DUAL FREQUENCY COAXIAL WAVEGUIDE FEED - DESIGN CALCULATIONS

G. Sankar, A. Praveenkumar
Antenna Systems Group. GMRT - TIFR. Pune.

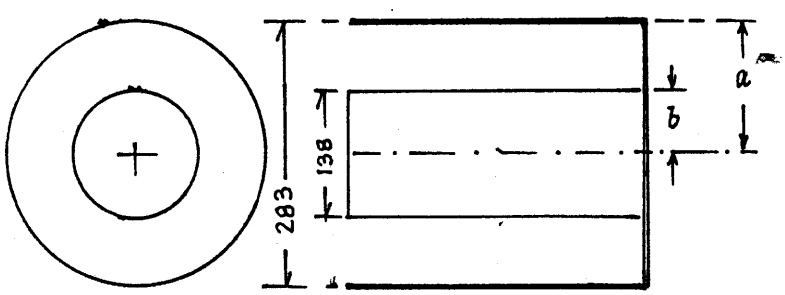
References : (i) L. Shafai & A.A. Kishk " Coaxial Waveguides as Primary Feeds for Reflector Antennas and their comparison with Circular Waveguides " AEÜ, Band 39 (1985) Heft I pp. 8-14

(ii) ~~Linvi~~ Livingston, M.L. " Multifrequency coaxial cavity apex feed " Microwave Journal (Oct. 1979) pp. 51-54.

1. SUMMARY :

The methodology adapted for the design is described in the next Section (:2). The results of those design calculation are presented here :

(1) 610 MHz COAXIAL FEED :



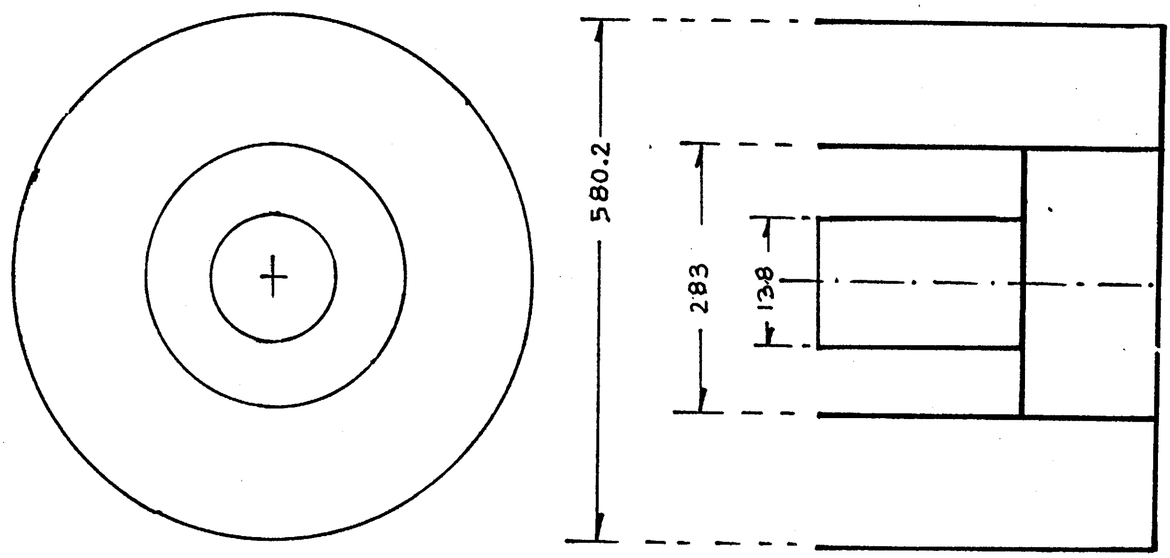
(Dimensions are in mm.)

Features :

- $\frac{a}{b} = 2.05$
- $a = 0.288\lambda$
- Cut-off freq. f_c for TE_{11}-mode: $f_c = 453$ MHz.

- The estimated 10-dB beamwidth (half-angle) will be 62.4° (i.e. the power level at the semi-rim angle of 62.674° of GMRT dish will be less than -10 dB ...)
- Radiation pattern of E and H planes will exhibit good match, resulting in higher ~~cross~~ polar ~~level~~ efficiency of the feed. In other words E and H pattern equalisation is achievable.

(2) 610 MHz / 233 MHz DUAL FREQ. FEED - Type A.



(Dimensions are in mm.)

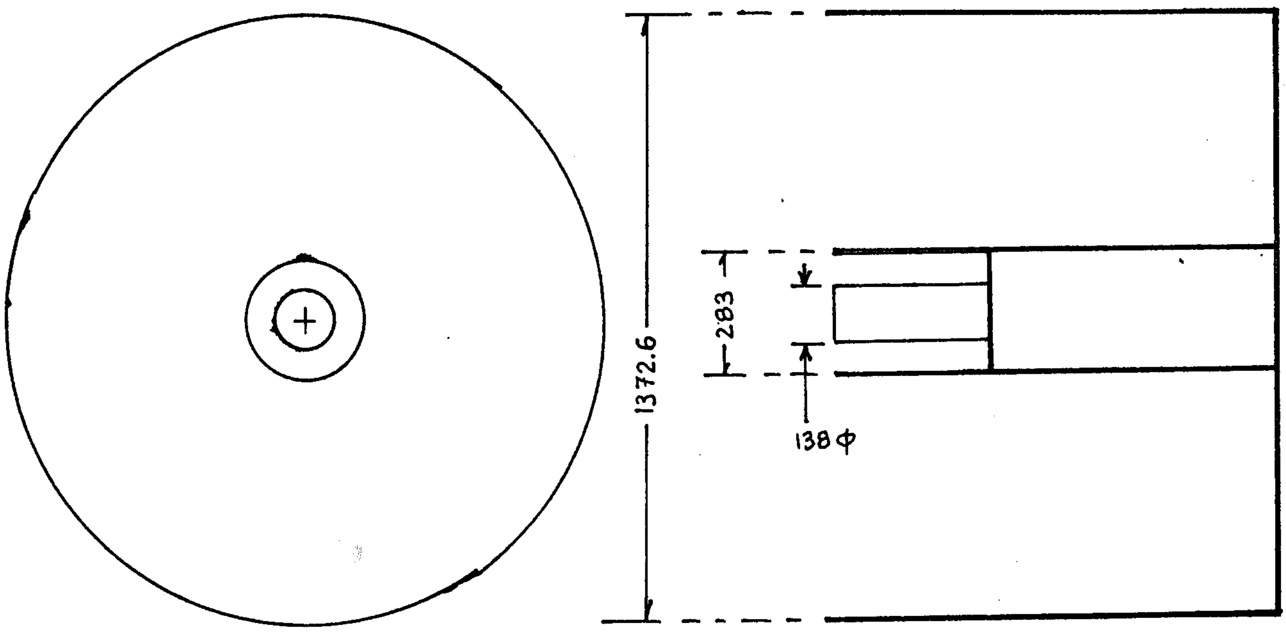
610 MHz Coaxial : Same as mentioned in (1) - page-1.

233 MHz Coaxial :

- $\frac{a}{b} = 2.05$
- $a = 0.255\lambda$
- Cut-off freq. f_c = 221 MHz.

  TOO- CLOSE TO THE OPERATING FREQUENCY.
- Could be used as a 327 MHz Coaxial feed, except for the objection that the two frequencies are very nearby and a possible mutual coupling may arise.
- E and H mismatch will be about 3° at 10 dB beamwidth half-angle.

(3) 610 MHz / 233 MHz DUAL FREQ. FEED - Type B



(All dimensions are in mm.)

610 MHz Feed : Same as mentioned in (1) - Page: 1

233 MHz Feed :

- $\frac{a}{b} = 4.85$
- $a = 0.533\lambda$
- Cut off freq. $f_c = 115$ MHz
- Could be effectively used for 150 MHz. also since the cut-off freq. is ~ 20% of 150 MHz.
- E and H mismatch will be ~10° at 10 dB beamwidth (half-angle)

2. DESIGN CALCULATIONS :

Ref. (i) cited above (Shafai & Kishk) gives the theoretically estimated 10 dB beam width (half-angle) for E and H planes vs. outer conductor radius expressed as fractions of the operating wavelength for a/b ratios of 3, 4 and 5.

A similar curve is presented in Ref. (ii) for $a/b = 2$, which is based on experimentally determined values. Here the 10dB width - full angle is plotted against the operating frequency normalised to the cut-off frequency f_c for TE_{11} mode excitation inside the cavity / annular space.

To compare both these graphs, the x-axis of plot of Ref (ii) should be transformed to a/λ. The conversion relation is given by

$$\frac{f}{f_c} = \frac{\lambda_c}{\lambda}$$

$$= \frac{\pi(m+1)}{m}\left(\frac{a}{\lambda}\right) \qquad \dots \qquad \dots (2.1)$$

where $m = a/b$

[The cut-off wavelength expression, $\lambda_c = \pi(a+b)$ is explained in Appendix - A].

Fig. 1 of this report is the combined plots, as per the above relation and requirement. For identical situation, the plots from Ref (i) - for coaxial feeds without choke are taken.

Our design aim is :

- Feeds should exhibit pattern symmetry and good match of E and H patterns ; Any mismatch will result in increased cross-polar losses.
- The 10 dB beamwidth half-angle should be less than the semi-rim angle of the GMRT-

- parabolic dish (62.674°) to minimise the spill-over loss†.

To attain the first design aim, the E- and H- pattern match - occuring at a single a/λ value for a given a/b ratio is read-out from Fig. (1). The 10-dB width and the a/λ values are plotted against a/b ratio as shown in Fig. (2).

610 - MHz FEED :

If we choose 10 dB beamwidth of 62.4° (lower than the rim- angle of GMRT dish by ~ 0.3°) the corresponding a/b ratio from Fig. 2 is 2.05 and a/λ = 0.288 .

For 610 - MHz, then

$$a = 0.288\,\lambda_{610}$$

$$= 141.5 \text{ mm.}$$

$$\text{Then, } b = 69 \text{ mm.}$$

$$\text{Cut-off frequency, } f_c = c/\lambda_c$$

$$= 453 \text{ MHz.}$$

For a of 0.288λ, the E and H mis- match at 10 dB will be ~ 0.5 deg. (from Fig. 1 ; $a/b = 2$)

Hence this design will satisfy both the requirements, spelt out in the previous page.

Dual Frequency Feed Design :

For the chosen lower freq. of 233 MHz, the inner conductor radius is fixed - being the outer conductor radius of 610 MHz. Feed.

For the required 10- dB half- angle of 62.4° a/b has to be 2.05. Therefore, the outer conductor radius, a will be

(... 6)

† Note: Westerbork's coaxial feed for 610 MHz. was designed for $f/D = 0.35$; i.e. Semi rim-angle $\psi_0 = 71.1°$. The measured 10 dB widths are: 63° for E - plane and 66° for H - plane. The corresponding spill-over loss was 9 %.
(Ref: Jenken et. al. " A Dual Freq. Dual Polarized Feed for Radio Astronomical Applications NTZ 1982 Heft 8 pp. 374- 376.)

2.05 times 141.5, viz. 290.1 mm.

i.e. $a = 0.255\lambda$.

From Fig. (1) for $a/b \simeq 2$, 0.255λ corresponds to a mis-match of $\sim 2.5^\circ$ between E and H patterns.

The cut off freq. f_c for 233 MHz feed is 221 MHz, which is very close to the operating frequency.

Let this particular design be designated as 'Type-A'. (See p.2)

If 327 MHz. is desired, this design could be utilised except for the fact that the two operating frequencies are very nearby.

The next option will be - one can look for the 10-dB width of 62.4° at $a/b \geqslant 3$. From Fig. 2 we see that at $a/b = 4.85$, the required width is attainable.

Since $b = 141.5$ mm,

$$a = 686.3 \text{ mm}$$

$$= 0.533\lambda$$

From Fig. 1 for $a/b = 5$ plot, one can deduce the E and H mis-match for $a/b = 4.85$ will be $\sim 9.5^\circ$

Cut-off frequency, f_c = 115 MHz.

Let this design be 'Type-B' (See p. 3)

Comparing these two designs, (having 10-dB width of 62.4°)

Design	Outer most cond. dia (2a) (mm.)	E and H mis-match @ 10dB	Cut-off Freq. f_c (MHz)	Remarks
'A'	580.2	$\sim 2.5^\circ$	221	◂ May be useful for 327 MHz ?
'B'	1372.6	$\sim 9.5^\circ$	115	◂ Could be used for 150 MHz. also; only exciting probe dimensions will vary

3. INCORPORATION OF CHOKES :

Fig. 3 illustrates the 10 dB BW half-angle and ~~ou~~ a/λ variation with a/b ratio for feeds with quarter-wavelength chokes (Ref. (1)). The table below gives the same - with and without chokes:

(For E and H matching pattern, at 10 dB level ...)

a/b ratio	10-dB BW - half angle (in deg.)		Outer-cond. radius/oper. wavelength ratio ; (a/λ)	
	Without Choke	With Choke	Without Choke	With Choke
2	62°	-	0.274	-
3	68.5°	63.3°	0.375	0.372
4	65°	61.7°	0.420	0.394
5	62°	60.8°	0.436	0.403

From Fig. 2 and Fig. 3 and from the above table, it is seen that the provision of chokes reduces the 10-dB beamwidth as well as the a/λ ratio.

A different look at the effect of choke on radiation pattern is to see the variation of f/f_c against a/b ratio. Fig. 4 presents such a plot. Ratio a/λ is converted to f/f_c, using Eqn. (2.1). One additional data point in the plot is derived from Westerbork's design (Ref. cited in foot-note of p. 5).

Within the frame work of the limited information available, it can be concluded that provision of chokes has influence on the performance only when $a/b \geqslant 3$.

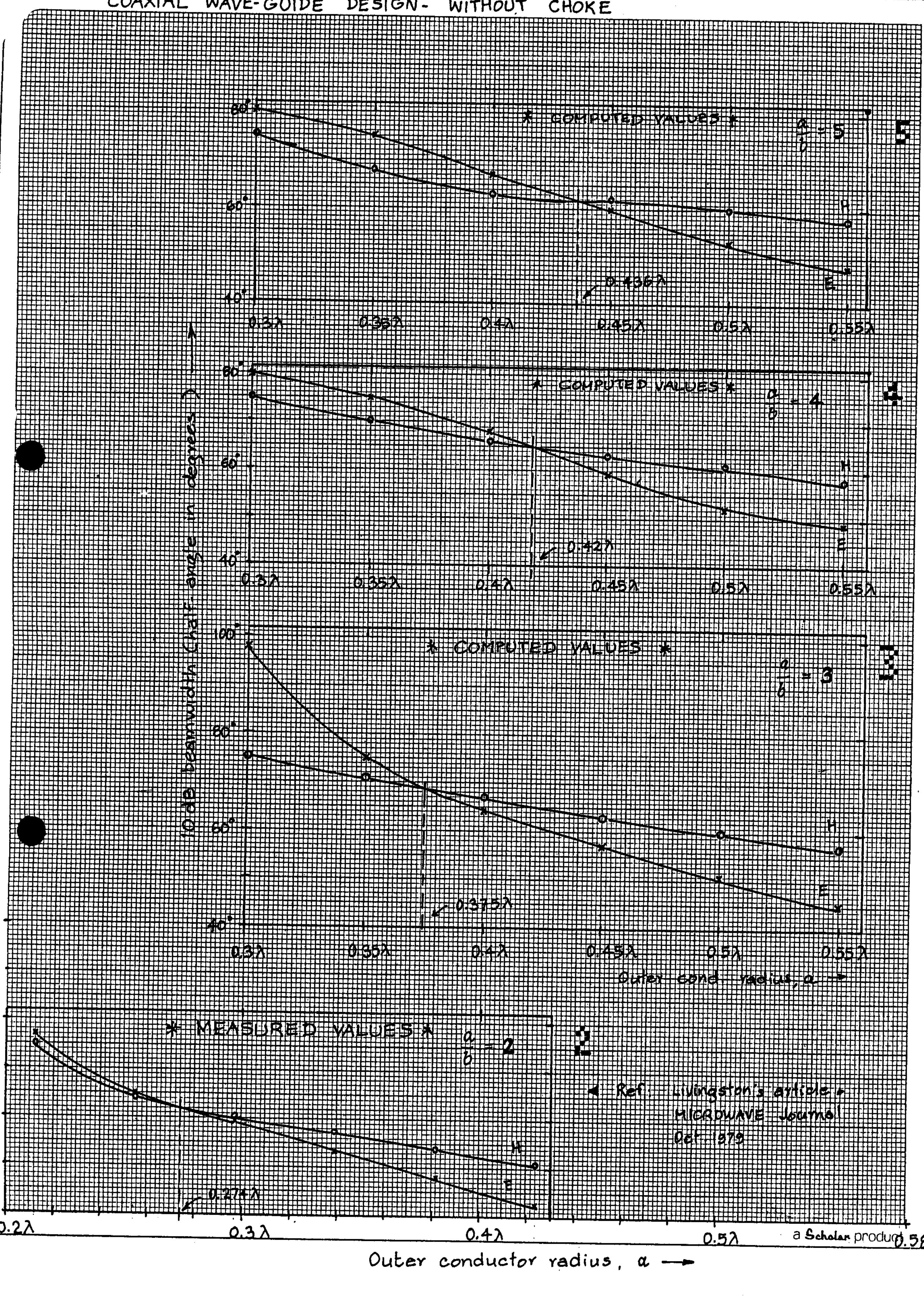
Hence in the designs considered, 610 MHz - feed alone will not have choke ; By experimental studies on 610 Feed for $a/b = 2.05$ the above conclusion can be verified.

Dual Freq. feeds of Type A & B will be provided with chokes on the outer most conductor alone ; 610 MHz - feed with choke may distort the electric field inside cavity in such a dual freq. feed.

* * * * * * *

Fig. 1

COAXIAL WAVE-GUIDE DESIGN- WITHOUT CHOKE



Outer conductor radius, a →

Ref: Shafai & Kishk 'Coaxial Waveguides as primary feeds' AEU Band 39 (1985)

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p3.(9)

Fig. 2

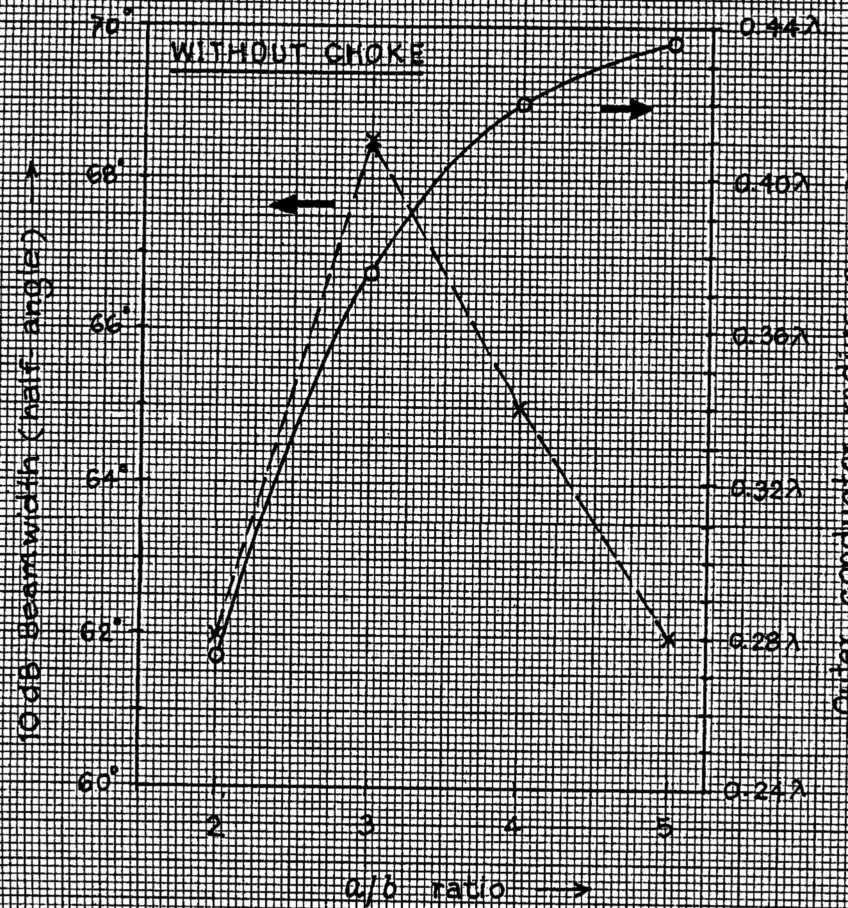
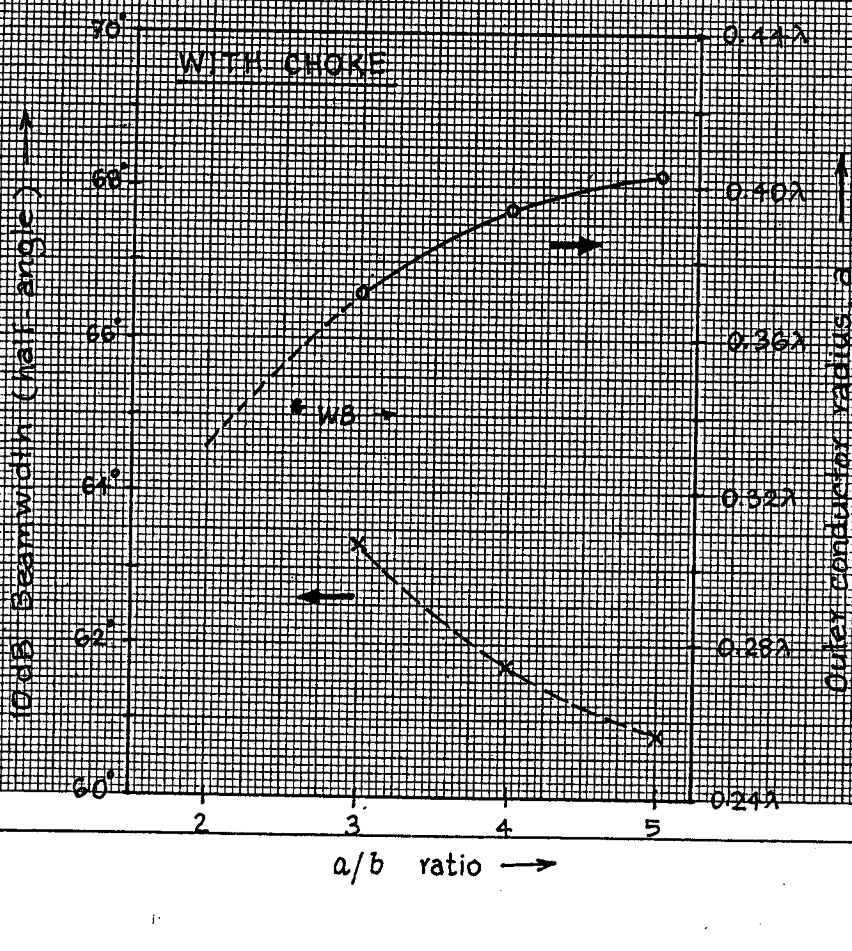
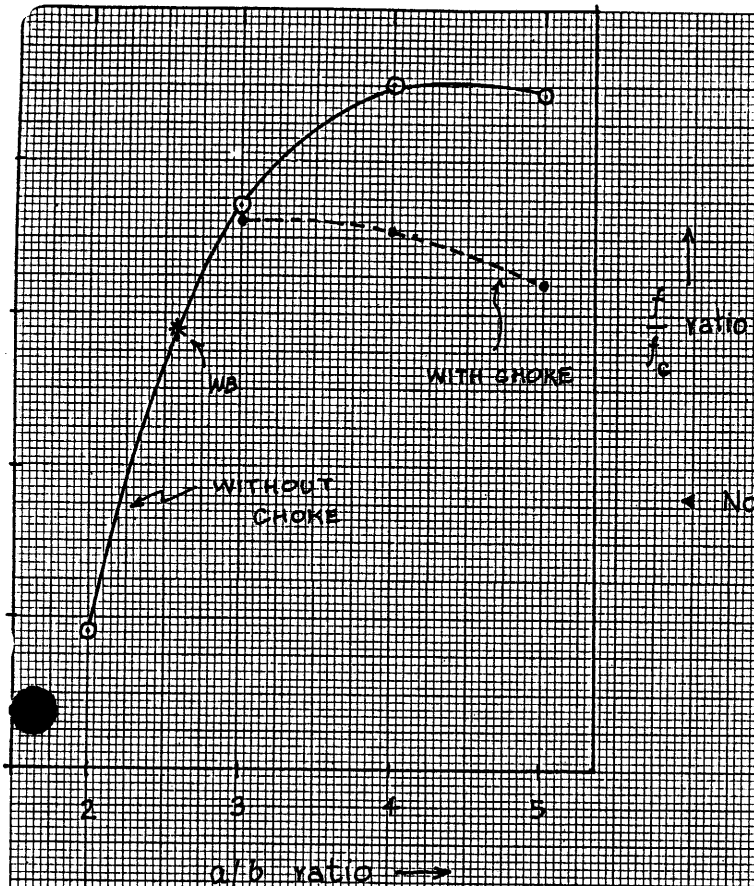


Fig. 3



◄ Note:
Point marked 'WB' is the outer cond. radius of the Westerbork design of 610 MHz coaxial feed.
The E & H patterns do not match; the measured 10 dB width is 66° for E-plane and 64.5° for H.

Fig. 4



The relationship between $\frac{f}{f_c}$ and $\frac{a}{\lambda}$ is given by :

$$\frac{f}{f_c} = \frac{\pi(m+1)}{m}\left(\frac{a}{\lambda}\right)$$

where $m = \frac{a}{b}$

◄ Note: Point marked 'WB' is the 610 MHz feed (made at GMRT, Pune) – <u>with</u> choke ; $a/b = 2.58$ and $f/f_c = 1.49$.

Appendix - A

Pages 77 to 79 of

N. Marcuvitz "Waveguide Handbook"

Peter Peregrinus Ltd. London UK (1986).

Please Note :

- The H-modes described here are the TE-modes as per Standard definitions. So H_{11} is equivalent to TE_{11}-mode.
- a - outer conductor radius

  b - inner conductor radius
- In Eqn. (42a), the factor $\frac{2}{(c+1)\chi'_{m1}}$ varies from unity to ~0.97 when c varies from 1.0 to 4.0:

If $k_1 = \frac{2}{(c+1)\chi'_{m1}}$, then

c	k_1
1.0	1.0
2.0	0.9847
3.0	0.9728
4.0	0.9732

... normalized in accordance with Eqs. (1·5) are deri e from scalar functions

$$\Psi_i = Z_m\left(\chi'_i \frac{r}{b}\right) \begin{matrix}\cos \\ \sin\end{matrix}\, m\phi, \tag{41}$$

where

$$Z_m\left(\chi'_i \frac{r}{b}\right) = \frac{\sqrt{\pi\epsilon_m}}{2} \frac{J_m\left(\chi'_i \frac{r}{b}\right) N'_m(\chi'_i) - N_m\left(\chi'_i \frac{r}{b}\right) J'_m(\chi'_i)}{\left\{\left[\frac{J'_m(\chi'_i)}{J'_m(c\chi'_i)}\right]^2 \left[1 - \left(\frac{m}{c\chi'_i}\right)^2\right] - \left[1 - \left(\frac{m}{\chi'_i}\right)^2\right]\right\}^{1/2}},$$

$$m = 0, 1, 2, 3, \cdots,$$

and $\chi'_i = \chi'_{mn}$ is the nth root of the derivative of the Bessel-Neumann combination $Z_m(c\chi'_i)$ with $c = a/b$. For $n = 1$ the quantities $(c+1)\chi'_{m1}$ are tabulated as a function of the ratio c in Table 2·4; for $n > 1$ the quantities $(c-1)\chi_{mn}$ are tabulated in Table 2·5.

Table 2·4.—First Root of $J'_m(c\chi')N'_m(\chi') - N'_m(c\chi')J'_m(\chi') = 0$
Tabulated in the form $(c+1)\chi'_{m1}$, $(m > 0)$

c \ m1	11	21	31
1.0	2.000	4.000	6.000
1.1	2.001	4.001	6.002
1.2	2.002	4.006	6.008
1.3	2.006	4.011	6.012
1.4	2.009	4.015	6.017
1.5	2.013	4.020	6.018
1.6	2.018	4.025	6.011
1.8	2.024	4.026	5.986
2.0	2.031	4.023	5.937
2.5	2.048	3.980	5.751
3.0	2.056	3.908	5.552
3.5	2.057	3.834	5.382
4.0	2.055	3.760	5.240

In terms of the tabulated values, the cutoff wavelength of an H_{m1}-mode can be expressed as

$$\left.\begin{aligned} \lambda''_{ci} &= \frac{2\pi}{(c+1)\chi'_{m1}}(a+b), \\ \Rightarrow \quad \lambda''_{ci} &\cong \frac{\pi(a+b)}{m} \qquad \text{for } m = 1, 2, 3, \cdots \end{aligned}\right\} \tag{42a}$$

and for an H_{mn}-mode as

$$\left.\begin{aligned} \lambda''_{ci} &= \frac{2\pi}{(c-1)\chi'_{mn}}(a-b), \\ \lambda''_{ci} &\cong \frac{2(a-b)}{(n-1)} \qquad \text{for } n = 2, 3, 4, \cdots \end{aligned}\right\} \tag{42b}$$

It is evident from these equations that the H_{11}-mode is the dominant H-mode in coaxial guide. The cutoff wavelength of the H_{01}-mode is identical with that of the E_{11}-mode, i.e., $\chi'_{01} = \chi_{11}$ and can be obtained from Table 2·3.

TABLE 2·5.—HIGHER ROOTS OF $J'_m(c\chi')N'_m(\chi') - N'_m(c\chi')J'_m(\chi') = 0$
Tabulated in the form $(c-1)\chi'_{mn}$, $(n > 1)$

c \ mn	02*	12	22	32	03*	13	23	33
1.0	3.142	3.142	3.142	3.142	6.283	6.283	6.283	6.283
1.1	3.143	3.144	3.148	3.156	6.284	6.284	6.287	6.290
1.2	3.145	3.151	3.167	3.193	6.285	6.288	6.296	6.309
1.3	3.150	3.161	3.194	3.249	6.287	6.293	6.309	6.337
1.4	3.155	3.174	3.229	3.319	6.290	6.299	6.326	6.372
1.5	3.161	3.188	3.27	3.40	6.293	6.306	6.346	6.412
1.6	3.167	3.205	3.32	3.49	6.296	6.315	6.369	6.458
1.8	3.182	3.241	3.4	3.7	6.304	6.333	6.419	6.56
2.0	3.197	3.282	3.5	......	6.312	6.353	6.47	6.67
2.5	3.235	3.396	......	......	6.335	6.410	6.6	7.0
3.0	3.271	3.516	......	......	6.357	6.472	6.8	
3.5	3.305	3.636	......	......	6.381	6.538	7.0	
4.0	3.336	3.753	......	......	6.403	6.606		

c \ mn	04*	14	24	34	05*	15	25	35
1.0	9.425	9.425	9.425	9.425	12.566	12.566	12.566	12.566
1.1	9.425	9.426	9.427	9.429	12.567	12.567	12.568	12.570
1.2	9.426	9.428	9.433	9.442	12.567	12.569	12.573	12.579
1.3	9.427	9.431	9.442	9.461	12.568	12.571	12.579	12.593
1.4	9.429	9.435	9.454	9.484	12.570	12.574	12.588	12.611
1.5	9.431	9.440	9.467	9.511	12.571	12.578	12.598	12.631
1.6	9.434	9.446	9.482	9.541	12.573	12.582	12.609	12.654
1.8	9.439	9.458	9.515	9.609	12.577	12.591	12.634	12.704
2.0	9.444	9.471	9.552	9.684	12.581	12.601	12.661	12.761
2.5	9.460	9.509	9.665	9.990	12.593	12.629	12.739	12.92
3.0	9.476	9.550	9.77	10.1	12.605	12.660	12.82	13.09
3.5	9.493	9.593	9.89	......	12.619	12.692	12.91	13.3
4.0	9.509	9.638	10.0	......	12.631	12.725	13.0	13.5

* The first nonvanishing root χ'_{0n} is designated as $n = 2$ rather than $n = 1$. The roots χ'_{0n+1} and $\chi_{1n}(n > 0)$ are identical.

The electric- and magnetic-field components of an H_{mn}-mode are given by Eqs. (1·4), (1·10), and (41) as

$$\left.\begin{aligned} E_r &= \pm V''_i \frac{m}{r} Z_m\left(\chi'_i \frac{r}{b}\right) \begin{matrix}\sin\\ \cos\end{matrix}\, m\phi, \\ E_\phi &= V''_i \frac{\chi'_i}{b} Z'_m\left(\chi'_i \frac{r}{b}\right) \begin{matrix}\cos\\ \sin\end{matrix}\, m\phi, \\ E_z &= 0, \end{aligned}\right\} \tag{43a}$$

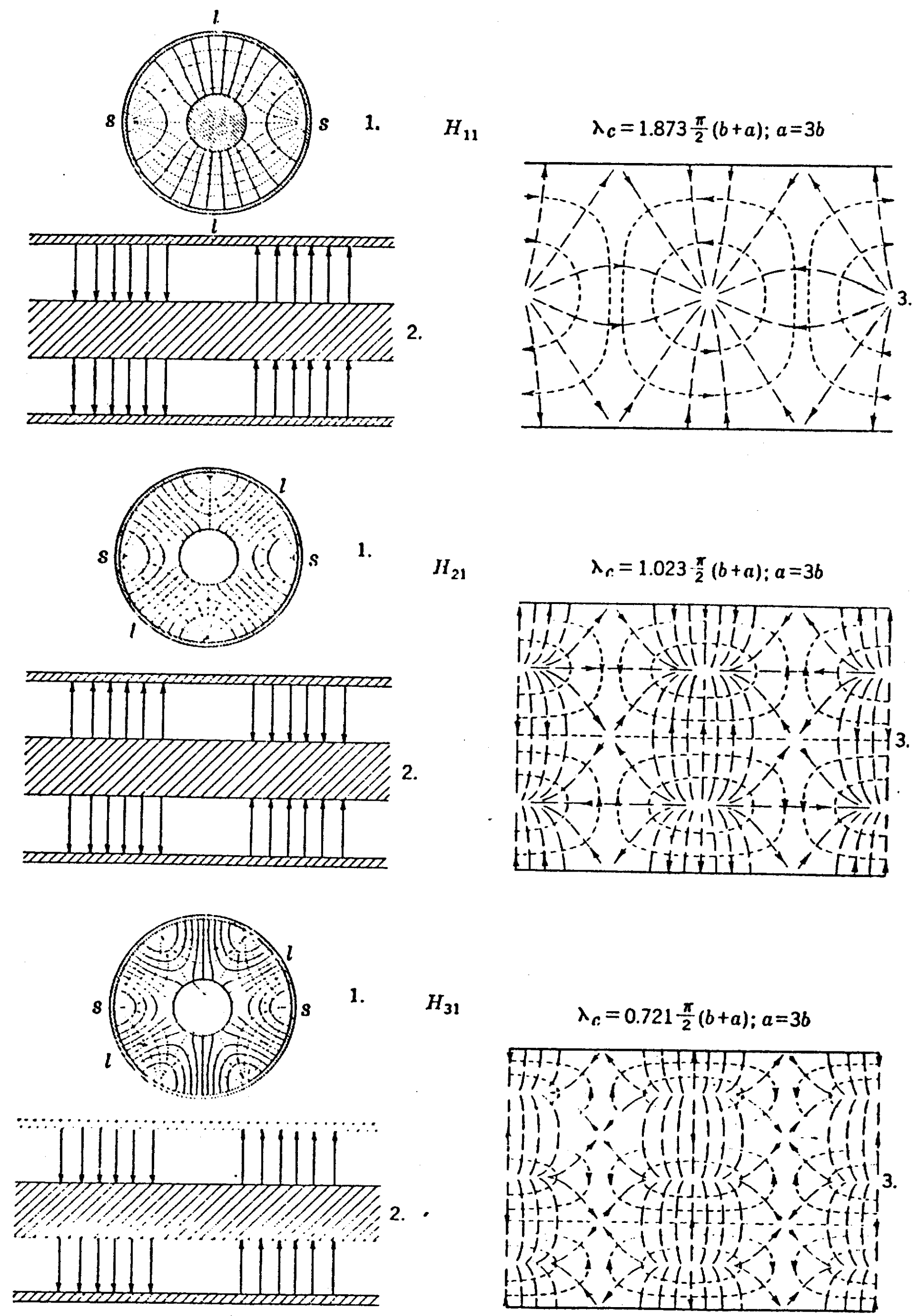


FIG. 2·9.—Field distribution for H-modes in coaxial waveguide.
1. Cross-sectional view
2. Longitudinal view through plane l-l
3. Surface view from s-s