsilon_2\cos(\psi_1+\psi_2)}{\varepsilon_0}\right] & \left[\frac{\varepsilon_1\varepsilon_2\sin(\psi_1+\psi_2)}{\varepsilon_0}\right] & \left[\frac{-\varepsilon_1\cos\psi_1+\varepsilon_2\cos\psi_2}{\varepsilon_0}\right. \\
\left[\frac{\varepsilon_1\sin\psi_1+\varepsilon_2\sin\psi_2}{\varepsilon_0}\right] & \left[\frac{\varepsilon_1\varepsilon_2\sin(\psi_1+\psi_2)}{\varepsilon_0}\right] & \left[\frac{1-\varepsilon_1\varepsilon_2\cos(\psi_1+\psi_2)}{\varepsilon_0}\right] & \left[\frac{-\varepsilon_1\sin\psi_1+\varepsilon_2\sin\psi_2}{\varepsilon_0}\right. \\
0 & \left[\frac{\varepsilon_1\cos\psi_1}{1+\varepsilon_1^2}-\frac{\varepsilon_2\cos\psi_2}{1+\varepsilon_2^2}\right] & \left[\frac{\varepsilon_1\sin\psi_1}{1+\varepsilon_1^2}-\frac{\varepsilon_2\sin\psi_2}{1+\varepsilon_2^2}\right] & \frac{1-\varepsilon_1^2\varepsilon_2^2}{\varepsilon_0^2}
\end{bmatrix}
$$

with $\varepsilon_0^2 = [(1+\varepsilon_1^2)(1+\varepsilon_2^2)]^{\frac{1}{2}}$

Antenna Polarisation Matrix for a single dish

Plots for 10 Diff. Declinations (−60 deg. to +90 deg) 12 values