Switching Frequencies for Walsh Functions for GMRT

A. Pramesh Rao
(22 Feb 90)

While it is generally accepted that we should have phase switching using Walsh functions in the GMRT, there seems to be some uncertainty as to what should be the frequencies of the Walsh functions. In his note of 17 Aug 89, N. Sunil suggests using a minimum sequency of 13 Hz implying a maximum sequency of 13x128=1664 Hz. While this is quite satisfactory to the astronomer, such high frequencies are likely to complicate the design of the GMRT receivers. The purpose of this note is to show that a maximum sequency of about 52 Hz is quite adequate to meet the astronomical requirements.

The fringe frequency of an interferometer depends on the baseline, the observing frequency and the declination and hour angle of the source. With the current coordinates of the GMRT, the highest fringe frequency is 9.514 Hz, which occurs at 18cm at zero declination for the E6-W6 baseline around transit. The fringe frequency is not antenna based but baseline based and arises only when signals from 2 antennas are correlated. To avoid interaction between the Walsh function and the fringe frequencies, and for good rejection of any CW stray signal that are picked up after the first point of phase switching the Walsh function frequency should be higher then the fringe frequency by at least 0.8 Hz (VLA Electronics Memo 122, "Phase switching in the VLA", A. R. Thompson, 1974). Adding this to the highest fringe frequency, we will use 12 Hz as a working number for the effective highest fringe frequency.

If s1 and s2 are the Walsh function sequencies of the of the 2 antennas forming the baseline with fringe frequency of 12 Hz, the product Walsh function at the output of the correlator has a sequency greater than |s1-s2| and should be equal to or greater than 12 Hz. If fo is the lowest sequency (ie the Walsh functions are defined over an interval T=1/fo) and we generate 64 Walsh functions having sequencies from fo to 64fo, it is fairly straight forward to allocate sequencies to stations such that the difference in the sequencies of the antennas at the ends of the arms (E6,W6,S6) is at least 16fo and the difference decreases proportionately as one goes towards the centre. Such schemes for allocating the sequencies have been given both by Thompson and by Sunil. Since the highest fringe frequency occurs for E6-W6, the baseline with the largest east west baseline, the implication of this is that the astronomical requirement that |s1-s2| should be greater than the fringe frequency boils down to 16fo > 12 Hz, giving fo > 0.75 Hz and 64fo, the maximum sequency , is of the order of 48 Hz. The maximum switching interval is then of the order of 10 ms which can be handled fairly easily. The interval over which the Walsh functions are defined is (1/fo) 1.333 s or less.

For actual astronomical observations one would like the actual integration period to be some integral multiple of the Walsh function period since then the Walsh functions are orthogonal and one gets the best performance. For synthesis observations, the typical integration periods are a few seconds to the order or minute. For this 1.333s is a rather inconvenient interval since integration periods of of ten or multiple of ten seconds cannot be achieved. For this reason it is slightly more useful to increase fo to 0.8 Hz which reduces the interval over which the Walsh functions are defined to 1.25 s and which supports integration periods of 5,10 and multiples of 5 and 10 seconds. With this the highest Walsh function frequency goes up to 64x0.8=51.2 Hz and the fastest switching interval reduces to a little less than 10 ms.

In actual practice, one may need 6 Walsh functions for each antenna (four for the the possible four IF channel and two for the two local oscillators). This increases the order of Walsh functions we need to use from 64 to 256. However, the point to be noted is that this does not necessarily have to increase the highest Walsh function frequency (or reduce the shortest switching interval). Quadrupling the number of Walsh functions can be achieved by decreasing fo by a factor of 4 or equivalently, increasing the period over which the Walsh functions are defined from 1.25 to 5s. Since the fringe frequency between the local oscillator signals and between the IF channels of the same antenna are zero, there are no constraints on them, and we can if required allocate sequential sequencies. In Appendix II we show a possible allocation of sequencies, giving 6 Walsh functions to each antenna. While in this table we have worried about avoiding square waves, no consideration has been given to avoiding 50 Hz and its sub harmonics (is the 50 Hz line frequency stable enough for such planning ?).

Some more thought has to be put into the matter before the final allocation of the Walsh function frequencies can be made. The purpose of this note was just to shows the range of actual switching frequencies required and to allay some fears that frequencies of the order of kiloherz would be required. With 256 Walsh function frequencies to play with there are many possible allocation schemes and there are bound to be schemes that give better performance than the configuration in Appendix II. However, what can be discussed and finalised is the actual value of fo, whether fo of 0.2 Hz is a satisfactory number, so that one can proceed with details of actual design.

APPENDIX 1

Coordinates and Walsh function frequencies for GMRT (1/antenna)

Given below are a possible set of Walsh function sequencies for the GMRT antennas. Given are the antenna name, x and y coordinates in metres (in the local survey units ie x to east and y to south) and the Walsh function sequency for each antenna in units of fo, the minimum sequency which is 0.8 Hz. Table 2 shows the maximum fringe frequency (at 18cm) on each baseline and the absolute difference between the Walsh function frequencies of the two antennas forming the baseline. As seen from the table, the Walsh frequency difference is higher than the fringe frequency by much more than the recommended 0.8 Hz on most baselines. On 3 baselines, s2-s4,w2-w3 & c3-c4 it is only 0.7Hz, but even this can be improved if we take into account the fact that there will be only 30 antennas in GMRT and we are here handling 32. The effect of the excess being only 0.7 Hz as against 0.8 Hz is marginal, reducing the rejection of correleted signals by a small factor.

Ant	x(m)	y(m)	Walsh seq
c0	-258.0	259.0	33.
c1	103.0	280.0	35.
c2	429.0	237.0	37.
c3	802.0	94.0	39.
c4	998.0	104.0	40.
c5	360.0	497.0	42.
c6	461.0	469.0	43.
c8	149.0	659.0	49.
c9	386.0	396.0	44.
c10	600.0	850.0	45.
c11	1034.0	575.0	47.
c12	266.0	907.0	15.
c13	847.0	1380.0	17.
c14	903.0	899.0	19.
e1	-1540.0	-100.0	51.
e2	-2690.0	-800.	53.
e3	-4190.0	-1750.0	55.
e4	-7590.0	-2750.0	58.
e5	-10090.0	-3350.	61.
e6	-11890.0	-4300.	63.
s1	-290.0	2750.	31.
s2	760.0	4950.0	21.
s3	-240.	7150.	29.
s4	-790.	9750.	23.
s5	510.	11600.	27.
s6	660.	14400.	25.
w1	1810.	-300.	13.
w2	3510.	-1300.	11.
w3	5710.	-2600.	09.
w4	7310.	-5200.	07.
w5	8410.	-8000.	05.
w6	11610.	-9250.	01.

GLOBAL MAXIMUM OF FRINGE FREQUENCY IS 9.5139 Hz

Matrix showing maximum fringe frequency on each baseline, lower left hand triangle, and |s1-s2| , upper right hand triangle.

Frequency in units of 0.1 Hz

	c0	c1	c2	c3	c4	c5	c6	c8	c9	c10	c11	c12	c13	c14	e1	e2	e3	e4	e5	e6	s1	s2	s3	s4	s5	s6	w1	w2	w3	w4	w5	w6
c0	0	16	32	48	56	72	80	128	88	96	112	144	128	112	144	160	176	200	224	240	16	96	32	80	48	64	160	176	192	208	224	256
c1	1	0	16	32	40	56	64	112	72	80	96	160	144	128	128	144	160	184	208	224	32	112	48	96	64	80	176	192	208	224	240	272
c2	3	1	0	16	24	40	48	96	56	64	80	176	160	144	112	128	144	168	192	208	48	128	64	112	80	96	192	208	224	240	256	288
c3	4	3	2	0	8	24	32	80	40	48	64	192	176	160	96	112	128	152	176	192	64	144	80	128	96	112	208	224	240	256	272	304
c4	5	4	2	1	0	16	24	72	32	40	56	200	184	168	88	104	120	144	168	184	72	152	88	136	104	120	216	232	248	264	280	312
c5	2	1	0	2	3	0	8	56	16	24	40	216	200	184	72	88	104	128	152	168	88	168	104	152	120	136	232	248	264	280	296	328
c6	3	1	0	1	2	0	0	48	8	16	32	224	208	192	64	80	96	120	144	160	96	176	112	160	128	144	240	256	272	288	304	336
c8	1	0	1	3	4	1	1	0	40	32	16	272	256	240	16	32	48	72	96	112	144	224	160	208	176	192	288	304	320	336	352	384
c9	3	1	0	2	3	0	0	1	0	8	24	232	216	200	56	72	88	112	136	152	104	184	120	168	136	152	248	264	280	296	312	344
c10	3	2	0	1	2	1	0	2	0	0	16	240	224	208	48	64	80	104	128	144	112	192	128	176	144	160	256	272	288	304	320	352
c11	5	4	2	0	1	3	2	4	3	2	0	256	240	224	32	48	64	88	112	128	128	208	144	192	160	176	272	288	304	320	336	368
c12	2	0	1	2	3	1	1	0	1	1	3	0	16	32	288	304	320	344	368	384	128	48	112	64	96	80	16	32	48	64	80	112
c13	4	2	0	2	2	1	0	2	1	0	1	2	0	16	272	288	304	328	352	368	112	32	96	48	80	64	32	48	64	80	96	128
c14	4	3	1	1	1	2	2	3	2	1	1	3	1	0	256	272	288	312	336	352	96	16	80	32	64	48	48	64	80	96	112	144
e1	5	7	8	9	10	8	8	7	8	8	10	7	9	10	0	16	32	56	80	96	160	240	176	224	192	208	304	320	336	352	368	400
e2	10	11	12	14	15	12	12	11	12	13	15	11	13	14	4	0	16	40	64	80	176	256	192	240	208	224	320	336	352	368	384	416
e3	15	17	18	20	21	18	18	17	18	18	20	17	19	20	10	6	0	24	48	64	192	272	208	256	224	240	336	352	368	384	400	432
e4	29	30	32	33	34	31	32	30	31	32	34	31	33	33	24	19	14	0	24	40	216	296	232	280	248	264	360	376	392	408	424	456
e5	39	40	42	43	44	41	42	40	41	42	44	41	43	43	34	29	24	10	0	16	240	320	256	304	272	288	384	400	416	432	448	480
e6	46	47	49	50	51	48	49	47	48	49	51	48	50	50	41	36	31	17	7	0	256	336	272	320	288	304	400	416	432	448	464	496
s1	3	4	4	6	6	4	4	3	4	4	6	3	5	5	2	7	13	27	37	44	0	80	16	64	32	48	144	160	176	192	208	240
s2	3	5	6	6	6	5	6	4	5	5	6	4	5	5	4	9	15	29	40	47	2	0	64	16	48	32	64	80	96	112	128	160
s3	9	9	10	10	11	9	9	9	9	9	10	9	9	9	6	1	6	22	33	40	6	5	0	48	16	32	128	144	160	176	192	224
s4	13	13	14	14	15	13	13	13	13	13	14	12	13	14	12	9	2	15	27	34	9	9	4	0	32	16	80	96	112	128	144	176
s5	14	15	15	15	15	15	15	14	15	14	15	14	14	14	10	5	2	19	31	38	10	9	4	4	0	16	112	128	144	160	176	208
s6	18	18	19	19	19	18	18	18	18	18	18	18	17	18	14	9	2	15	27	35	14	12	8	0	4	0	96	112	128	144	160	192
w1	8	7	6	4	3	6	6	7	6	5	3	6	4	4	14	18	24	38	48	55	9	8	13	17	17	20	0	16	32	48	64	96
w2	15	14	13	11	10	13	13	14	13	12	10	13	11	11	20	25	31	45	55	62	16	14	19	23	21	24	7	0	16	32	48	80
w3	24	23	22	20	19	22	22	23	22	21	19	22	20	20	29	34	40	54	64	71	25	22	27	31	28	30	16	9	0	16	32	64
w4	31	30	29	27	26	29	29	30	29	28	26	30	28	27	36	41	47	60	70	78	32	30	35	38	35	37	23	16	7	0	16	48
w5	37	35	34	33	32	34	34	35	34	34	32	35	33	33	42	46	52	65	75	82	38	35	40	44	41	43	29	22	13	6	0	32
w6	50	48	47	45	45	47	47	48	47	46	45	48	46	45	54	59	65	78	88	95	51	48	53	56	53	54	41	34	25	18	13	0
	c0	c1	c2	c3	c4	c5	c6	c8	c9	c10	c11	c12	c13	c14	e1	e2	e3	e4	e5	e6	s1	s2	s3	s4	s5	s6	w1	w2	w3	w4	w5	w6

APPENDIX II

Possible distribution of Walsh functions (6/antenna)

Given below are a possible distribution of Walsh functions giving 6 Walsh functions per antenna (2 for the local oscillators and 4 for the IF channels). Since the LO signals have no fringes in them the Walsh sequencies can be arbitrarily chosen (we only avoid square waves). Similarly, there are no fringes between IFs from the same antenna and so the Walsh function sequency difference can be arbitrarily small. Between antennas the Walsh function sequency difference should exceed the maximum fringe frequency by at least 0.8 Hz. To accommodate 180 Walsh functions, we need to work with a set of 256 Walsh functions as compared to 64 in Appendix I. This is achieved by reducing fo 0.8 Hz to 0.2 Hz which increases the period over which the Walsh functions are defined from 1.25 s to 5s. The highest frequency in this scheme is the same as that in Appendix I. Since from one antenna to the next, the sequency of the Walsh functions increases by at least 4, the astronomical performance of this configuration, (excess of Walsh function sequency difference over the highest fringe frequency) is as good as in the previous case.

name	LO1	LO2	IF1	IF2	IF3	IF4
c0	129	130	131	132	133	134
c1	137	138	139	140	141	142
c2	145	146	147	148	149	150
c3	153	154	155	156	157	158
c4	151	152	159	160	161	162
c5	165	166	167	168	169	170
c6	163	164	171	172	173	174
c8	193	194	195	196	197	198
c9	119	120	175	176	177	178
c10	135	136	179	180	181	182
c11	185	186	187	188	189	190
c12	57	58	59	60	61	62
c13	65	66	67	68	69	70
c14	73	74	75	76	77	78
e1	201	202	203	204	205	206
e2	209	210	211	212	213	214
e3	217	218	219	220	221	222
e4	229	230	231	232	233	234
e5	241	242	243	244	245	246
e6	249	250	251	252	253	254
s1	121	122	123	124	125	126
s2	81	82	83	84	85	86
s3	113	114	115	116	117	118
s4	89	90	91	92	93	94
s5	105	106	107	108	109	110
s6	97	98	99	100	101	102
w1	49	50	51	52	53	54
w2	41	42	43	44	45	46
w3	33	34	35	36	37	38
w4	25	26	27	28	29	30
w5	17	18	19	20	21	22
w6	3	5	6	7	9	10