Methodology of Computing the Coordinates of the Rope-truss Anchoring and Connecting points, given the Mesh-width

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Internal Technical Report : **RAC-PD-01/89**

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1 Introduction :

The coordinate system being followed is identical to what TCE had used, as shown in Fig.1. Because of the symmetry, the anchoring point on any one of the parabolic arm (Node 1) and the Connecting block (connecting the top and bottom wire ropes) of the Rope Truss (Node 2) – their coordinates alone are needed to be computed.

2 Calculation of Node 1 Coordinates :

All these points lie along the parabolic arm and they should be positioned at the 14–nodes of the arm. The inter–block distance W_g is given by

$$W_g = \frac{w_m}{\cos(\phi 1_j)} \tag{2.1}$$

where

w_m – supplied mesh width (for which the Rope trusses should be built to match)

$\phi 1_j$ – the angle between the line connecting Anchor block at 'j' and '$(j+1)$',[where j – being the Rope truss index, running from 1 to 14 ; from Hub to Rim of the Parabolic dish] and the x-axis projection on the mesh panel's plane.

`Note :  The angle` $11.25°$ `is the angle between the` x`-axis and the line projected on the` x–y `plane of the inter-anchor block connecting line, whose length is` W_g`.  Angle` $\phi 1$ `varies from` $9.6°$ `to` $10.9°$ `(for` $11.25°$ `on the` x–y `plane)`

For $j = 1$, the coordinates are found out from the geometry of the hub and the paraboloidal constraint. For the rest of the points ($j \neq 1$), a convenient system of coordinates is used, as

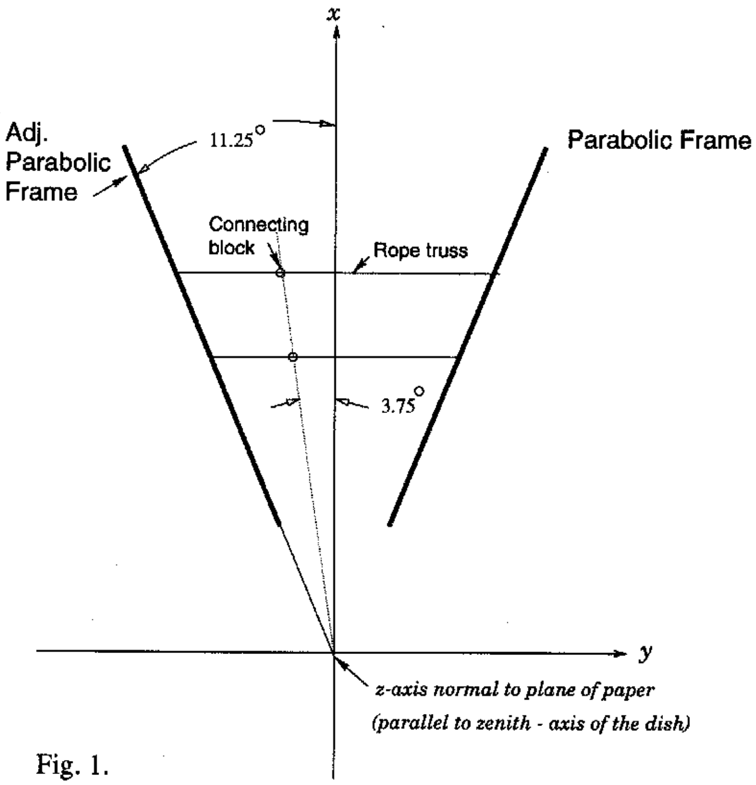

Fig. 1.

shown in Fig.2.

The $\hat{x}$–$\hat{z}$ plane contains the parabolic curve:

$$\hat{z} = \frac{\hat{x}^2}{4f} \tag{2.2}$$

The relations between the two system of coordinates,viz.,(x, y, z) and $(\hat{x}, \hat{y}, \hat{z})$ are :

$$z \equiv \hat{z} \tag{2.3}$$

$$y = \hat{x}\sin(11.25°) \tag{2.4}$$

$$x = \hat{x}\cos(11.25°) \tag{2.5}$$

The inter–block distance W_g in the $\hat{x}$-$\hat{z}$ coordinate system is given by

$$W_g{}^2 = (\hat{x}_{j+1} - \hat{x}_j)^2 + (\hat{z}_{j+1} - \hat{z}_j)^2 \tag{2.6}$$

Points j and $j+1$ should satisfy the parabola equation– Eqn.(2.2) with $f = 18.54$ m. Combining this with Eqn.(2.6),

$$(\hat{x}_{j+1} - \hat{x}_j)^2 + \left(\frac{\hat{x}^2_{j+1}}{4f} - \hat{z}_j\right)^2 = W_g{}^2 \tag{2.7}$$

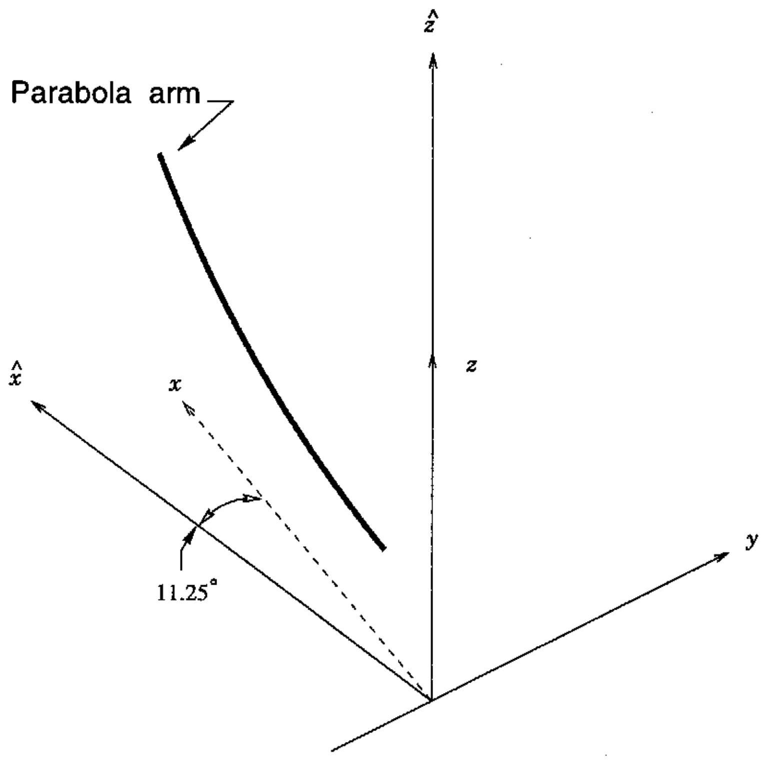

Fig. 2.

Starting from $j = 1$,
$\hat{x}_j, \hat{z}_j$ and W_g are known quantities. Substituting these in Eq.(2.7), $\hat{x}_{j+1}$ can be solved through an iterative process, which can be repeated till $j = 14$.
Using Eq.(2.3) to Eq.(2.4), x, y, z can also be computed.
The local normal angle, θ_j with respect to the z –axis is given by

$$\theta_j = \arctan\left(\frac{\hat{x}_j}{2f}\right) \tag{2.8}$$

Solution of Eq.(2.7) and subsequent calculation of θ_j as per the above equation completes the Node 1 coordinates computation.

3 Rope Truss Plane :

Angle θ_j defines the inclination of the Rope Truss (RT) plane to the z – axis. Node 1's any general point defined by x_j, y_j, z_j will be satisfying the local normal RT plane's equation, defined w.r.to the x, y, z system of coordinates.
To formulate the RT normal plane's equation, let us go back to $(\hat{x}, \hat{y}, \hat{z})$ system of coordinates. The equation of the RT plane can be conveniently written in intercept form. Since all the RT planes will be parallel to the y –axis, the y –intercept b will be ∞.

Considering the RT plane at point 'j' of the parabola,as shown in Fig.3,

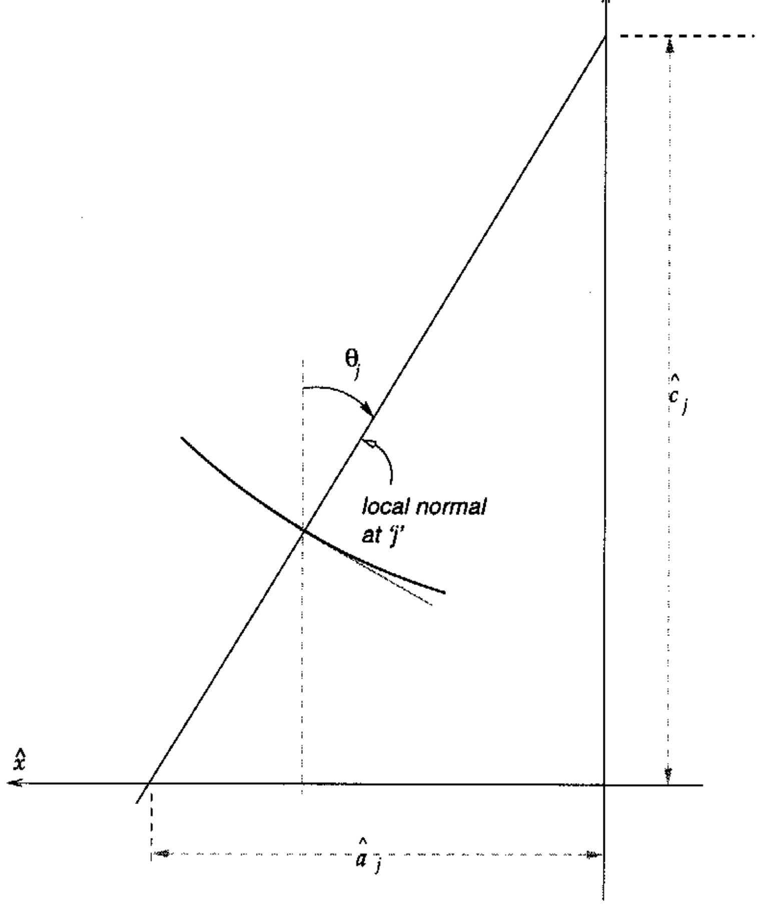

Fig. 3.

$$\hat{c}_j = \hat{z}_j + \frac{\hat{x}_j}{\tan\theta_j} \tag{3.1}$$
$$\hat{a}_j = \hat{x}_j + \hat{z}_j \tan\theta_j \tag{3.2}$$

Re–transforming $(\hat{x}, \hat{y}, \hat{z})$ system to x, y, z system, the respective intercepts a, b, c on the x, y, z axes becomes

$$a_j = \hat{a}_j \cos 11.25° \tag{3.3}$$
$$b_j = \infty \tag{3.4}$$
$$c_j = \hat{c}_j \tag{3.5}$$

Hence the equation of the RT plane passing thorough (x_j, y_j, z_j) is given by

$$\frac{x_j}{a_j} + \frac{z_j}{c_j} = 1 \tag{3.6}$$

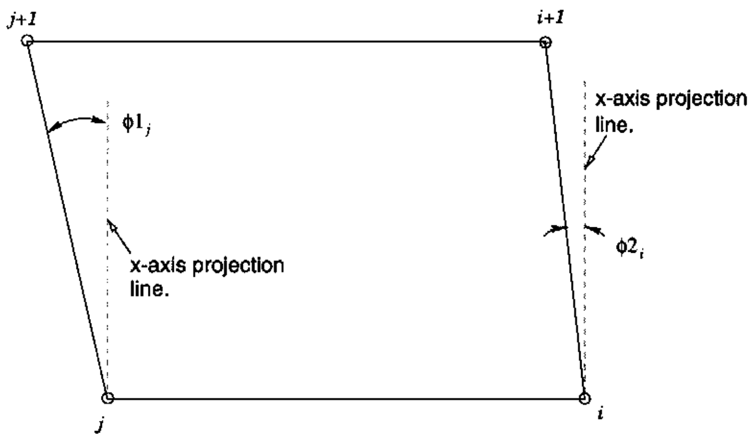

Fig. 4.

3.1 Node 2 Coordinates estimation :

Any point of the Node 2 should satisfy the following conditions:

- The point of Node 2 should satisfy the RT plane equation, viz., Eqn.(3.6) for the corresponding j value.

- The point should lie on the parabola described at the Node 2 – line i.e., a line inclined by 3.75° to the x –axis on the $x-y$ plane. In other words, if x_i, y_i, z_i are the coordinates of the Node 2 point (corresponding to index j on the Node 1 – line),

$$\frac{{x_i}^2 + {y_i}^2}{z_i} = 4f \tag{3.7}$$

- Any two adjacent RT plane's top line (i.e. the line connecting the anchor–block at 'j' of Node 1 to the connecting–block at 'i' of Node 2) should be parallel to each other

Conditions (or constraints) 1 & 2 have been written down in explicit mathematical form as shown in Eqn.(3.6) and Eqn.(3.7). To do the same for Constraint 3, consider Fig.(4).

Similar to the definition of angle $\phi 1_j$, another angle $\phi 2_i$ is defined at Node 2 line: It is the angle between the line joining adjacent Node 2 points i and $(i+1)$ and the x –axis line projection on to the containing $j, (j+1), i$ and $(i+1)$ points (or the mesh–plane ,in short ...). Estimation of $\phi 2_i$ is outlined in Appendix–**A**.
Coordinates of the Node 2 point for $i = 1$ can be computed initially as per the geometry of the hub ,paraboloidal constraint and RT plane constraint [vide. Appendix–**B**]. To simplify the notations used, Fig.5 illustrates the points and angles of Fig.4 in the simpler form:

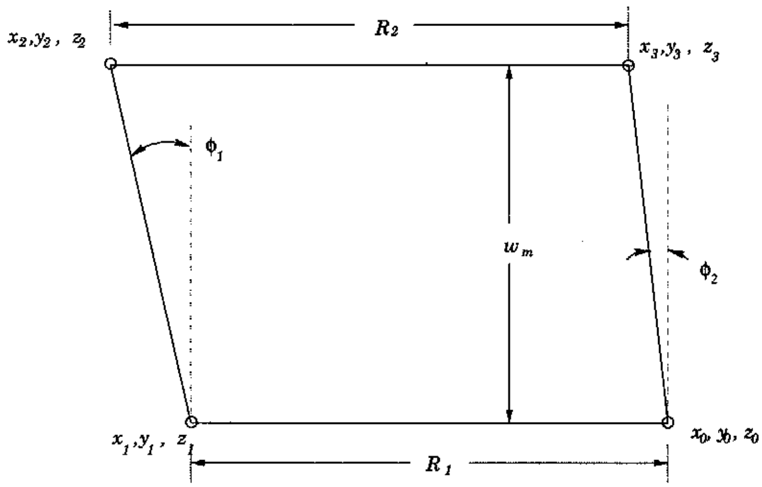

Fig. 5.

$$\begin{aligned} j &\rightarrow 1 \\ (j+1) &\rightarrow 2 \\ i &\rightarrow 0 \\ (i+1) &\rightarrow 3 \\ \phi1_j &\rightarrow \phi_1 \\ \phi2_i &\rightarrow \phi_2 \end{aligned}$$

The known quantities here are:
(x_1, y_1, z_1),(x_2, y_2, z_2),(x_0, y_0, z_0) and R_1, which is given by

$$R_1 = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2} \tag{3.8}$$

From the above geometry shown in Fig.5, R_2 can be calculated in terms of the known quantities :

$$R_2 = R_1 + w_m(\tan\phi_1 - \tan\phi_2) \tag{3.9}$$

Let the direction cosines of line R_1 be (l_1, m_1, n_1) and of line R_2 be (l_2, m_2, n_2).
The angle η between lines R_1 and R_2 is given by

$$\cos\eta = l_1 l_2 + m_1 m_2 + n_1 n_2 \tag{3.10}$$

Since R_2 is parallel to R_1, the above eqn.reduces to

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 1 \tag{3.11}$$

Substituting for l_1, m_1, n_1

$$l_1 = \frac{x_0 - x_1}{R_1}$$
$$m_1 = \frac{y_0 - y_1}{R_1}$$
$$n_1 = \frac{z_0 - z_1}{R_1}$$

Eqn.(3.11) reduces to

$$(x_0 - x_1)(x_3 - x_2) + (y_0 - y_1)(y_3 - y_2) + (z_0 - z_1)(z_3 - z_2) = R_1 R_2 \qquad (3.12)$$

Rewriting Eqn.(3.6) & Eqn.(3.7) as per the simplified notation,

$$\frac{x_3}{a_j} + \frac{z_3}{c_j} = 1 \qquad (3.13)$$
$${x_3}^2 + {y_3}^2 - 4f(z_3 - s) = 0 \qquad (3.14)$$

In the above equation the quantity 's' is the amount of shift in z –axis required to 'bring–out' the parabola from the tube–centre to the anchor block mean position level, as dictated by the theodolite survey method, while fabricating and during (subsequent) check–out.
Now combining Eqn.(3.12) to Eqn.(3.14), a quadratic in x_3 is formed (vide Appendix–**C**). Solving for x_3 and substituting it in Eqn.(3.13) z_3 can be computed. Similarly y_3 can be found out from Eq.(3.12).

Formation and solving the quadratic in x_3 for $i > 1$ and $i \leq 14$ by successive steps completes the solution for all Node 2 points. To check the solutions, the constraints (1) thro' (3) cited in Sec.(2.1) could be evaluated by substituting x_i, y_i, z_i.

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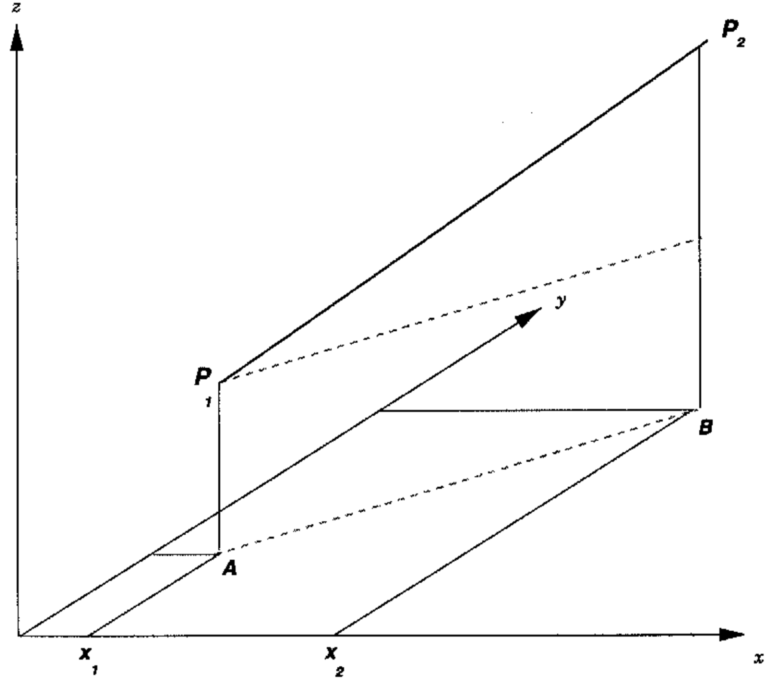

Fig. A1.

A Appendix – A

Steps to compute the relationship between an angle within two lines in 3D–space and the angle within the *projected lines* on one of the coordinate plane are outlined here.

Direction cosines of a projected line :

Refer Fig.A1. AB is the projection of line P_1P_2 on the $x-y$ plane.P_1's coordinates are x_1, y_1, z_1 and that of P_2 are x_2, y_2, z_2.Let α, β, γ be the direction angles of line P_1P_2.

$$\cos\alpha = \frac{x_2 - x_0}{P_1P_2} \quad (A.1)$$

$$\cos\beta = \frac{y_2 - y_0}{P_1P_2} \quad (A.2)$$

$$\cos\gamma = \frac{z_2 - z_0}{P_1P_2} \quad (A.3)$$

if α',β',γ' be the direction angles of the line AB, then

$$\cos\alpha' = \frac{x_2 - x_0}{AB}, = \frac{x_2 - x_0}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \quad (A.4)$$

$$\cos\beta' = \frac{y_2 - y_1}{AB} \quad (A.5)$$

$$\cos\gamma' = 0 \quad (A.6)$$

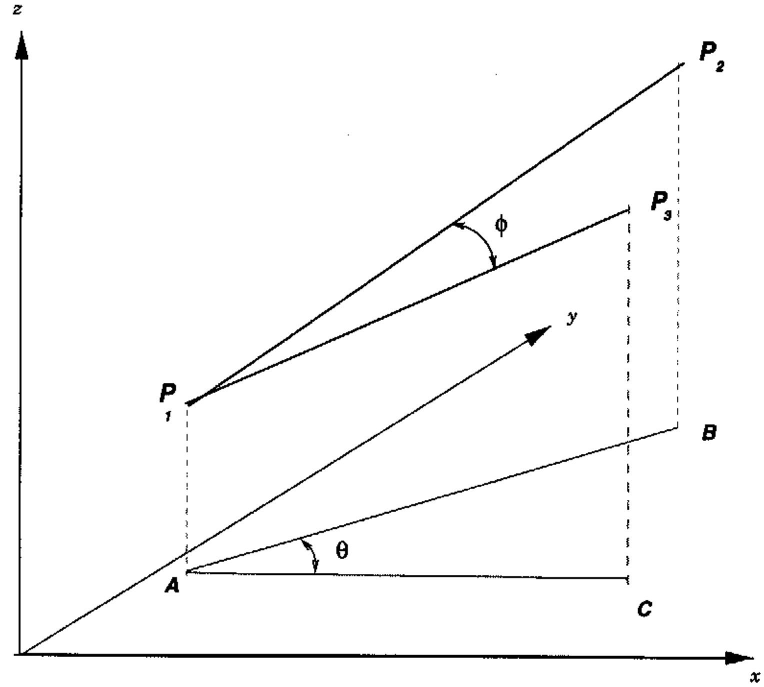

Fig. A2.

,since AB is normal to z – axis.
Rewriting the above by sustituting from Eqn.(A.1) to (A.3),

$$\begin{aligned}\cos\alpha' &= \frac{x_2 - x_1}{\sqrt{(P_1P_2)^2\cos^2\alpha + (P_1P_2)^2\cos^2\beta}} \\ &= \frac{x_2 - x_1}{P_1P_2}\cdot\frac{1}{\sqrt{\cos^2\alpha + \cos^2\beta}} \\ &= \frac{\cos\alpha}{\sqrt{\cos^2\alpha + \cos^2\beta}} \qquad \text{(A.7)}\end{aligned}$$

Similarly

$$\cos\beta' = \frac{\cos\beta}{\sqrt{\cos^2\alpha + \cos^2\beta}} \qquad \text{(A.8)}$$

Since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, Eqns. (A.7) and (A.8) can be simplified to

$$\cos\alpha' = \frac{\cos\alpha}{\cos\gamma} \qquad \text{(A.9)}$$

$$\cos\beta' = \frac{\cos\beta}{\cos\gamma} \qquad \text{(A.10)}$$

Angle between projected lines :

Consider now two lines in space, subtending an angle ϕ between them, as shown Fig.A2. Let the direction cosines of P_1P_2 be $\cos\alpha_1, \cos\beta_1, \cos\gamma_1$ and that of P_1P_3 be $\cos\alpha_2, \cos\beta_2, \cos\gamma_2$. Angle ϕ, being the angle between P_1P_2 and P_1P_3, is given by

$$\cos\phi = \cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2 + \cos\gamma_1\cos\gamma_2$$

If θ is the angle between the corresponding projections AB and AC of P_1P_2 and P_1P_3 respectively, then

$$\cos\theta = \cos{\alpha'}_1\cos{\alpha'}_2 + \cos{\beta'}_1\cos{\beta'}_2 \tag{A.11}$$

From Eq.(A.9)& (A.10),

$$\begin{aligned}\cos\theta &= \frac{\cos\alpha_1}{\sin\gamma_1}\cdot\frac{\cos\alpha_2}{\sin\gamma_2} + \frac{\cos\beta_1}{\sin\gamma_1}\cdot\frac{\cos\beta_2}{\sin\gamma_2} \\ &= \frac{\cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2}{\sin\gamma_1\sin\gamma_2}\end{aligned}$$

i.e.,

$$\cos\theta = \frac{\cos\phi - \cos\gamma_1\cos\gamma_2}{\sin\gamma_1\sin\gamma_2} \tag{A.12}$$

Computation of angles $\phi1_j$ & $\phi2_j$:

Angles $\phi1_j$ can be calculated from the known coordinates (x_j, y_j, z_j) and $(x_{j+1}, y_{j+1}, z_{j+1})$

$$\tan\phi1_j = \frac{y_{j+1} - y_j}{\sqrt{(x_{j+1} - x_j)^2 + (z_{j+1} - z_j)^2}} \tag{A.13}$$

Angle $\phi2_j$ has to be computed using Eq.(A.12). In doing so, an approximation simplifies lengthier steps : The line connecting i and $i+1$ (on the Node 2 – line) is assumed to be normal to the local normal of the parabolic curve at 'i' ; i.e., i to $i+1$ line is inclined to the z – axis by an angle $(\frac{\pi}{2} - \theta_j)$.
Hence

$$\begin{aligned}\cos\gamma_1 &= \sin\theta_j \\ &= \cos\gamma_2\end{aligned}$$

From Eqn.(A.12)

$$\begin{aligned} \cos\phi 2_j &= \cos 3.75^\circ \sin\gamma_1 \sin\gamma_2 + \cos\gamma_1 \cos\gamma_2 \\ &= (\cos\theta_j)^2 \cdot \cos 3.75^\circ + (\sin\theta_j)^2 \end{aligned} \tag{A.14}$$

[This approximation results in an error of $\pm 0.002^\circ$ or ± 7.2 arcseconds]

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B Appendix – B

Estimation of Node 2 coordinates for $i = 1$:

Let (x_2, y_2, z_2) be the coordinates of the Node 2 point for $i = 1$. This point will be placed on the Hub–circle. As pointed out in pp. 5 – 6, this point should satisfy the following conditions :

(i). Local–normal plane condition :

$$\frac{x_2}{a_j} + \frac{z_2}{c_j} = 1 \tag{B.1}$$

(ii). Paraboloidal condition :

$${x_2}^2 + {y_2}^2 - 4f(z_2 - s) = 0 \tag{B.2}$$

(iii). The line connecting Anchor block at $j = 1$ and the Node 2 point in question $(i = 1)$ should be parallel to the y – axis. If d_1 is the line length (between $j = 1$ and $i = 1$), then

$$\frac{y_1 - y_2}{d_1} = 1 \tag{B.3}$$

where
y_1 is the y –coordinate of $j = 1$ point (Anchor bock).
From the Hub–geometry d_1 can be estimated as

$$d_1 = 2R_h \cdot \sin\left(\frac{\theta}{2}\right) \tag{B.4}$$

where $\theta = 7.5°$ and R_h is the Hub–radius.
So y_2 can be found out from Eqn.(B.3) as d_1 is known.

Substituting Eqn.(B.1) in (B.2) ,eliminating z_2 and further substituting y_2's value, a quadratic in x_2 results. Solution of the quadratic and subsequent solution of Eqn.(B.1) yields x_2 and z_2. Hence (x_i, y_i, z_i) for $i = 1$ are estimated in this method.

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C Appendix – C

Solution of x_3 :

Eqn.(3.12) can be re–written as follows :

$(x_0 - x_1)(x_3 - x_2) + (y_0 - y_1)(y_3 - y_2) + (z_0 - z_1)(z_3 - z_2) = R_1 R_2$

i.e.,

$$p_x(x_3 - x_2) + p_y(y_3 - y_2) + p_z(z_3 - z_2) = R_1 R_2 \tag{C.1}$$

where $p_x = (x_0 - x_1)$; $p_y = (y_0 - y_1)$; and $p_z = (z_0 - z_1)$.
Re-arranging terms,

$$\begin{aligned} p_x x_3 + p_y y_3 + p_z z_3 &= R_1 R_2 + p_x x_2 + p_y y_2 + p_z z_2 \\ &= K_1 \end{aligned} \tag{C.2}$$

From Eqn.(3.13),

$$z_3 = c_j \left(1 - \frac{x_3}{a_j}\right) \tag{C.3}$$

Substituting Eq.(C.3) in Eq.(C.2)

$$p_x x_3 + p_y y_3 + p_z c_j \left(1 - \frac{x_3}{a_j}\right) = K_1$$

i.e.,

$$\begin{aligned} y_3 &= \frac{K_1 - p_z c_j}{p_y} + \frac{x_3}{p_y} \cdot \left(\frac{p_z c_j}{a_j} - p_x\right) \\ &= q_1 + q_2 x_3 \end{aligned} \tag{C.4}$$

where

$$q_1 = \frac{1}{p_y}(K_1 - p_z c_j) \tag{C.5}$$

and

$$q_2 = \frac{1}{p_y}\left(\frac{p_z c_j}{a_j} - p_x\right) \tag{C.6}$$

Substituting Eqns.(C.4) and (C.3) in Eqn.(3.14),and simplifying further

$$A{x_3}^2 + Bx_3 + C = 0 \tag{C.7}$$

where

$$A = 1 + {q_2}^2 \tag{C.8}$$
$$B = 2q_1q_2 + \frac{4fc_j}{a_j} \tag{C.9}$$
$$C = 4f(s - c_j) + {q_1}^2 \tag{C.10}$$

Solving the quadratic eqn.(C.7) yields x_3.
Substituting x_3's solution in Eqn.(C.3) gives z_3 and similarly in Eqn.(C.4) gives y_3.