# CHAPTER 4

# ANGULAR SIZE - FLUX DENSITY RELATION AND COSMOLOGY

## 4.1 INTRODUCTION

Radio sources offer an important arena for observational cosmology because of their large distances. The observed large scale properties of radio sources are affected both by the geometry of the Universe and the evolution of these sources with cosmological epoch. The present scatter in the observational data arising from the intrinsic distributions of source properties and the uncertainties in their evolution allows a large number of possible world models. Still, the continuing availability of information on a large number of radio sources situated at cosmological distances promises to restrict the range of acceptable models considerably. The recent efforts in observational cosmology are inclined towards an understanding of the evolutionary effects since their influence on observations dominate over the geometric differences between world models.

The use of radio sources in observational cosmology began with the source counts, whose study initiated about two decades ago by Ryle and his colleagues at Cambridge gave evidence for an evolutionary Universe. This conclusion got further support from the discovery of microwave background radiation by Penzias and Wilson (1965). The conclusions derived from source counts are sometimes questioned because of the uncertain distances of radio sources, possible anisotropies in their distribution and the conversial nature of the origin of redshifts of quasars. These doubts may be expected to be settled in the near future in view of the remarkable progress recently being witnessed in the radio source surveys and optical identifications, as reviewed extensively at the IAU Symposium No. 74 on Radio Astronomy and Cosmology (Jauncey 1977).

Apart from the source counts, several other cosmological tests are possible with radio sources as described in Chapter 1. In particular, the angular sizes ($\theta$) now being available from high resolution observations of large number of radio sources, provide an important input to the cosmological tests apart from the flux densities (S). There is an appreciable scatter in the angular size data arising from the distribution of their linear sizes and projection effects. However, as shown by Swarup (1975) by a comparison of weak sources observed at Ooty by lunar occultations with the stronger sources observed by other workers, the median value of angular sizes ($\theta_m$) is well correlated with S. The median angular size was found to decrease with flux density indicating that the weaker sources have statistically smaller angular sizes and possibly attain a steady value of about 10 arc sec at $S_{408} \sim 1$ Jy. This result was combined by Kapahi (1975) with the angular size counts $N(\theta)$ of the 3 CR sources to obtain an independent evidence for the evolution of both luminosities and linear sizes of radio sources.

In this chapter, an improved $\theta_m(S)$ diagram is presented by including the angular sizes of 119 more sources observed at Ooty during 1973-74 and incorporating the angular size data on 3CR sources from recent literature. In addition, the detailed $N(S,\theta)$ distributions are compared with the predictions of both Steady State and evolutionary world models by a formal chi-square test. The analysis shows that the angular size data can only be explained by assuming evolution in both luminosity and linear sizes of radio sources.

## 4.2 ANGULAR SIZE - FLUX DENSITY RELATION

4.2.1 Observational data: The data used for the present analysis consist of 513 sources outside the galactic plane ($|b^{II}| > 10^{o}$) whose details are given below:

(a) 62 sources with $S_{408} \geq 16.5$ Jy (Wyllies' scale) are from the All-Sky Catalogue covering 10.2 sr of the sky (Robertson 1973).

(b) 195 sources belonging to the 3CR catalogue form the sample in 4.2 sr (Mackay 1971; Longair and Gunn 1975), including 27 sources from the above All-Sky sample. The data for these sources have generally been taken from the recent observations made with the Cambridge 5-km radio telescope (Pooley and Henbest 1974; Riley and Pooley 1975; Jenkins et al. 1977).

(c) The remaining 283 sources were obtained from the lunar occultation observations made at 326.5 MHz with the Ooty Radio telescope during 1970-71 and 1973-74. The sample includes all the sources observed during this period such that at least two strip-scans were available along directions separated by more than $30^{o}$ for each source. For these sources, $\theta$ was defined to be the 'largest angular size' - the component-separation of double sources or the maximum observed angular size for single sources (Miley 1971; Swarup 1975). The data were taken from the following lists:

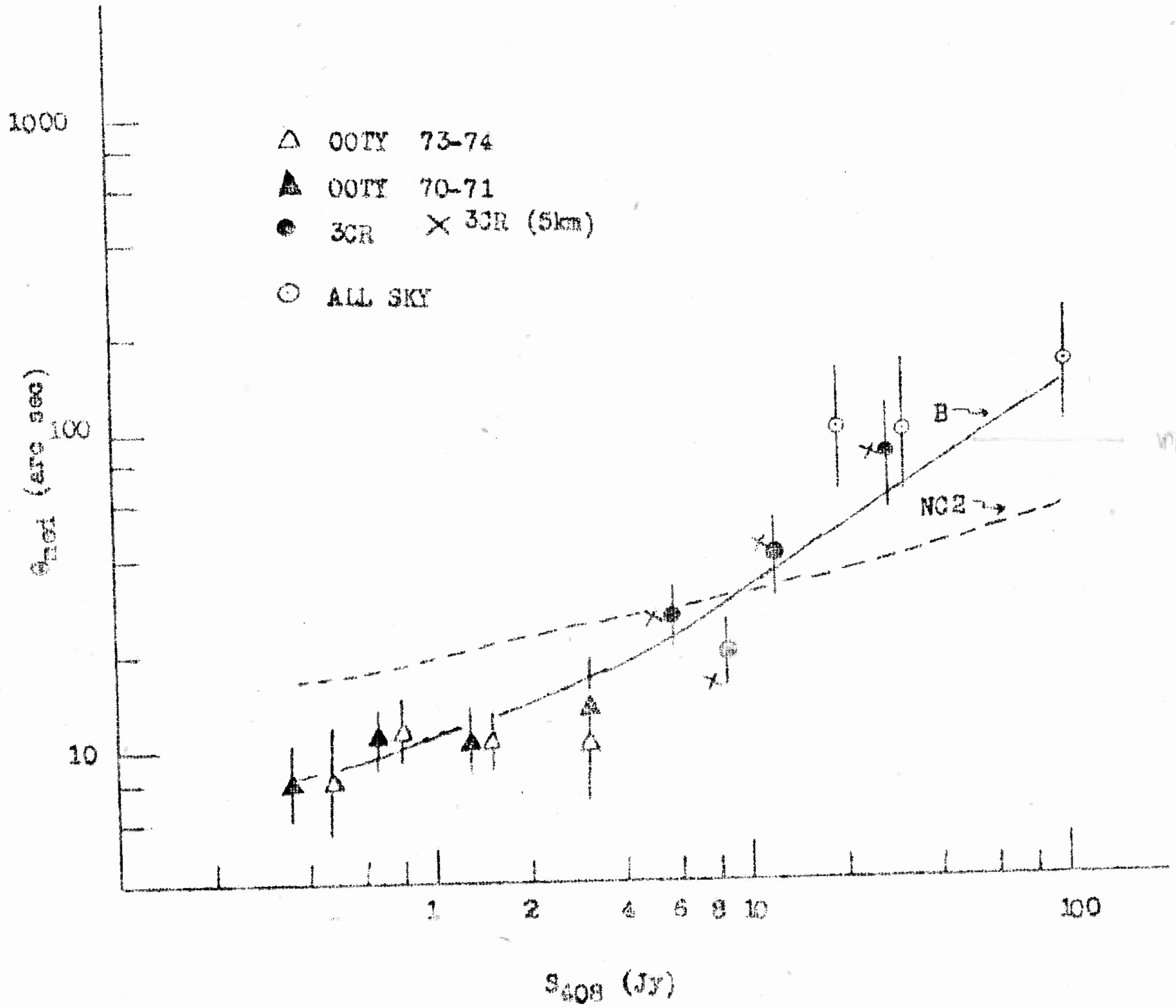


Figure 4.1 : $\theta_m(S)$ for the Ooty, 3CR and All-Sky samples along with the predictions of model B (solid lines) and the Steady State model NC2 (broken lines)

Swarup et al. (1971a)
Kapahi (1971)
Kapahi et al. (1972)
Kapahi et al. (1973)
Kapahi et al. (1973a)
Joshi et al. (1974)
Kapahi et al. (1974)
Subrahmanya and Gopal-Krishna (1977)
Singal et al. (1977)
Venkatakrishna and Swarup (1977).

4.2.2 $\theta_m(S)$ Relation : As shown by Swarup (1975), the median value of angular sizes is well correlated with flux density implying that weaker sources are statistically smaller in angular size. This was used by Kapahi (1975, 1975a) along with the angular size counts $N(\theta)$ for 3CR sources to derive the evolutionary parameters for an assumed model of radio luminosity and size functions in Einstein de-Sitter Universe. Their analyses were based on the comparison of the angular sizes of a smaller sample of 163 Ooty sources observed during 1970-71 with those of stronger sources.

The $\theta_m(S)$ relation for the present sample of 513 sources is shown in Figure 4.1 along with the predicted curves for evolutionary and Steady State models discussed below (models B and C) with the optimum choice of parameters. The revised observational points are in very good agreement with those derived earlier (Swarup and Subrahmanya 1977).

As shown by Kapahi (1975, 1975a) it is not possible to explain the $\theta_m(S)$ relation in terms of a simple Steady State model. But the recent model of Narlikar and Chitre(1977) with a luminosity-dependent linear size predicts an acceptable relation without any need for evolution. However, it will follow from the discussion given in Sections 4.3.3 and 4.3.4

that this model is not supported by a more detailed consideration of the available data on the source counts or angular size counts.

4.2.3 N(S,θ) Distribution: Since the $\theta_m(S)$ relation does not reveal the actual distribution of the angular sizes of sources, a rigorous test of a model should compare its predictions with the observed N(S,θ) distribution - the number of sources in specified intervals of S and θ. For this purpose, the data on the 513 sources in our sample have been grouped into 8 ranges of S and in each of these, 17 bins of angular sizes have been considered with $\log \theta \leqslant 0$, 0.2, 0.4, ..., 2.8, 3.0 and $> 3.0$. The observed numbers in these 136 bins are compared with model predictions.

The model predictions of N(S,θ) depend on the Radio Luminosity Function (RLF) and the Radio Size Function (RSF). RLF, denoted by $\eta(P,z)$ is the spatial density per unit interval of P, of the sources of luminosity P at a given epoch z, and can be factored into

$$\eta(P,z) = \eta_o(P) \cdot F(P,z) ,$$

where $F(P,z)$ is the evolution function and $\eta_o(P)$ is the local luminosity function. In the absence of evolution, $F(P,z) = 1$ by definition.

The RSF, denoted by $\varphi(\ell)$ is the fraction per unit size interval of the sources (of a given luminosity) having a projected linear size $\ell$ at the epoch z. This distribution arises from projection effects and the various factors responsible for the intrinsic size distribution like the spread in the initial conditions of formation of a source, and the physical processes governing the expansion and confinement of the source during its evolution. Since most of these factors are not well understood at present, we can only consider a simplified model consistent with the local size function which can be approximately inferred from the limited sample of nearby sources.

If we know the RLF and RSF, we can compute the number of sources $N(>S, >\theta)$ with flux density $>S$ and angular size $>\theta$ as:

$$N(>S, >\theta) = \int_0^\infty \eta_o(P)\,dP \int_0^{z(P,S)} F(P,z)\,dV \int_{\ell(\theta,z)}^\infty \varphi(\ell)\,d\ell$$

The relations $dV(z)$, $z(P,S)$ and $\ell(\theta,z)$ are defined by the geometry in the assumed world model (see e.g. Weinberg 1972). In this chapter we have used only the Einstein de-Sitter and Steady State geometries for which these relations are as follows:

| | | Einstein de-Sitter | Steady State |
|---|---|---|---|
| $\dfrac{dV/dz}{4\pi(c/H)^3}$ | = | $\dfrac{4[1-(1+z)^{-\frac{1}{2}}]^2}{(1+z)^{3/2}}$ | $z^2/Y^3$ |
| $\dfrac{(H/c)^2 \cdot P/S}{(1+z)^{1+\alpha}}$ | = | $[1-(1+z)^{-\frac{1}{2}}]^2$ | $z^2$ |
| $\dfrac{(c/H)\theta}{\ell(1+z)}$ | = | $\dfrac{1}{2[1-(1+z)^{-\frac{1}{2}}]}$ | $1/z$ |

## 4.3 COMPARISON OF OBSERVATIONS OF N(S,θ) WITH MODEL PREDICTIONS

In this Section, the observed angular size - flux density distributions are compared for 3 types of models of which two (A and B) are evolutionary models using Einstein de-Sitter geometry and the other (C) uses the Steady State geometry.

4.3.1 Model A : In this model, the local luminosity function is approximated by a 3-slope power law and a power-law density evolution is assumed only for high luminosities. This form was suggested by Kapahi(1977) as an improvement over his earlier model (Kapahi 1975). The local size function for model A is taken from Kapahi(1977) who determined it from the observed distribution of projected linear sizes of nearby galaxies. The luminosity and size functions and the optimum parameters of this model determined by the chi-square analysis described in Section 4.3.4 are given in Table 4.1. The N(S,θ) distributions predicted by the model with optimum choice of parameters are shown in Figure 4.2 along with the observed numbers. The crosses indicate the number of sources for which definite values of θ were available and the vertical bars indicate the number for which only upper limits were given. The predicted distributions (solid curves) are seen to agree reasonably well with the observations. However, it may be noted here that the luminosity function used in this model does not reproduce the observed source counts particularly at low flux densities (Swarup 1977).

Table 4.1 : The Parameters of Model A

(a) Local Luminosity Function:

$$\eta_o(P) \propto \begin{cases} P^{-\gamma_o} , & P_o \leqslant P \leqslant P_1 \\ P^{-\gamma_1} , & P_1 \leqslant P \leqslant P_m \\ P^{-\gamma_2} , & P_m \leqslant P \leqslant P_u \\ 0 , & P < P_o \text{ or } P > P_u \end{cases}$$

with $P_o = 10^{-5} P'$ , $P_1 = 0.04 P'$ ) $P' = 10^{26}$ W/Hz/sr

$P_m = 2\ P'$ , $P_u = 200\ P'$ ) at 178 MHz

$\gamma_o = 1.25$ , $\gamma_1 = 2.3$ , $\gamma_2 = 2.9$

(b) Density Evolution:

$$F(P,z) = \begin{cases} 1 , & P < P_m \text{ and } z \leqslant z_c \\ (1+z)^{\beta} , & P \geqslant P_m \text{ and } z \leqslant z_c \\ 0 , & z > z_c , \text{ where} \\ & z_c \text{ is the redshift cutoff} \end{cases}$$

with $\beta = 5.5$ , $z_c = 3.0$

(c) Radio Size Function (RSF):

$\varphi(\ell) = (1/\ell_o) \cdot \exp(-\ell/\ell_o)$ , where

$\ell_o(z) = \ell_o(0) \cdot (1+z)^{-n}$

with $\ell_o(0) = 0.3$ Mpc ; $n = 1.0$

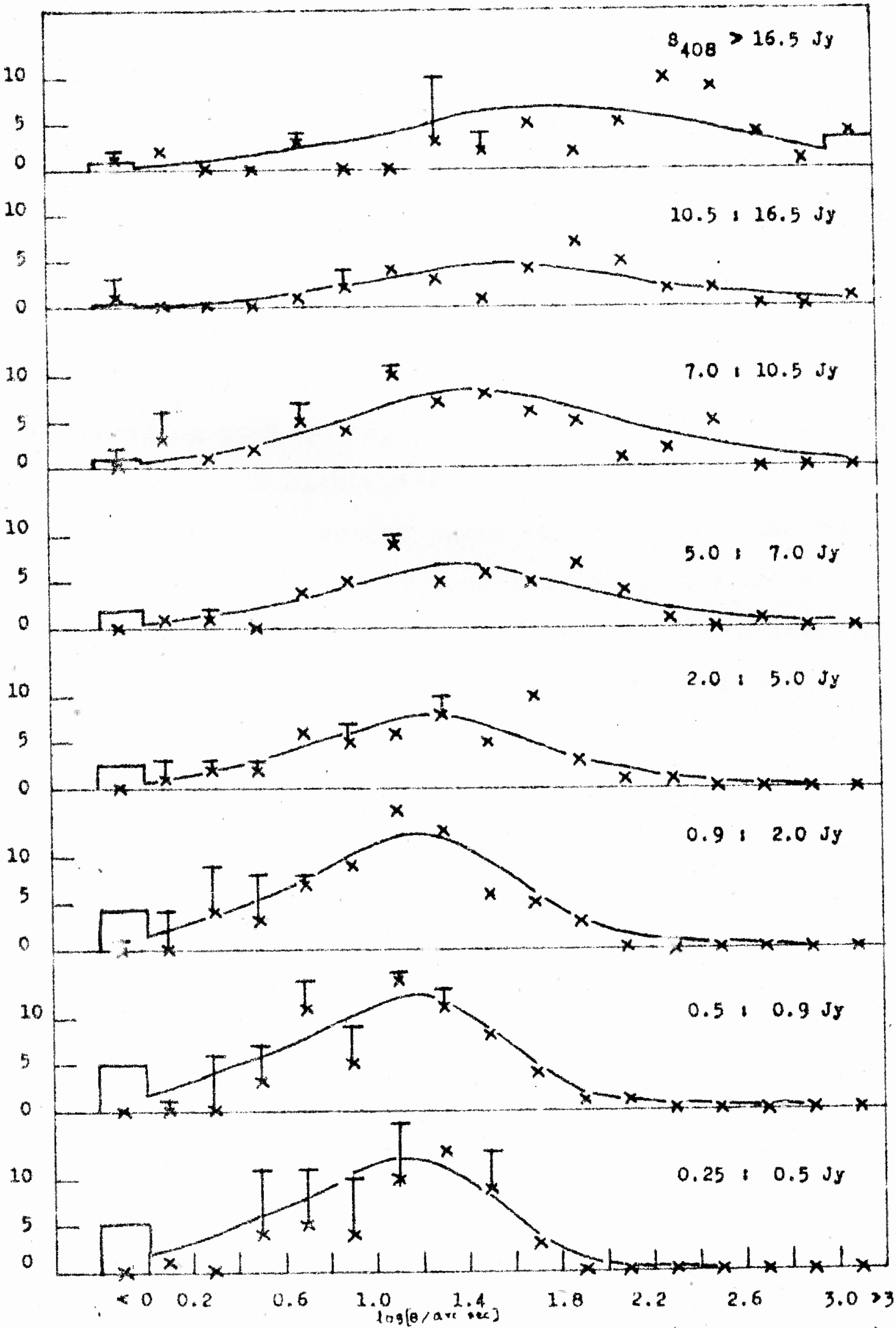


Fig. 4.2 : Comparison of N(S,θ) predicted by Model A (solid curves) with the observed distributions of the angular sizes of the 513 sources in 8 ranges of S. The observed number of sources is indicated by crosses (definite values of θ) and vertical lines (upper limits).

4.3.2 Model B : The simple evolutionary scheme of model A has two shortcomings: (a) the resulting RLF has a discontinuity at $P_m$ , and (b) the assumed RLF cannot reproduce the observed n(S) satisfactorily for any choice of parameters (Wall et al. 1977). Both these limitations are overcome in model B by choosing a luminosity (corresponding to model 4b of Wall et al. 1977) which is known to give a good agreement with n(S) ranging down to $S_{408} \sim 10$ mJy. Details of model B are given in Table 4.2 along with the optimum choice of parameters. In order to get an agreement with the observed $N(\theta)$ in this model, we found it necessary to introduce a dependence of linear size on P. Since the available data on redshifts of galaxies are not sufficient to determine the complete luminosity - linear size relation, we used a simple form of this dependence. We have chosen the exponential size function as in model A but have assumed that the e-folding size is 0.4 Mpc for $P > P_1$ and 0.1 Mpc for $P < P_1$. These are within about 20 per cent of the 37-percentiles inferred from the distribution of luminosities and linear sizes of nearby galaxies with measured redshifts as given by Kapahi(1977).

The $N(S,\theta)$ distributions predicted by model B for the optimum choice of parameters are shown in Figure 4.3 along with the observed counts as in the previous figure. It can be seen from the figure that the model agrees reasonably well with the observations.

Table 4.2 : The Parameters of Model B

---

(a) Local Luminosity Function: This is determined from the luminosity distribution of a complete sample of sources with $S_{408} \geq 10$ Jy as given in Wall et al. (1977).

(b) Density Evolution:

$$F(P,z) = \begin{cases} \exp[m(P)\cdot(1-t/t_o)] , & z \leq z_c \\ 0 \quad \text{if} \quad z > z_c \end{cases}$$

where

$$m(P) = \begin{cases} 0 , & P \leq P_1 \\ \dfrac{M(\log P - \log P_1)}{\log P_2 - \log P_1} , & P_1 \leq P \leq P_2 \\ M , & P \geq P_2 \end{cases}$$

and t is the 'cosmic time', given by

$$t/t_o = (1+z)^{-3/2}$$ for Einstein de-Sitter geometry.

Optimum values:

$P_1 = 10^{25}$ W/Hz/sr
$P_2 = 10^{27.3}$ W/Hz/sr ) at 408 MHz

$M = 11.0$ ; $z_c = 3.5$

(c) Radio Size Function : This is the same as that in model A, but linear size is assumed to depend on P as follows:

$$\ell_o(0) = \begin{cases} 0.1 \text{ Mpc} , & P < P_1 \\ 0.4 \text{ Mpc} , & P \geq P_1 \end{cases}$$

Optimum value of $n = 1.4$

---

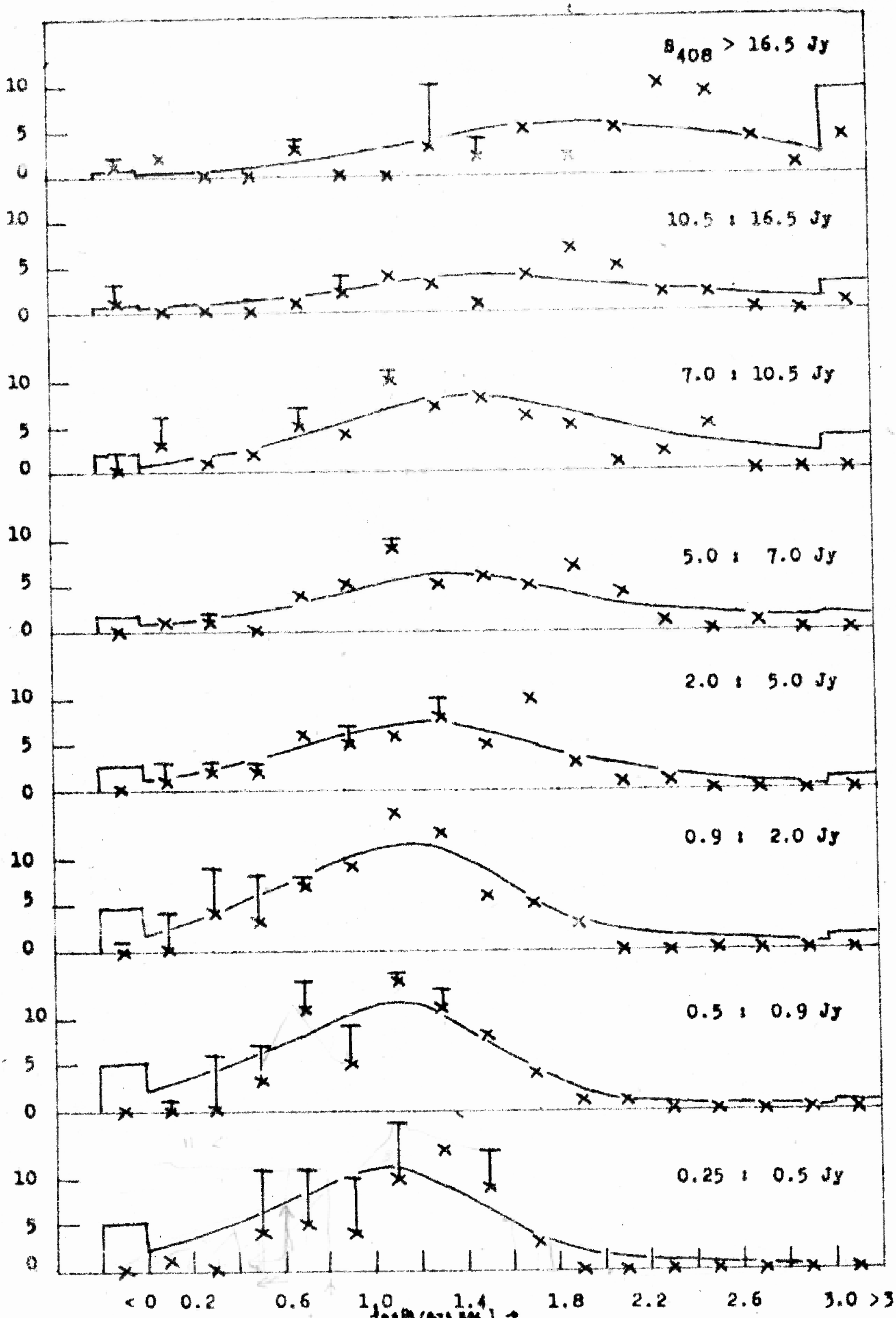


Fig. 4.3 : Comparison of N(S,Θ) predicted by Model B (solid curves) with the observed distributions of the angular sizes of the 513 sources in 8 ranges of S. The observed number of sources is indicated by crosses (definite values of Θ) and vertical lines (upper limits).

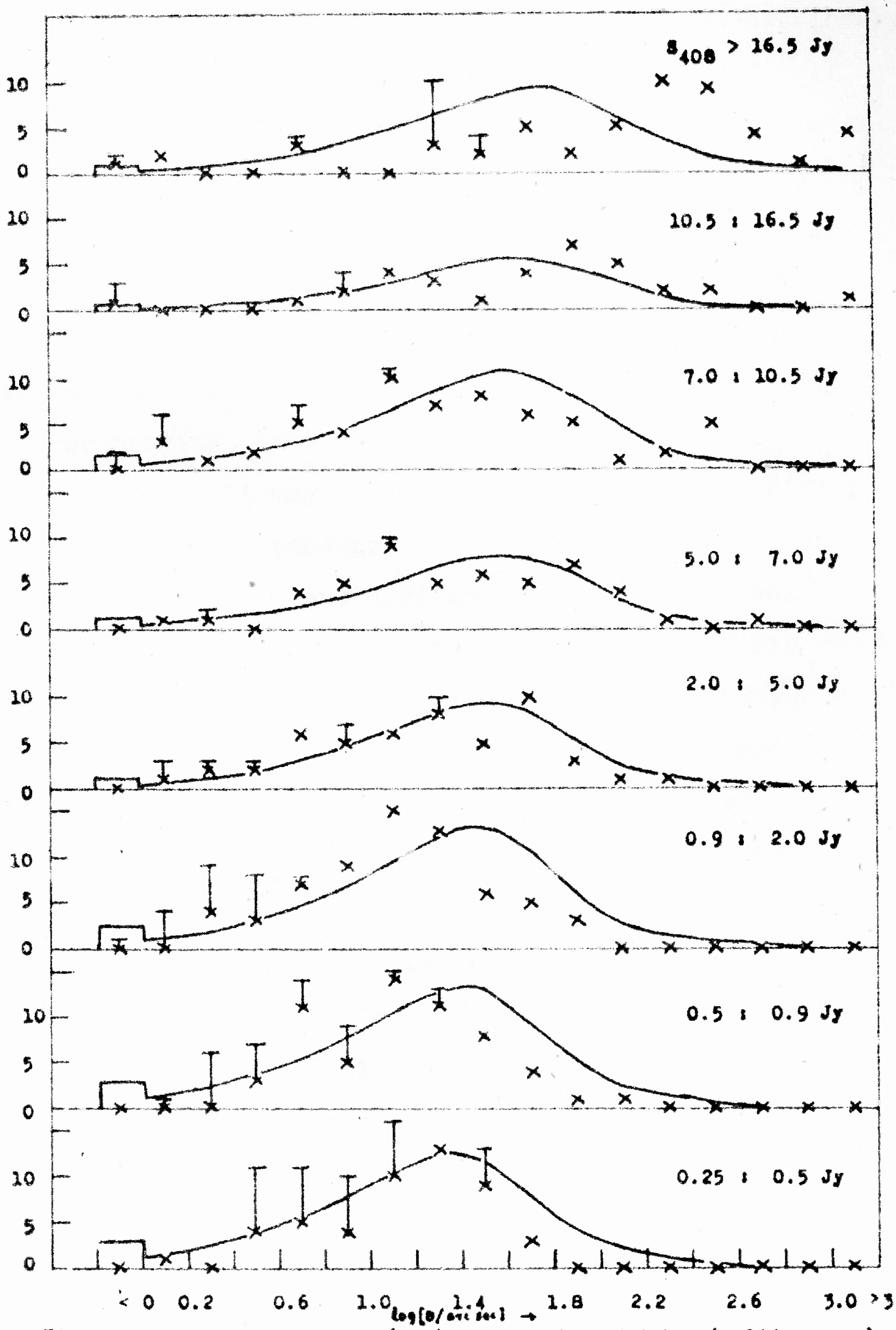


Fig. 4.4 : Comparison of N(S,θ) predicted by Model C (solid curves) with the observed distributions of the angular sizes of the 513 sources in 8 ranges of S. The observed number of sources is indicated by crosses (definite values of θ) and vertical lines (upper limits).

4.3.3 Model C : In this model we consider a non-evolutionary model using the Steady State geometry. Recently, Narlikar and Chitre(1977) have examined the possibility of explaining the angular size statistics in the Steady State model by introducing a dependence of linear size on luminosity. In model C, we consider the two cases studied by them: the original set of parameters suggested in Narlikar and Chitre (1977), and the revised set suggested in the ERRATUM to their paper. For brevity, these will be referred to as NC1 and NC2 respectively. It may be noted that NC1 and NC2 differ only in the assumed dependence of linear size on P. The details of model C (NC1 and NC2) are summarised in Table 4.3. The predictions of this model were compared by Narlikar and Chitre(1977) with the observed angular sizes of 298 sources taken from the earlier sample used by Swarup(1975) and Kapahi(1975). For this purpose, they grouped the data into 16 bins of angular size and flux density intervals and performed a chi-square test to compare their model with that of Kapahi(1975) using a linear size evolution given by $n = 1.5$ for his model. From this comparison, they concluded that a non-evolutionary model can fit the angular size data with a significance level comparable to that of the evolutionary model of Kapahi(1975) with $n = 1.5$. We have critically examined the model proposed by Narlikar and Chitre(1977) and comment as follows:

(a) The assumed luminosity function is inconsistent with the local luminosity function which is reasonably well established by the recent identification of a complete sample of 3CR sources (Fanti and Perola 1977 ; Wall et al. 1977). Further, it is well known that the Steady State model cannot be reconciled with the source counts in any known form. This has been recognised by the authors themselves.

(b) In the absence of a luminosity - linear size dependence, the Steady State model was ruled out by Kapahi(1977) by considering the $N(\theta)$ for the 3CR sources. The authors pointed out that this situation is not true for NC1 which predicts a slope of -1.34 for log N - log $\theta$ in the region of 100 to 1000 arc sec as against the observed value of $-1.1 \pm 0.25$. However, NC2 predicts a slope of -1.5 which is inconsistent with the observations.

(c) For the NC1 model, the authors had originally claimed that that the chi-square value for the 16 bins considered by them was 24.85. However, our computation for their model did not check with this conclusion and a discussion with the authors revealed that there was a computational oversight in the numbers given in their paper. The true value of $\chi^2$ for NC1 is about 40, which gives a significance level less than 0.1 per cent for the model and hence rules it out. This has been pointed out by the authors in an erratum to their paper where they suggested the use of NC2 which has a significance level of about 10 per cent as inferred from the $\chi^2$ for their 16 bins. However, this model is ruled out by the consideration of $N(\theta)$ as discussed in the previous paragraph. The computed $N(S,\theta)$ distributions for NC2 are shown in Figure 4.4 along with the observed distributions as in the previous figures. From this it is clear that the agreement of the model with the observations is by no means reasonable. This is also apparent from the chi-square analysis described in the next section which gives a significance level less than $10^{-4}$ for NC2.

Thus the conclusion of the authors with their 16 bins comprising of sources from a smaller sample than the present is not supported by a more detailed comparison.

Table 4.3 : The Parameters of model C

---

(a) Radio Luminosity Function:

$$\eta(P) \propto \begin{cases} P^{-1.9} , & 0.001 \leqslant P/P_m \leqslant 1.0 \\ P^{-2.1} , & 1.0 \leqslant P/P_m \leqslant 200 \\ 0 & \text{otherwise,} \end{cases}$$

where $P_m = 10^{26}$ W/Hz/sr at 178 MHz

(b) Radio Size Function: The assumed size function is similar in form to the local size function of Kapahi(1975), i.e., the actual linear sizes of the sources are assumed to decrease linearly with the size with a maximum linear size of $\ell_o$. For projected linear sizes, this gives:

$$\varphi(\ell) = (2/\tau)\cdot[\text{arc cos } \tau - \tau \ln(1 + \sqrt{1/\tau^2 - 1})]$$

with $\tau = \ell/\ell_o$

The linear size is assumed to depend on luminosity as:

$$\ell_o = \begin{cases} s_1 , & P/P_m \leqslant 0.1 \\ s_2 , & 0.1 < P/P_m < 1.0 \\ s_3 , & P/P_m \geqslant 1.0 \end{cases}$$

with $s_1 = 0.5$ Mpc , $s_2 = 0.6$ Mpc ,
and $s_3 = 0.75$ Mpc for NC1 ; and
$s_1 = s_2 = 0.4$ Mpc , $s_3 = 0.75$ Mpc for NC2.

---

Table 4.4 Chi-square values for the three models discussed in the text

| No. of bins | Model A | | | | | Model B | | | | Model C (NC2) |
|---|---|---|---|---|---|---|---|---|---|---|
| | n=0.8 | 1.0 | 1.1 | 1.2 | 1.5 | n=1.0 | 1.2 | 1.4 | 1.6 | |
| 65 | 79 | 71 | 73 | 78 | 117 | 98 | 86 | 84 | 93 | 199 |
| 40 | 57 | 47 | 48 | 51 | 81 | 77 | 63 | 58 | 62 | 162 |
| 27 | 33 | 24 | 24 | 27 | 54 | 50 | 37 | 32 | 35 | 118 |
| 21 | 30 | 21 | 20 | 22 | 47 | 40 | 27 | 22 | 23 | 125 |

Table 4.5 Chi-square values for some standard significance levels

| No. of bins | Significance levels | | | | |
|---|---|---|---|---|---|
| | 0.25 | 0.10 | 0.05 | 0.01 | 0.005 |
| 65 | 71.2 | 78.9 | 83.7 | 93.3 | 96.9 |
| 40 | 44.5 | 50.7 | 54.6 | 62.4 | 65.5 |
| 27 | 30.4 | 35.6 | 38.9 | 45.6 | 48.3 |
| 21 | 23.8 | 28.4 | 31.4 | 37.6 | 40.0 |

<u>4.3.4 Chi-Square Analysis</u> : For a statistical evaluation of the agreement of the three models discussed above with the observations, we used a chi-square analysis similar to that used by Swarup and Subrahmanya(1977) and Narlikar and Chitre(1977). The $\chi^2$ is defined as :

$$\chi^2 = \sum_{\text{all bins}} (n_o - n_e)^2 / n_e$$

where $n_o$ and $n_e$ are respectively the observed and expected numbers for the assumed model for the bin in question. In order to obtain a stable statistic, adjacent $\theta$-bins were merged together to obtain 4 different groupings of bins which contained a total of 65, 40, 27 and 23 bins such that the minimum expected number of sources in a bin for model B (n = 1.4) was respectively 5, 10, 15 and 20 for these groupings. The merging was done uniformly starting from the largest angular size such that the criterion of minimum expected number was reached. However, the bins with the lowest values of $\theta$ ( $<$ 4 arc sec) in the Ooty sources which had a large fraction of unresolved sources were merged even if the original expected numbers exceeded the stipulated minimum.

Since the area of sky covered by the Ooty survey is not known accurately, it is necessary to introduce empirically the 8 normalisation constants required to get the expected numbers in each S-range. This could be done in two ways: (a) one can require that the total expected number of sources in <u>each</u> S-range should equal the observed number -- this implies a choice of the area as given by the reasonably well-established n(S) relation which is quite justifiable since the Ooty survey is unbiased and the n(S) relation is based on complete surveys; (b) alternatively, the normalisation can be determined by minimising $\chi^2$ under the condition that the total expected number of sources in all the bins should equal the observed number (513 for our sample). For our data, these two methods gave almost the same normalisations and we found no need to differentiate between the two methods. Hence, in the subsequent discussion, we will assume that the second criterion (minimum $\chi^2$) has been used and that the number of degrees of freedom for a grouping consisting of K bins is K-1.

The results are summarised in Table 4.4 which gives the $\chi^2$ values for all the four groupings of bins mentioned above for the three models A, B and C. For convenience of judging the fit, we have given in Table 4.5 the $\chi^2$ values corresponding to 5 significance levels from 0.005 to 0.25 for the four groupings chosen by us. It is clear from Table 4.4 that the Steady State model is not supported by the N(S,θ) data and the models A and B are consistent with the observations for n = 1.0 and 1.4 respectively. Further, if one considers the source counts, models A and C are rejected. Thus the only acceptable model of those discussed above is model B with n = 1.4 ± 0.2.

4.3.5 Effects of Observational Limitations : The various selection effects in the observations of angular sizes have been discussed by Kapahi (1975) who has shown that they are not likely to alter the statistics significantly. In this section, we consider two effects specially applicable to the Ooty data. First, since the Ooty sources have generally been scanned only along 2 or 3 position angles, the inferred 'maximum' angular size for single sources is actually the projection of major axis along a direction of scan and hence is smaller than the true value. Secondly, it is possible that some large diameter sources have escaped detection in the occultation records of weak sources.

The projection effects are considerably reduced in our sample by restricting the Ooty sources to those having scans along directions differing by more than $30^{\circ}$. However, one can compute the effect of random projections on the observed $N(S,\theta)$ in a simple way. For this purpose, we will consider the case in which there are only two scans separated by $2\alpha$ which vary uniformly from $2\alpha_1$ to $2\alpha_2$ such that $\alpha_1+\alpha_2 = \pi/2$. If $\mu$ is the angle between the major axis and the bisector of two scans, and has a uniform probability distribution between 0 and $\pi$, the probability of getting a particular value of $\varphi = |\mu-\alpha|$ is proportional to the number of times this angle can be realised, and is given by :

$$p(\varphi) = \frac{\int_{\alpha_1}^{\alpha_2} d\alpha \int_0^{\pi} \delta(|\mu-\alpha| - \varphi)\,d\mu}{\int_{\alpha_1}^{\alpha_2} d\alpha \int_0^{\pi} d\mu}$$

$$= \begin{cases} 2/\pi & , \quad \varphi \leq \alpha_1 \\ \dfrac{2(\alpha_2-\varphi)}{\pi(\alpha_2-\alpha_1)} & , \quad \alpha_1 \leq \varphi \leq \alpha_2 \\ 0 & , \quad \varphi \geq \alpha_2 \end{cases}$$

In our case, $\alpha_1 = \pi/12$ ($=15^\circ$) ; $\alpha_2 = 5\pi/12$ and using this relation we can get the observed differential counts $n'(\theta)$ from the true distribution $n(\theta)$ as :

$$n'(\theta_o) = \int_0^\infty n(\theta_o+\theta)\cdot g(\theta)\,d\theta$$

where

$$g(\theta) = \frac{\theta(\theta+2\theta_o)}{\theta_o(\theta+\theta_o)^2}\, p(\text{arc cos}\frac{\theta_o}{\theta+\theta_o})$$

By using a similar analysis, it was shown by Swarup and Subrahmanya (1977) that such effects do not affect the shape of $N(S,\theta)$ appreciably, but only lead to a slight overestimation of $n$.

In order to examine the possibility of missing large diameter sources in the Ooty sample, a list of sources was prepared from the Molonglo and Bologna catalogues with $S_{408} > 0.5$ Jy which were expected to have been occulted during the period of observations of the Ooty sample. This resulted in a sample of 43 sources with a median flux density of about 1.2 Jy. Out of these, 41 sources had already been recorded at Ooty including 6 or 7 sources with $\theta > 40$ arc sec (Swarup and Subrahmanya 1977). This checks with the number expected from our optimum model (model B with $n = 1.4$) which predicts about 16 per cent for the sources with $\theta > 40$ arc sec in the flux-density range 0.5 to 2 Jy. Thus it is unlikely that we have missed detection of significant number of weak sources with large diameters.

## 4.4 DISCUSSION OF RESULTS

In spite of the wide spread in the distributions of luminosities and linear sizes of radio sources, and the difficulties in determining these distributions observationally even for the present epoch, it is possible to use the $N(S,\theta)$ statistics to discriminate between some of the world models and evolutionary schemes. The analysis presented above has shown that it seems necessary to postulate the evolution of both the number density (or/and luminosity) and linear sizes of radio source population with cosmic epoch in order to explain the available statistics on the angular sizes and flux densities. Even though the available data are insufficient to determine the geometry of the Universe, they do rule out the Steady State model since it cannot accommodate the cosmological evolution of radio sources.

For the assumed model of linear size evolution, the optimum evolution parameter inferred above by us has been $n = 1.4 \pm 0.2$. This agrees with the value obtained by Katgert(1977) by analysing the $\theta_m(S)$ data going down to $S_{408} \sim 10$ mJy using a luminosity function similar to that in our model B, but with a Friedmann model with $q_o=0$. However, the inferred optimum value of $n$ may not be independent of the assumed luminosity function as evident from the fact that the value of n inferred from the earlier studies of $N(S,\theta)$ by Swarup and Subrahmanya(1977) and Kapahi(1977) was 1.0, the same as that for our model A. However, these are based on a luminosity evolution which is inconsistent with the known $n(S)$ relation (Swarup 1977).

The possibility of linear size evolution was inferred by Miley (1971) and Wardle and Miley (1974) from the observed Euclidean behaviour ($\theta \propto 1/z$) of the upper envelope of the $\theta(z)$ relation for quasars. Recently, however, Riley et al. (1977) have inferred from a comparison of quasars in the 3C and 4C surveys that the data are consistent with an <u>absence</u> of evolution of the linear sizes of quasars. However, this does not conflict our results which are based on both radio galaxies and quasars with the fraction of quasars in the sample being much smaller.

The origin of linear size evolution in terms of physical processes is not well understood. It should probably result from the interaction of the expanding radio source clouds with the surrounding intergalactic medium and the microwave background radiation. There have been some theoretical predictions of size evolution of the form $(1+z)^{-n}$ by considerations of such interactions. For instance, by considering the ram-pressure confinement and inverse Compton losses of the expanding radio clouds against the microwave background, Rees and Setti (1968) obtained an evolution with n=1.5. The same result is also obtained in the relativistic beam model of Blandford and Rees (1974). It is interesting to note that the value of n=1.4 inferred by us is consistent with these theoretical predictions. However, it should be noted that the details of mechanisms involved in these models are still to be investigated.

It is also possible that the apparent linear size evolution partly results from plausible conditions ignored in the analysis of data. Jackson (1973) attempted to explain the $\theta(z)$ relation by assuming a correlation between luminosity and linear size of radio sources. Observationally, even though no correlation seems to be apparent between the luminosity and linear sizes of the 3CR radio galaxies (Mackay 1973; Kapahi 1977), some correlation has been noticed for quasars by Riley et al. (1977). It is interesting to recall here that in our optimum model (B) we could obtain a reasonable fit to $N(S,\theta)$ only by postulating a dependence of linear sizes on luminosity even though the assumed dependence may have been oversimplified.

It has often been pointed out (e.g. Roeder 1975) that an apparent size evolution could also arise from the assumed homogeneity in a truly inhomogeneous Universe. However, as shown by Katgert (1977) for the 'Swiss-Cheese model' which regards galaxies as point masses in a uniform intergalactic material (Dyer and Roeder 1972, 1973, 1974), the assumption of homogeneity may only lead to an over-estimation of $n$ by less than 0.4 for $q_0 = 1$.

It is also likely that the radio source spectra, whose distributions have been ignored completely in our analyses, may significantly influence the $\theta$ - $S$ relation. For a complete investigation of the evolution of radio sources, it is necessary to include all these effects in the model calculations.

In summary, we conclude that a proper statistical analysis of the $\theta - S$ data on a large sample of sources can profitably be used to restrict the range of acceptable forms of intrinsic distributions of radio source properties and their evolution, and possibly the geometric effects also. This should provide useful input to the physical investigation of the origin and evolution of radio sources. Although our present understanding of these phenomena is still uncertain, it seems reasonably clear that the observations of angular sizes and flux densities of extragalactic radio sources imply the existence of cosmological evolution.

# CHAPTER 5
# SUMMARY AND COMMENTS

Shall any gazer see with mortal eyes,
 Or any searcher know with mortal mind?
Veil after veil will lift - but there must be
 Veil after veil behind

Edwin Arnold: THE LIGHT OF ASIA

IN this thesis, two topics have been discussed which are related to the study of radio sources from their lunar occultations. First, a new method has been suggested for restoring the strip-brightness distribution of a source from its occultation profile. Secondly, a statistical analysis of the available angular size data obtained from lunar occultation and other methods is made with a view to making cosmological inferences.

## 5.1 OPTIMUM DECONVOLUTION METHOD (ODM)

The smoothing nature of convolution leads to a loss of information beyond a resolution limit which depends on the nature of smoothing function and the observational signal-to-noise ratio. The essential philosophy of ODM described in Chapter 2 is to use available a priori information on the nature of the source in order to retrieve a part of this 'lost' information. This is in contrast with the classical methods which seek a general solution with minimum assumptions on the nature of the solution to be obtained. A valuable prior information usually available is that the solution should be positive. A simple iterative algorithm has been suggested in this thesis for incorporating positiveness which can also be used for more general constraints like upper and lower bounds on the solution. An 'optimum' solution is attempted by using this algorithm in a least squares procedure along with some standard techniques of constrained minimisation like the Lagrange multiplier method. This method is more readily applicable for lunar occultations than the other existing super-resolution techniques like the 'Maximum Entropy Method' (Frieden 1972 , Ables 1974). Although our method has been called an 'optimum' method because it can absorb a variety of prior information, it is more appropriate to regard it as illustrating a scheme for the optimum method, rather than as being a universally optimum method. In fact, we do not believe that such a universally optimum deconvolution method exists.

Application of ODM to lunar occultations is described in Chapters 2 and 3, where it is also compared with Scheuer's method which is the most readily applicable 'conventional' method for this problem. In an iterative scheme like ODM, the efficiency depends on a proper choice of the empirical parameters required in the method which are related to our a priori knowledge of the solution. Several guidelines have been suggested in Chapter 2 for choosing these parameters in the application to lunar occultations. Our experience has shown that it is often possible to obtain a significantly higher resolution and a more objective interpretation by using ODM in place of conventional methods. The method has also been applied to the lunar occultations of 63 weak radio sources with a median flux density of about 0.5 Jy at 327 MHz, described in Chapter 3.

The possibility of super-resolution by using ODM in lunar occultations has two-fold advantages. First, a better determination of angular sizes is possible, particularly for weak radio sources which are more important for cosmological studies but cannot be restored normally with high resolutions because of poor signal-to-noise ratio. Secondly, ODM can be used to study the finer details of radio source structure like compact heads, central components, bridges and weak extensions which provide useful insight into the physical processes responsible for radio emission. In this thesis, we have restricted ourselves to the use of angular size statistics to make cosmological inferences.

## 5.2 COSMOLOGY FROM ANGULAR SIZE STATISTICS

A statistical analysis of the data on the angular sizes ($\theta$) and flux densities (S) of 513 extragalactic radio sources with $S_{408} \sim 0.3$ Jy has been presented in Chapter 4. The observed $N(S,\theta)$ distributions have been compared with the predictions of some standard world models using Einstein de-Sitter and Steady State geometries. In order to get a reasonable agreement with the data, it became necessary to introduce cosmological evolution of both the number (or luminosity) and linear sizes of radio sources. By assuming the linear sizes to evolve as $(1+z)^{-n}$, we found that the best fit to the above data was obtained with $n = 1.4 \pm 0.2$. This value of n is consistent with other determinations from the $\theta$ - z data (e.g. Wardle and Miley 1974) and some theoretical models on the interaction of expanding radio clouds with the surrounding medium and the microwave background in the ram-pressure confinement model (e.g. de Young 1971).

Even though the angular size data clearly require an evolving luminosity function, the available data permit a wide range of evolutionary schemes. This range can be restricted considerably by using the source counts as in our model B where we have assumed the luminosity function given by Wall et al. (1977) which predicts the observed source counts satisfactorily down to $S_{408} \sim 10$ mJy. It is interesting that for this model, we could get a reasonable fit to $N(S,\theta)$ only after assuming a dependence of linear size on luminosity as well as redshift. We have only considered a simple dependence requiring the local size function to have an e-folding size of 0.1 Mpc for the non-evolving low-luminosity sources as against 0.4 Mpc for the evolving high-luminosity sources. It may be recalled that since the latter are mostly seen at high redshifts, their linear size is still lower than that of the low luminosity sources because of the $(1+z)^{-n}$ dependence. The best fit is found for $n = 1.4 \pm 0.2$. Admittedly, the assumed luminosity dependence is oversimplified and it is likely that the inferred value of n is influenced by the assumed dependence. By extending the $N(S,\theta)$ data to weaker sources and improving the angular sizes of unresolved sources, one can hope to put restrictions on the possible luminosity - size dependence. These restrictions will provide constraints on the physical models of the radio sources predicting the sizes of sources in the course of their evolution.

The present work shows that in spite of the wide scatter in the intrinsic properties of radio sources, a proper statistical analysis of their angular sizes and flux densities can lead to useful cosmological inferences. The influence of scatter in the data can be significantly reduced by considering larger samples obtained by high resolution studies of weak extragalactic radio sources.

# REFERENCES


Ables J.G., 1974, Astron. Astrophys. Suppl. 15, 383

Biraud Y., 1969, Astron. Astrophys. 1, 124

Blanco V.M., Graham J.A., Lasker B.M. and Osmer P.S., 1975, Astrophys. J., 198, L63

Blandford R.D. and Rees M.J., 1974, Mon. Not. RAS 169, 395

Bolton J.G., 1966, Nature 211, 917

Bracewell R.N. and Roberts J.A., 1954, Aust. J. Phys. 7, 615

Burbidge G.R. and Narlikar J.V., 1976, Astrophys. J. 205, 329

Dahlquist G. and Bjorck A., 1974, Numerical Methods, (Transl. Anderson N.), Prentice-Hall

De Young D.S., 1971, Astrophys. J. 167, 541

De Young D.S., 1976, Ann. Rev. Astron. Astrophys. 14, 447

Dixon R.S.,1970, Astrophys. J. Suppl. Ser. 20, 1

Dyer C.C. and Roeder R.C., 1972, Astrophys. J. 174, L115

Dyer C.C. and Roeder R.C., 1973, Astrophys. J. 180, L31

Dyer C.C. and Roeder R.C., 1974, Astrophys. J. 189, 167

Ekers R.D. and Miley G.K., 1977, p. 109 in Jauncey(1977)

Evans D.S., 1970, Astron. J. 75, 589

Fanti R. and Perola G.C., 1977, p. 171 in Jauncey(1977)

Frieden B.R., 1972, J. Opt. Soc. Amer. 62, 511

Gopal-Krishna, 1976, Ph.D. thesis, Bombay University

Hazard C., 1961, Nature 191, 58

Hazard C., 1962, Mon. Not. RAS 124, 343

Hazard C., 1972, Astrophys. Lett. 11, 139

Hazard C., 1976, Methods of Experimental Physics, Vol. 12C, (Ed.) Meeks M.L., Academic Press, Chapter 4.6

Hazard C., Mackay M.B. and Shimmins A.J., 1963, Nature 197, 1037

Hewish A., Readhead A.C.S. and Duffet-Smith P.J., 1974, Nature 252, 657

Hogbom J.A., 1969, Rep. URSI 16th Gen. Assembly, Ottawa

Hogbom J.A., 1974, Astron. Astrophys. Suppl. 15, 417

Hoglund B., 1967, Astrophys. J. Suppl. Ser. 15, 61

Hoskins D.G., Murdoch H.S., Hazard C., and Jauncey D.L., 1972, Aust. J. Phys. 25, 559

Hoyle F., 1959, Paris Symposium on Radio Astronomy, (Ed.) Bracewell R.N., Stanford University Press, p.529

Hoyle F. and Burbidge G.R., 1966, Nature 210, 1346

Jackson J.C., 1973, Mon. Not. RAS 162, 11P

Jauncey D.L., 1975, Ann. Rev. Astron. Astrophys. 13, 23

Jauncey D.L., 1977, (Ed.)'Radio Astronomy and Cosmology' IAU Symp. 74, D. Reidel Publ.

Jaynes E.T., 1968, Trans. IEEE SSC-4, 227

Jenkins C.J., Pooley G.G. and Riley J.M.,1977, Mem. RAS(in press)

Joshi M.N., Kapahi V.K., Gopal-Krishna, Sarma N.V.G. and Swarup G., 1973, Astron. J. 78, 1023

Kapahi V.K., 1971, Nature Phys. Sci. 234, 49

Kapahi V.K., 1975, Mon. Not. RAS 172, 513

Kapahi V.K., 1975a, Ph.D. thesis, Bombay University

Kapahi V.K., 1977, p.119 in Jauncey(1977)

Kapahi V.K., Gopal-Krishna and Joshi M.N., 1974, Mon. Not. RAS 167, 299

Kapahi V.K., Joshi M.N. and Gopal-Krishna,1972, Astrophys. Lett. 11, 155

Kapahi V.K., Joshi M.N. and Kandaswamy J., 1973a, Astrophys. Lett. 14, 31

Kapahi V.K., Joshi M.N., Subrahmanya C.R. and Gopal-Krishna,1973, Astron. J. 78, 673

Katgert P., 1977, Ph.D. thesis, Leiden University

Krishnan T., 1971, Highlights of Astronomy, (Ed.) De Jager, D. Reidel Publ., p.646

Lang K.R., 1969, Astrophys. J. 158, 1189

Longair M.S., 1971, Rep. Prog. Phys. 34, 1125

Longair M.S. and Gunn J.E., 1975, Mon. Not. RAS 170, 121

Lucy L.B., 1974, Astron. J. 79, 745

Mackay C.D.,1971, Mon. Not. RAS 154, 209

Mackay C.D., 1973, Mon. Not. RAS 162, 1

Miley G.K., 1971, Mon. Not. RAS 152, 477

Munro R.E.B., 1972, Aust. J.Phys. Astrophys. Suppl. No. 22

Murdoch H.S., McAdam W.B. and Hunstead R.W., 1974, Nature 248, 491

Narlikar J.V. and Chitre S.M., 1977, Mon. Not. RAS 180, 525 (also see the ERRATUM to this paper)

Nather R.E. and McCants M.M., 1970, Astron. J. 75, 963

Nguyen-Quang-Rieu, 1969, Astron. Astrophys. 1, 128

Paatero P., Manninen S.,and Paakkari T., 1974, Philos. Mag. 30, 1281

Penzias A.A. and Wilson R.W., 1965, Astrophys. J. 142, 419

Phillips D.L., 1962, J. Assoc. Comp. Mach. 9, 84

Pooley G.G. and Henbest S.N., 1974, Mon. Not. RAS 169, 477

Pooley G.G. and Ryle M., 1968, Mon. Not. RAS 139, 515

Rees M.J. and Setti G., 1968, Nature 219, 127

Riley J.M., Longair M.S. and Hooley A.A., 1977, p.133 in Jauncey (1977)

Riley J.M. and Pooley G.G., 1975, Mem. RAS 80, 105

Robertson J.G., 1973, Aust. J. Phys. 26, 403

Roeder R.C., 1975, Astrophys. J. 196, 671

Ryle M., 1959, Paris Symp. on Radio Astronomy, (Ed.) Bracewell R.N., Stanford Univ. Press, p.523

Scheuer P.A.G., 1962, Aust. J. Phys. 15, 333

Scheuer P.A.G., 1965, Mon. Not. RAS 166, 513

Singal A.K., Gopal-Krishna and Venugopal V.R., 1977, (to be published)

Subrahmanya C.R., 1975, Bull. Astron. Soc. India 3, 13

Subrahmanya C.R. and Gopal-Krishna, 1977 (to be published)

Sutton J.M., 1966, Ph.D. thesis, University of Sydney

Sutton J.M., Davies I.M., Little A.G. and Murdoch H.S., 1974, Austr. J. Phys. Astrophys. Suppl. No. 33, 1

Swarup G., 1975, Mon. Not. RAS 172, 501

Swarup G., 1977, Bull. Astron. Soc. India 5, 36

Swarup G., Kapahi V.K., Sarma N.V.G., Gopal-Krishna, Joshi M.N. and Rao A.P., 1971a, Astrophys. Lett. 9, 53

Swarup G., Sarma N.V.G., Joshi M.N., Kapahi V.K., Bagri D.S., Damle S.H., Ananthakrishnan S., Balasubramanian V., Bhave S.S. and Sinha R.P., 1971, Nature Phys. Sci. 230, 185

Swarup G. and Subrahmanya C.R., 1977, p.125 in Jauncey (1977)

Terrel J., 1975, Nature 258, 132

Turchin V.F., Kozlov V.P., Mahevich M.S., 1970, Sov. Phys. Uspekhi 13, 681

Twomey S., 1963, J. Assoc. Comp. Mach. 10, 97

Twomey S., 1965, J. Franklin Inst. 279, 95

Van de Kamp P., 1967, Principles of Astrometry, Freeman

Von Hoerner S., 1964, Astrophys. J. 140, 65

Von Hoerner S., 1965, Astrophys. J. 142, 1265

Von Hoerner S., 1973, Astrophys. J. 186, 741

Venkatakrishna K.L. and Swarup G., 1977 (to be published)

Wall J.V., Pearson T.J. and Longair M.S., 1977, p.269 in Jauncey (1977)

Wardle J.F. and Miley G.K., 1974, Astron. Astrophys. 30, 305

Watts C.B., 1963, The Marginal Zone of the Moon, (Naut. Almanac Office, U.S. Naval Obs.)

Weinberg S., 1972, Gravitation and Cosmology, Wiley

Wernecke S.J., 1975, presented at the topical meeting on Image Processing for 2-D and 3-D Reconstruction from Projections, August 4-7, 1975, Stanford University.