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AN OPTIMUM DECONVOLUTION METHOD FOR LUNAR OCCULTATIONS AND COSMOLOGICAL STUDIES USING ANGULAR SIZES OF RADIO SOURCES

By

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TO

MY PARENTS

SYNOPSIS

This thesis describes a new method of deconvolution and its application to the lunar occultation studies of radio sources. It also discusses some cosmological inferences made from a study of the angular size - flux density relation of extragalactic radio sources. The results are based on a comparison of the angular sizes of weak radio sources observed by the lunar occultation method at Ooty with those of stronger sources observed by other workers.

The most popular method of restoration of brightness distribution of a radio source from lunar occultation is to convolve the observed occultation profile with a suitably chosen restoring function as suggested by Scheuer. He showed that the achievable resolution depends only on the observational signal-to-noise ratio and not on the effects of diffraction fringes. Analysis of lunar occultation data is a special case of deconvolution problem. In general, it is possible to improve the resolution in any deconvolution process if *a priori* information is available on the nature of the solution. An important prior information is the fact that the solution, being the brightness distribution across a source, must be positive everywhere. This has been incorporated into the new method presented in this thesis, which is found to provide higher resolution than that possible with classical methods. The 'Maximum Entropy Method' recently described by Frieden and by Ables also provides higher resolution by incorporating prior knowledge, but is not readily adaptable to the case of lunar occultations, for which the new method is computationally simpler.

The method presented in the thesis is essentially a generalised least squares procedure (minimising the variance of the difference between the observed and computed data), but it is adapted to conform to *a priori* information and provides an optimum solution to the problem. The main feature of the method is a simple iterative algorithm for incorporating positiveness. This algorithm is used in the least squares procedure along with some existing techniques of constrained minimisation to form an Optimum Deconvolution Method (ODM).

Incorporation of prior knowledge is illustrated for the following cases: (a) functions whose effects are to smooth the data (like the receiver time constant and effects of lunar limb irregularities) are absorbed into the definition of 'point-source response' appearing in the convolution integral as the kernel; (b) known values for some functions of the solution (like the area or flux-density) are introduced as constraints by the classical method of Lagrange multipliers and (c) bounds on the solution, (like positiveness) are introduced by the simple iterative algorithm described in the next iteration. The smoothness of the solution, an inevitable requirement of deconvolution, is introduced by minimising the variance of second derivatives.

The method also allows for a determination of the background variations like baseline drifts by expressing them as linear functions of some parameters, e.g., a polynomial. Restoration is also possible with a specified resolution by first smoothing the data with a Gaussian of suitable width(β_1) and then partly compensating for it by smoothing the point-source response with a Gaussian of smaller width (β_2). The resolution is then given by $(\beta_1^2 - \beta_2^2)^{\frac{1}{2}}$. Experience with a large number of computer simulations and occultations observed with the Ooty Radio Telescope has demonstrated the superiority of ODM over conventional methods.

This thesis consists of three parts. After a brief general discussion of the deconvolution problem, ODM is described in detail. All empirical parameters appearing in the method have been discussed and the form most suitable for lunar occultations is emphasised. For the purpose of illustration, it is compared with Scheuer's method for (a) about 150 computer simulations of lunar occultation data with Gaussian noise, and (b) about 75 observations made with the Ooty radio telescope. These comparisons have shown that ODM generally provides better resolution than conventional methods. An average resolution-improvement by about a factor of 2 over Scheuer's method was inferred from the computer simulations. Such an improvement was also found for many of the actual occultation data considered. However, sometimes, if the parameters of ODM are not chosen properly, it may amplify the effects of noise in some regions. Guidelines have been given in the thesis for choosing the parameters in the case of lunar occultations.

The second part comprises a study of 63 weak extragalactic radio sources from their occultations observed at Ooty. Optical identifications have also been attempted for all these sources with the aid of the Palomar Sky Survey Prints. Data were first analysed by Scheuer's method, which has so far been used for mass-analysis of lunar occultation data at Ooty. However, restoration has also been performed by ODM for all these sources and significant differences, wherever found, have been pointed out. Because of the super-resolution of ODM many more sources could be recognised as double than were possible with Scheuer's method.

The last part of the thesis contains some cosmological investigations of the angular sizes of extragalactic radio sources. It is an extension of the recent works of Swarup and Kapahi which have provided an independent evidence for the evolution of radio sources. Rather than confining to the relation between the median values of angular size (θ) and flux density (S), and the angular size counts for the 3C sources only, we have considered histograms of $n(S,\theta)$ of 513 sources divided into 8 ranges of S and 17 ranges of θ . These are selected from the 3CR Survey, Robertson's All Sky Survey and the Ooty occultation survey. Latest available data from the 5 km-radio telescope at Cambridge and recent occultation data from Ooty have been included.

The detailed histograms of $n(S,\theta)$ have been compared with the predictions of Steady State and Einstein de-Sitter cosmological models. The data cannot be explained in terms of the Steady State model and they also indicate the presence of strong evolutionary effects. If a simple 3-slope power law is assumed for the local luminosity function, and a density evolution of the form $(1+z)^{\beta}$ for the high-luminosity source and a linear size evolution of the form $(1+z)^{-n}$ are assumed, a chi-square analysis of the computed curves versus observed histograms leads to $\beta = 5.5 \pm 0.5$ and $n = 1.0 \pm 0.2$. It is shown that these conclusions are unlikely to be affected by observational uncertainties. Consistency of the assumed luminosity function with the known source counts, $n(S)$, is also discussed.

It is concluded that the distribution of angular size and flux density of extragalactic radio sources provides an independent evidence for the evolution of radio sources. The derived evolution parameters agree with the values determined by $n(S)$ and $\theta(z)$ relations. It is known that the intrinsic properties of radio sources have a wide scatter. However, it seems that by considering detailed statistics of angular size, flux density and identification content of extragalactic radio sources, useful cosmological inferences can be made.

A man said to the Universe,
"Sir, I exist!"
"However" replied the Universe, "The fact has not created in me
a sense of obligation"

STEPHEN CRANE

"Nature loves to hide"

HERACLITUS

O world invisible, we view thee,
O world intangible, we touch thee,
O world unknowable, we know thee,
Inapprehensible, we clutch thee!

FRANCIS THOMSON

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The successful operation of the Ooty Radio Telescope is a result of the sincere efforts of dedicated personnel at the Radio Astronomy Centre, Ootacamund and I am deeply indebted to all of them. The optical identifications required an extensive use of photographic reproduction which was provided by Shahul Hameed and Jan at Kodaikanal, Upadhyaya and Acharya at Bombay, and Prem Kumar at Ooty.

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CHAPTER 1

INTRODUCTION

1.1 RADIO SOURCES AND COSMOLOGY

The potential of extragalactic radio astronomy in determining the geometry of the Universe was recognised about two decades ago by Ryle (1959) and others with the discovery of strong radio sources at relatively high redshifts implying large distances. In spite of the initial promises, the study of radio sources did not reveal much about the geometry of the Universe, since their overall properties were found to evolve strongly with cosmological epoch. Hence the emphasis in observational cosmology during the last few years has been directed more towards an understanding of the evolution of the radio source population than towards settling fine details about the geometry of the Universe.

<u>1.1.1 Cosmological Tests with Radio Sources</u>: In order to use radio sources as probes for cosmological investigations, four basic observables are available to us which are related to distance. These are the flux density (S), angular size (θ), spectral index (α) and the redshift of the associated optical object (z). The distributions of radio sources as a function of these variables depends on the geometry of the Universe (cosmological model) and the intrinsic properties of the sources, the latter being epoch-dependent for an evolving Universe. The general procedure is to assume different combinations of models for the geometry of the Universe and intrinsic properties of radio sources, and try to separate the two effects by a statistical analysis of the data.

The basic variables and some possible cosmological tests are listed in Table 1.1. The superscripts used in the Table indicate some of the relevant references given in the footnotes. The use of spectral index for cosmological investigations is not discussed in this thesis.

Table 1.1 : Cosmological Tests using Radio Sources

Test \ Variable	S	θ	z	(S)
1. Counts	$N(S)$[1]	$N(\theta)$[2]	$N(z)$[3]	
2. Correlations	$S(\theta)$[4]	$\theta(z)$[5,6,7]	$z(S)$[8,9]	
3. 2-dimensional Distributions	$N(S,\theta)$[10]	$N(\theta,z)$[11]	$N(S,z)$[12]	
4. Generalisation	$N(S,\theta,z)$[10]			

References:

1. Jauncey (1975);
2. Kapahi (1975a);
3. Burbidge and Narlikar (1976);
4. Swarup (1975);
5. Hoyle (1959);
6. Wardle and Miley (1974);
7. Hewish et al. (1974);
8. Bolton (1966);
9. Hoyle and Burbidge (1966);
10. Swarup and Subrahmanya (1977)
11. Swarup and Subrahmanya (in preparation)
12. von Hoerner (1973).

<u>1.1.2. The Counts of Radio Sources:</u> The first cosmological test employed in radio astronomy and being pursued even now involves the source counts, i.e., to study the number of sources (N(S) above a certain flux density S, as a function of S (Pooley and Ryle 1968; Longair 1971; von Hoerner 1973; Wall et al. 1977). There has now been a general agreement on

the behaviour of this curve determined by surveys made at various frequencies. It is now generally agreed that the observed counts imply cosmic evolution of the radio source population. The somewhat established general features of this evolution are that

(a) it is confined to the sources of high luminosity ($P_{178} \gtrsim 10^{26}$ W/Hz/sr); and

(b) the evolution is quite strong, implying that the number density of sources (or their luminosities) at large redshifts ($z > 1$) exceed the present numbers by several orders of magnitude.

However, there is yet no general agreement on questions like the form of evolution or whether it is frequency-dependent, and so on.

The main difficulty in our understanding the N(S) relation comes from the fact that there is a wide range of intrinsic luminosities of the radio sources - broader than about four orders of magnitude spanned by the observed flux densities. One has to know accurately the local luminosity function, i.e., the density of sources in a given luminosity range in the nearby space, in order to predict the evolutionary properties correctly. A wide range of models are still permissible within the existing uncertainties of the local luminosity function and the geometry of the Universe. However, with greatly improved surveys now available, and with the recent developments in the optical identification and determination of redshifts, the luminosity distributions are now known with much greater certainty than they were until a few years ago (Fanti and Perola 1977).

<u>1.1.3 The Radio 'Hubble Relation'</u> : The relation z(S) for radio sources is blurred by too much scatter in the data Hoyle and Burbidge 1966). This is expected from the spread in the intrinsic luminosities. In addition, the expected relation is critically dependent on the nature of the luminosity function. It may even happen that the luminosity function is such that the z(S) relation may not show any correlation. In such a case, the need for including other distance-dependent properties like the angular structure and spectra in addition to just fluxes and numbers becomes imperative (von Hoerner 1973). Observationally, there is likely to be a selection effect arising from the difficulties in the optical identification of weaker sources and their redshift measurements.

<u>1.1.4 Angular Size - Flux Density Relation</u> . A related test using angular sizes is to study $\theta(S)$. This requires a knowledge of the angular sizes of a large number of sources over a wide range of flux densities. The first detailed analysis of the angular sizes was made by Swarup (1975) from a comparison of the data obtained from the lunar occultations using the Ooty Radio Telescope with the available data on the stronger sources. His analysis showed that the median value of angular sizes is correlated with flux densities. Kapahi (1975,1975a) combined the $\theta_{med}(S)$ for all sources with the $N(\theta)$ for the 3CR sources and showed that the angular size data provide an independent means for deriving the radio luminosity function and its evolution. In addition, his analysis showed that the observed data implied the evolution of physical sizes of all radio sources as $(1+z)^{-n}$, with n between 1 and 1.5.

The analysis of the angular size - flux density data was continued by Swarup and Subrahmanya (1977) by studying the N(S,Θ) relation with a larger sample of weak radio sources and making a detailed statistical analysis of the data and their compatibility with some standard world models. This analysis has been extended further in Chapter 4 of this thesis. In addition to confirming the earlier results of Swarup and Kapahi, this analysis puts more stringent limits on the evolutionary requirement of the physical sizes of radio sources and also shows that the conclusions are not affected by simple considerations of inhomogeneities in the world models.

1.1.5 The Need for High Resolution Studies of Weak Sources : The cosmological investigations of angular sizes of radio sources discussed above have shown the need for studies of larger samples of weak radio sources. The effects of double smoothing of the data by the spread in the intrinsic sizes as well as luminosities can possibly be overcome by a proper statistical analysis of large samples. Further, the median values of angular sizes seem to have reached a constant value of about 8-10 arcsec for sources with $S_{408} \lesssim 1$ Jy (Swarup 1975; Ekers and Miley 1977). But the data are not sufficient to conclude unambiguously whether this value represents an asymptote or a minimum in the variation of angular sizes with decreasing flux densities. It is important to settle this question in order to decide between the various world models. This calls for the attainment of high resolutions of the order of a few seconds of arc for a large number of weak radio sources, say flux densities lower than about 1 Jy at about 300 or 400 MHz. Several aperture synthesis radio telescopes are now available for achieving resolutions of several seconds of arc at centimetre wavelengths. However, till recently their use has been restricted for mapping only the relatively strong radio sources.

1.2 HIGH RESOLUTION FROM LUNAR OCCULTATIONS

It was demonstrated by Hazard (1961, 1962) in the early sixties that it is possible to achieve high resolutions of the order of a second of arc at metre wavelengths by employing the technique of lunar occultations. The overall size of radio sources is more or less independent of frequency but their flux density increases with wavelength, making it advantageous to study them at metre wavelengths. It is difficult to obtain similar resolutions at metre wavelengths by interferometry since this would require baselines of a few hundred kilometres. In addition, the method of lunar occultation enables the determination of both compact and extended features in a source which is important to understand their physical nature. But the effective use of lunar occultations for a high resolution survey requires a steerable radio telescope of large collecting area. The Ooty Radio Telescope was specially designed for a survey of weak radio sources by observing their lunar occultations (Swarup et al. 1971). Even though this method restricts the survey to regions close to the ecliptic, it is possible to obtain large samples of weak radio sources because of the high sensitivity of the Ooty telescope. Lunar occultations of more than a thousand sources with S_{327} ranging down to 0.2 Jy have been observed at Ooty. The potential of this telescope for cosmological studies is likely to be enhanced further by the present efforts towards (a) increasing its sensitivity by a factor of about 4 and (b) using improved data analysis for obtaining higher resolution by making use of *a priori* information such as the fact that the brightness distribution should be positive, which is the main topic of the next chapter.

The methods of observation and analysis of the data from lunar occultations have been reviewed extensively in literature, e.g. von Hoerner (1964), Sutton (1966), Hazard (1976). The procedures followed at Ooty have been described by Swarup et al. (1971a), Kapahi et al. (1973), Kapahi (1975a) and Gopal-Krishna (1976). Hence only a brief account of the important aspects is given below.

1.2.1 Lunar Occultations : In its orbit around the Earth, the Moon moves in a plane which is inclined to the ecliptic at an angle of about 5°. In a cycle of 18.6 years, the Moon occults only the sources which lie in an 11°-wide belt centred on the ecliptic. The total area of the sky thus covered by the Moon is about one steradian.

1.2.2 The Occultation Curve of a Source : During an occultation, the radiation coming from a celestial source will suffer diffraction as it is intercepted by the limb of the Moon. As time progresses, successive portions of the diffraction pattern are observed on the ground. The nature of this diffraction depends on the angular structure of the source and the angular size of the first Fresnel zone at the distance of the Moon (D), which is given by $\sqrt{\lambda/D}$ for a wavelength λ, and is about 10 arcsec for $\lambda = 1$ m. The curvature of the Moon's limb is insignificant over a Fresnel zone and hence the lunar limb can be treated as an infinite straight edge within the observational errors.

The resulting diffraction pattern or the occultation curve observed at a wavelength λ is given by the convolution of the brightness distribution $b(\theta)$ of the source along the line joining the Moon's centre to the source and the point source response. The point source response is the Fresnel diffraction pattern $p_\lambda(\theta)$, given by

$$p_\lambda(\theta) = \tfrac{1}{2}\left[\{1+C(\theta/\theta_0)\}^2 + \{1+S(\theta/\theta_0)\}^2\right],$$

where C and S are the Fresnel integrals, given by

$$C(x) = \int_0^x \cos\frac{\pi u^2}{2}\,du\ ,\quad S(x) = \int_0^x \sin\frac{\pi u^2}{2}\,du$$

and the angular scale $\theta_0 = \sqrt{\lambda/2D}$ is about 7.2 arcsec for a wavelength of 0.92 m at which the Ooty telescope operates. Figure 1.1 shows a plot of $p_\lambda(\theta)$ as a function of θ/θ_0. The diffraction effects can be neglected if the angular sizes of the components of a source are much greater than θ_0.

The true limb profile of the Moon is irregular, with deviations from the mean limb of up to about 3 arcsec. However, the effects of these irregularities on the shape of the occultation curve are usually negligible except for the grazing

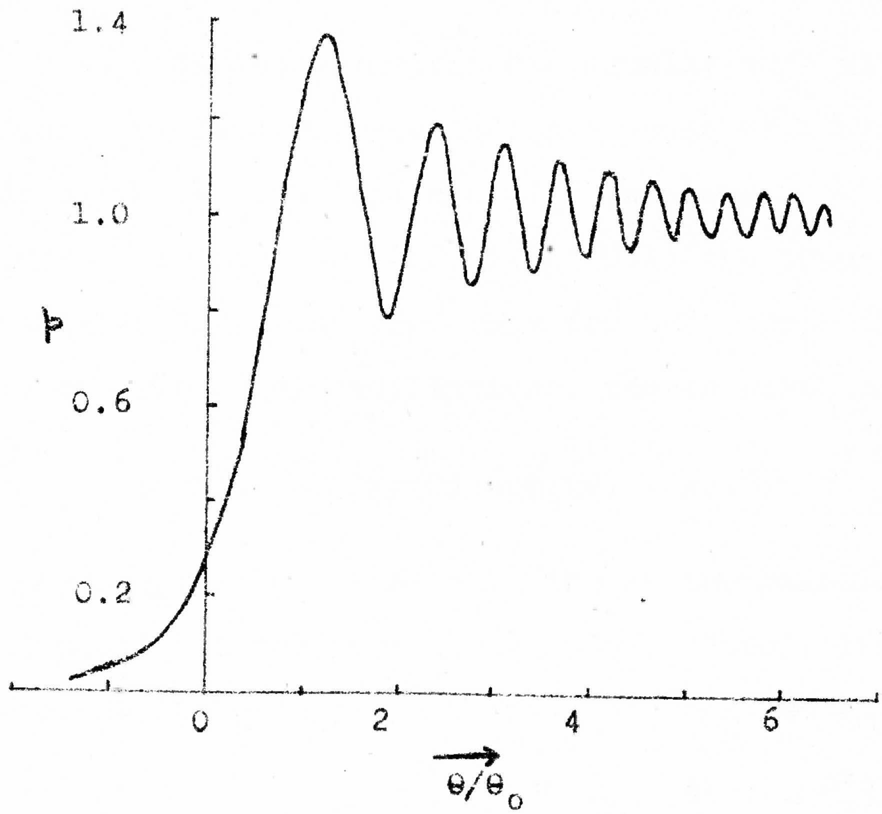

Figure 1.1 : Lunar occultation response of a monochromatic point source as a function of angular distance from the edge of geometric shadow. The angular scale of the pattern is given by $\theta_o = \sqrt{(\lambda/2D)} = 7.2$ arc sec for $\lambda = 0.92$ m.

occultations, and will not be considered in this thesis. A treatment of these effects can be found in Evans (1970).

The observations are usually made with a receiver of finite bandwidth. For the usual case of a symmetric passband of width $\Delta\lambda$ centred around a wavelength λ, such that $\Delta\lambda/\lambda << 1$ (say less than about 0.2) the true $p(\theta)$ is significantly different from that for the monochromatic case only in the region of oscillations, and is given by

$$p(\theta) = p_\lambda(\theta) \cdot g_\lambda(\theta)$$

where $g_\lambda(\theta)$ is essentially the cosine transform of the band-shape of the receiver (Lang 1969). Thus for a symmetric rectangular passband of width $\Delta\lambda$ centred at λ,

$$g_\lambda(\theta) = \frac{1}{\pi} \frac{\theta_o^2}{\pi\theta^2} \frac{\lambda}{\Delta\lambda} \sin\left(\frac{\Delta\lambda}{\lambda} \cdot \pi \frac{\theta^2}{\theta_o^2}\right)$$

The bandwidth effects are negligible for sources of angular sizes greater than about $\theta_o\sqrt{\Delta\lambda/\lambda}$. Effects of bandwidths of other shapes have been considered by Scheuer (1965), Lang (1969) and Krishnan (1970).

1.3 RESTORATION OF BRIGHTNESS DISTRIBUTION FROM THE OCCULTATION CURVE

In the next few sections a brief review of the various methods of restoring the strip brightness distribution $b(\theta)$ of a source from its occultation curve will be given. The methods treated in this section (except model-fitting which has a limited application) can all be regarded as classical methods in the sense that they aim at retrieving only the information contained in the occultation curve. Better results can be achieved by considering *a priori* information available about the source but not given by the occultation curve, which is the subject of the next chapter.

1.3.1 Library Methods and Model Fitting : If the source has a simple structure which can be fully described in terms of a few parameters, it can be retrieved by obtaining the parameters by a least squares fit to the observations. In other words, the parameters are evaluated by requiring a minimum value for the variance of residuals, the deviations of the computed occultation curve from the observed curve. This method is usually employed in optical astronomy in the analysis of the occultation data of stars since the sources can usually be modelled in terms of a few parameters, say $\lesssim 5$ (Nather and McCants 1970).

A simplification of this procedure can be called the library method, which involves using a library of theoretical occultation curves of some standard structures of sources, and comparing them with the observed curves to determine the nearest standard structure agreeing with the observations.

Some of the earliest occultations of radio sources were analysed by this method (Hazard 1962; Hazard et al. 1963).

1.3.2 Lobe Analysis : During an occultation, the Moon can be considered to act as a variable baseline interferometer whose baseline is given by the instantaneous distance of the lunar limb from the line joining the source to the observer. If each lobe in the diffraction fringe is approximated locally by a sinusoid, the amplitude and period of the lobe give respectively the amplitude and phase of one Fourier component of the source brightness distribution.

In the presence of noise, the main difference between an ideal variable baseline interferometer and the occultation is that the signal-to-noise ratio in an occultation curve decreases monotonically for higher frequency Fourier components. In fact, it is this decrease in the signal-to-noise ratio which sets a limit to the resolution which can be obtained by using the occultation technique. A detailed treatment of this method in practical situations has been given by Lang (1969).

Both lobe analysis and model fitting techniques are adequate provided there is already some idea of the source structure and the source is not too complex. For complex sources, and particularly for sources with several components of unequal sizes, these methods are not practical. For such cases, the most powerful classical method is that due to Scheuer, which is discussed in the next section.

1.3.3 Scheuer's Method : Scheuer (1962) showed that the brightness distribution of a source as seen with a fan beam $g(\theta)$ can be easily recovered from the observed occultation profile by simply convolving it with a restoring function,

$$- p''(-\theta) * g(\theta) ,$$

which is a convolution of the second derivative of the point source response with a smoothing function $g(\theta)$ representing the equivalent fan beam. Smoothing is necessary in order to overcome the divergence of the second derivative. Scheuer also showed that the attainable resolution is limited only by the signal-to-noise ratio of the observations. In particular, the restoring accuracy does not depend directly on the frequency of observation and is the same as that in the absence of diffraction.

Scheuer's method is attractive because of the ease with which it can be applied and because it can be applied without any knowledge of the structure of the source. This has been used for the reduction of the occultations of several hundred radio sources observed at Ooty. A detailed discussion of the restoring accuracy in terms of the observational signal-to-noise ratio and the instrumental parameters like time constant and bandwidth has been given by von Hoerner (1964) and reviewed by Hazard (1976). The expressions for restoring accuracy are given in the next section.

1.3.4 Restoring Accuracy for Scheuer's method : In this section we summarise the important formulae for the restoring accuracy of Scheuer's Method. It is assumed that the smoothing function used in the restoring function is a Gaussian of half-power-width β_r which defines the resolution. The observational signal-to-noise ratio for an integration of 1 arcsec denoted by q_o, corresponds to an integration time of about 3 sec for a typical occultation. Also, q_r denotes the signal-to-noise of the restored profile for a point source. The measured values of the position, half-power-width and flux of the source (assumed to be a Gaussian) are denoted by θ, β and F respectively. The expressions relating the accuracy of the restored parameters are given below:

(1) Signal-to-noise ratio on the restored output:

$q_r = 0.692 q_o \sqrt{\beta_r}$, β_r in arcsec

(2) Accuracy of position, width and flux: In the following formulae, Δ denotes the rms error in the quantity denoted by the symbol immediately following it:

$$\frac{\Delta\Theta}{\beta} = 0.8/q_o\sqrt{\beta_r} = 0.6/q_r$$

$$\Delta\beta/\beta = 1.4/q_r$$

$$\Delta F/F = 1.4/q_r$$

The diameter of the source φ is given by $\sqrt{\beta^2-\beta_r^2}$ and its rms error $\Delta\varphi$ is given by:

$$1+\Delta\varphi/\varphi = (1+2\beta\Delta\varphi/\varphi^2)^{\frac{1}{2}} = [1+\frac{3}{q_r}(1+\beta_r^2/\varphi^2)]^{\frac{1}{2}}$$

3) Minimum attainable resolution (β_s) defined as that giving $q_r = 5$:

$$\beta_s = 52/q_o^2$$

4) Instrumental limitations on the achievable resolution:

Bandwidth limitation:- $\beta_r \gtrsim 0.51\sqrt{B\lambda}$ arcsec for a rectangular passband of width B expressed as a percentage of the frequency and λ is in metres.

Finite size of the antenna:- $\beta_r \gtrsim d/D = 0.05d$ arcsec with the antenna size d expressed in 100m (D is the distance to the Moon).

1.4 THE PRESENT WORK

A detailed description of a new method suggested for deconvolution from noisy data is given in the next chapter. The difficulties of deconvolution arising from the loss of information which results from the smoothing nature of the convolution are briefly discussed. The main philosophy of the new method, referred to as the Optimum Deconvolution Method (ODM), is to make use of *a priori* information available on the solution. This enables one to retrieve a part of the information that was lost in the convolution process. Chapter 2 gives a detailed description of the method and its application to the lunar occultation data. From the analysis of a number of computer simulations and 20 occultations observed at Ooty, it is concluded that ODM can provide significantly better resolution, often by a factor of 2, and a restored profile which can be interpreted more objectively than the corresponding profile from Scheuer's method.

The results on the lunar occultation observations and optical identifications of 63 radio sources, mostly in the flux range 0.25 to 1 Jy at 327 MHz, are presented in Chapter 3. Angular structures of the sources have been derived using both Scheuer's method and ODM. A comparison between ODM and Scheuer's method is presented for 14 of these occultations by illustrating the brightness distributions restored by these two methods.

The cosmological investigations of angular sizes of radio sources are presented in Chapter 4. It contains a statistical analysis of the $N(S,\theta)$ distribution for a total of

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The cosmological investigations of angular sizes of radio sources are presented in Chapter 4. It contains a statistical analysis of the $N(S,\theta)$ distribution for a total of

513 extragalactic radio sources including 283 sources in the range S_{408} = 0.25 to 5 Jy taken from the Ooty Occultation survey and the remaining sources from the 3CR and All-Sky surveys. The histograms of N(S,θ) distributions are compared with the theoretical predictions of Einstein de-Sitter and the Steady State cosmologies. It is found that the data are inconsistent with the predictions of the Steady State model and indicate the presence of strong evolutionary effects in both the numbers and physical sizes of radio sources. The effects of inhomogeneities in the Universe are also considered and it is shown that the conclusions are not significantly affected.