CHAPTER 5

BESSEL TRANSFORMS

In section 2.11 we have seen that the Bessel transform of the temporal acf of the intensity fluctuations shows the Fresnel modulation much more clearly than the power spectrum. The interest in detecting the Fresnel modulation lies in the fact that if we see it we can directly estimate z/V^2 and since the value of z can be assumed without too much error, we can, from single station observations, measure the velocity of the solar wind. Lovelace et al. (1970) have computed the Bessel transforms for 8 observations of CTA 21 and on 3 occasions they observed deep minima which they could interpret as due to Fresnel filtering and could use to estimate the velocity of the solar wind. To study the usefulness of this technique for systematically studying the velocity of the solar wind, we have computed the Bessel transforms for all our observations. We find that the Fresnel modulation can be seen only very rarely and so this method is not of much value as a regular probe of the solar wind. This result was not very surprising since if the Fresnel modulation were present, they should have shown up, though with smaller amplitude, on the power spectrum, and the power spectra on most days show no modulation.

We have not attempted to directly measure the Bessel transform from the temporal acf mainly because the Bessel transform, unlike the Fourier transform, does not have the property that the transform of a convolution integral is the product of the individual transforms (Sneddon (1951)). The measured intensity fluctuations are smoothed by the R-C time constant and so the observed temporal acf is the scintillation acf convolved with the acf of the time constant. Thus the measured acf cannot directly be used for calculating the Bessel transform. The correct thing to do would be to compute the power spectrum of the intensity fluctuations, correct the spectrum for the effect of the time constant, take the inverse Fourier transform to get the scintillation acf and use this to calculate the Bessel transform using equation (2.81). This is too involved a procedure and we have preferred to estimate the Bessel transform by using the property that it is equal to the Abel transform of the temporal power spectrum. The power spectra of the scintillations had already been computed and were stored on magnetic tape and so all one had to do was to read them back into the computer and calculate the Abel transform.

The direct computation of the Abel transform using the definition

$$G(f) = -\frac{1}{\pi}\int_f^{\infty} dx\, \frac{dP(x)}{dx} \Big/ (x^2 - f^2)^{1/2} \tag{5.1}$$

is complicated by the presence of a singularity in the integrand at $x = f$. This can be removed by integrating by parts to give

$$G(f) = \frac{1}{\pi}\int_f^{\infty} dx\, \sqrt{x^2 - f^2}\; \frac{d}{dx}\left[\frac{P'(x)}{x}\right] \tag{5.2}$$

where we have assumed that the power spectrum and its derivatives vanish at $x = \infty$. This method was tested by feeding noise free functions whose Abel transforms could be analytically calculated and by seeing if the computed transform agreed with the theoretical transform. One such function was the power spectrum of the intensity fluctuations in the thin screen model given by equation (2.84) whose Abel transform is given by equation (2.85). The estimates of the Abel transform got by numerically integrating equation (5.2) were generally in poor agreement with the theoretical estimates and showed a marked tendency to go negative. Assuming that this is due to the sidelobes in the transform we have, following the suggestion made by Lovelace et al. (1970), computed the pre whitened Bessel transform $f G(f)$ which has a smaller range of values and so is less susceptible to the affects of the sidelobes. Unlike Lovelace et al. who started from the acf we have calculated the pre whitened Bessel transform from the power spectrum itself using a modified form of the Abel transform. Since the power spectrum and the Bessel transform are a Abel transform pair, they are related by definition (Bracewell (1965)).

$$P(f) = 2 \int_f^\infty \frac{G(x)\, x}{\sqrt{x^2 - f^2}}\, dx \tag{5.3a}$$

$$G(f) = -\frac{1}{\pi} \int_f^\infty \frac{P'(x)}{\sqrt{x^2 - f^2}} \tag{5.3b}$$

we have to get an expression for $f G(f)$. Let us define a function $g(f)$ whose derivative $g' = \frac{dg}{df}$ is equal to $f G(f)$. Substituting for $x\, G(x)$ in equation (5.3a) and multiplying both sides by $-\frac{1}{2\pi}$ we get

$$-\frac{P(f)}{2\pi} = -\frac{1}{\pi} \int_f^\infty \frac{g'(x)}{\sqrt{x^2 - f^2}}\, dx$$

which is similar to equation (5.3b) showing that $g(x)$ and $-\frac{P(x)}{2\pi}$ form an Abel transform pair. So $g(f)$ can be got from equation (5.3a) with $-\frac{P(f)}{2\pi}$ in the place of $G(f)$ giving

$$g(f) = -\frac{1}{\pi} \int_f^\infty \frac{P(x)\, x}{\sqrt{x^2 - f^2}}\, dx = \frac{1}{\pi} \int_f^\infty P'(x) \sqrt{x^2 - f^2}\, dx$$

where we have integrated by parts and assumed that $P(x)$ goes to zero at $x = \infty$. The pre whitened Bessel transform is given by

$$f\,G(f) = g'(f) = \frac{1}{\pi}\frac{d}{df}\left[\int_f^\infty dx\,(x^2 - f^2)^{1/2}\,\frac{dP(x)}{dx}\right] \tag{5.4}$$

Numerical evaluation of equation (5.4) gave quite satisfactory results for the Abel transform and this expression was used to calculate the Bessel transforms of all the data. Power spectra with a resolution of 0.025 Hz were read into the computer from the magnetic tape and their Abel transforms computed. The Abel transforms were smoothed by taking running means to give a fairly stable spectrum with resolution of 0.1 Hz.

The Bessel transforms were examined for the Fresnel modulation which could be identified by its characteristic signature of a train of minima at frequencies which are proportional to $\sqrt{n}$. The Bessel transforms of 6 or fewer minutes of data showed quite a number of deep minima which occurred at random frequencies and were due just to the noise. Transforms of 9 or more minutes of data are more stable but even these rarely show minima at frequencies proportional to $\sqrt{n}$. Of the 50 or so Bessel transforms computed only on two occassions did we find a sequence of minima at frequencies proportional to $\sqrt{n}$. In figure 20 we show the Bessel transform for the source PKS 2203-18 on March 1971. The first minimum is at 0.7 Hz. Using the expression for the first null

$$f_B = V/\sqrt{z\lambda} \tag{5.5}$$

and setting $z = 0.9$ A.U., we get $V = 260$ km s^{-1}. Examination of figure 20 shows that even on this day it is debatable whether the minima are really

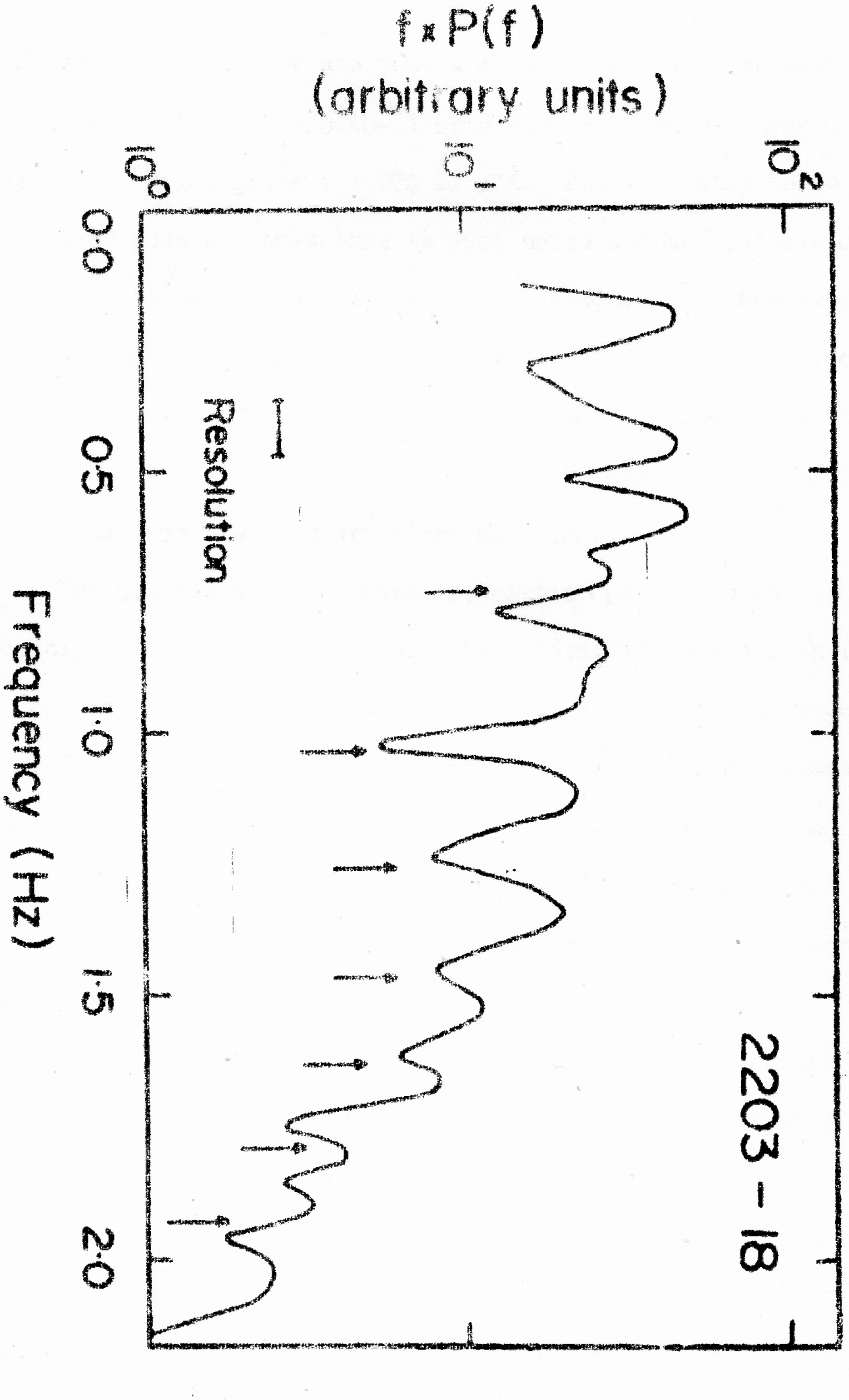

Figure 20. Bessel transform of PKS 2203-18 on 8 March 1971. The arrows indicate the position of the expected minima for V = 280 km s^{-1}.

due to Fresnel filtering or are just a chance occurrence of noise peaks. The Bessel transform of PKS 0115-01 on 6 May, 1971 shows three deep minima going as $\sqrt{n}$. which gives V = 370 km s^{-1}. But with only three minima it is again quite possible that this is just noise and the occurrence of the minima at the proper frequencies is due to chance. Thus the evidence in favour of our having seen Fresnel filtering effects is not very convincing and the only conclusion we can draw is that under typical conditions no Fresnel modulation is seen. Ward et al. (1972) have reached a similar conclusion based on their observations at 80 MHz.

The absence of the Fresnel modulation cannot be attributed to lack of frequency resolution in the Bessel transform. From equation (5.5), for V = 350 km s^{-1} and z = 1 A.U. the first null in the transform should occur at 0.9 Hz. Subsequent minima should occur at $\sqrt{n} \times 0.9$ Hz where n = 2,3.... With our resolution of 0.1 Hz we should easily see 4 or 5 minima if they are present. Since we cannot explain the absence of modulation by instrumental effects, we have to examine the various mechanisms which reduce the Fresnel modulation.

In section 2.11 we have seen that the amplitude of the Fresnel modulation in the Bessel transform decreases if the irregularities are elongated in the direction of the solar wind velocity and that the amplitude is given by $\left(\cos\theta_x / \cos\theta_y\right)^{1/2}$ where $\tan\theta_i = \frac{2z}{z_{0i}} = \frac{2z}{k a_i^2}$. In the symmetrical case $\theta_x = \theta_y$ and there is 100% modulation. The ratio of the scale sizes parallel and perpendicular to the solar wind has a value between 1 and 2. Let us take the worst case and assume it is equal to 2. The scale size estimated from the second moment is roughly equal to a_x the scale in the direction of the solar wind. Let us assume that a_x and a_y have the values 100 km and 50 km respectively which is compatible with the observations of the second moment and the assumed anisotropy of ratio of 2. Assuming an average value for z equal to 0.9 A.U., these scale sizes give $\theta_x = 76^o$ and $\theta_y = 86^o$ and the amplitude of the modulation reduces to about 55% which should still be observable. The actual anisotropy ratio is likely to be less than 2 and so it seems unlikely that the absence of the modulation in the observed spectra is caused purely by the elongation of the irregularities.

The thickness of the scattering medium also reduces the observable modulation in the Fourier and Bessel transforms. In the weak scattering regimes we can regard the extended medium to be a collection of thin screens at different distances from the observer. The observed power spectrum is the sum of the spectra produced by the various screens. Since the screens are at different distances from the observer, they produce Fresnel modulation with the nulls occurring at different frequencies which causes the modulation in the observed spectrum to be smeared out. If we consider the medium to be uniform and to extend from $z + \frac{L}{2}$ to $z - \frac{L}{2}$ the observed Fourier transform is given by (Salpeter (1967))

$$\begin{aligned} M_z^2(\underline{q}) &= 4\,\underline{\Phi}^2(\underline{q})\,L^{-1}\int_{z-L/2}^{z+L/2} dz'\,\sin^2\left(\frac{z'q^2}{2k}\right) \\ &= 2\,\underline{\Phi}^2(\underline{q})\left[1 - \frac{2k}{q^2L}\sin\left(\frac{Lq^2}{2k}\right)\cos\left(\frac{zq^2}{2k}\right)\right] \end{aligned} \qquad (5.6)$$

This expression is not of much use in computing the expected power spectra for two reasons. Firstly the IP medium cannot be regarded as a uniform slab of material since the r.m.s. phase fluctuation produced by the medium is proportional to r^{-2} and so varies along the line of sight to the source.

Secondly, to convert the spatial frequencies to temporal frequencies we must know the velocity of the solar wind perpendicular to the line of sight. If we assume that the radial velocity is constant, the component perpendicular to the line of sight varies with distance because of varying projection effects and this increases the smearing. We can take these two effects into account and express the expected power spectrum as an integral but it is not possible to get an analytical solution to the integral. Ward et al. (1972) have taken these two effects into account and have estimated by numerical calculations the power spectrum at various elongations. They have assumed that scale sizes increases as $r^{1.5}$ as suggested by Readhead (1971) but since the assumed dependence of the scale size will not substantially affect the smearing, we will simply quote their results. Their computations show that for $p > 0.6$ A.U. there is considerable smearing but for smaller elongations the Fresnel modulation should be seen on the power spectra. In the Bessel transform the modulation will be even larger and so simple geometrical smearing alone cannot explain the absence of the modulation in the observed Bessel transform.

The presence of a random component in the velocity of the solar wind also reduces the amplitude of the modulation. The measured velocity of the solar wind fluctuates considerably with time about its mean value and so it is reasonable to assume that at any given time the velocity of the solar wind will vary with position. In the above calculation it was assumed that the only variation in the velocity was due to projection effects. If we assume that in addition to the projection effect there is a random variation of velocity with position, the smearing of the modulation increases. The magnitude of this effect depends on the magnitude of the random component in the velocity. Ward et al. (1972) have computed the power spectrum with a 10% peak to peak random variation in the velocity and they find that the modulation is considerably reduced. Armstrong et al. (1972) have suggested that the fluctuating component in the velocity may be 25% or more in which case, the modulation would be completely washed out. To make quantitative estimates of this effect one must know the origin of the velocity fluctuations. If we assume that it is due to the velocity varying with position one has to know the typical scale size of the velocity fluctuation since if the scale size is much larger than 0.2 or 0.3 A.U. which is the typical effective width of the medium, the smearing effect of the random velocity is negligible. If however the random velocities are due to the presence of turbulence in the solar wind, in which case the velocity field varies with both space and time, then the time scale of the fluctuation also plays a role in smearing the modulation. If the time scale is larger than the typical averaging time of about 10 minutes, the time variations do not smear the modulation. One could in principle test the importance of time variations by averaging the Bessel transform over less number of minutes and seeing if the modulation becomes prominent. However this is not a practical proposition because when we reduce the length of averaging the statistical stability of the spectrum reduces and the transform becomes too noisy to make any conclusion. If the solar wind is turbulent it is quite likely that in each of the thin screens that constitute the extended medium, the pattern is no longer frozen in and as discussed in section 2.11b, this also smooths out the Fresnel modulation.

Thus the absence of the modulation in the observed spectrum cannot be explained by anisotropy of the irregularities or by geometrical smearing effects. In order to explain the absence it seems to be necessary to introduce a randomly varying component in the velocity of the solar wind. From our observations we cannot decide whether the solar wind velocity varies with position or with time or whether there is an intrinsic spread in the velocity at each point in space and time.

CHAPTER 6

STRUCTURE OF THE SCINTILLATING SOURCES

In section 2.9 we have seen that the spatial power spectrum for an extended source is related to the visibility function of the source by

$$M^2_{ext}(\underline{q}) = \left| V\left(\frac{z}{2\pi}\underline{q}\right)\right|^2 M_{pt}(\underline{q}) \quad (6.1)$$

which shows that we can estimate the visibility function of the source by dividing the measured spatial spectrum by that of a point source. However what we measure is the temporal power spectrum which is related to the spatial spectrum by

$$P(f) = \frac{2\pi}{V}\int_{-\infty}^{\infty} M^2_{ext}\left(\frac{2\pi f}{V}, q_y\right) dq_y \quad (6.2)$$

Because of the integration over q_y, the visibility function cannot be directly estimated from the observed spectra and the only way one can estimate the structure of the source is by comparing the observed spectra with spectra computed for different source models. Since comparison of the detailed power spectra is difficult, we have preferred to estimate the source structure by using only the width of the spectrum as measured by the second moment f_2. In cases where the observed m and f_2 are compatible with more than one model for the source, we have interpreted the source in terms of the simplest model.

If we assume that the irregularities in the medium are frozen into the screen, the second moment f_2 is related to q_2, the second moment of the spatial spectrum, by

$$f_2 = V/2\pi\ell = V q_2/2\pi$$

Using this, and the fact that the scintillation index is the same for both the spatial and temporal fluctuations, we can use the results of section 2.9, where we have derived m and q_2 for different source models. We will describe all the sources by giving the diameter of the scintillating component, assuming it to be a Gaussian, and by giving the fraction of the total flux in the scintillating component.

For a source with a Gaussian brightness distribution of the form $P(\theta) = e^{-\theta^2/2\theta_0^2}$, the observed m and f_2 are given by

$$m = m_0 \left[1 + 2\left(z\theta_0/\ell \right)^2 \right]^{-1/2} \tag{6.3}$$

$$f_2 = f_{2_0} \left[1 + 2\left(z\theta_0/\ell \right)^2 \right]^{-1/2} \tag{6.4}$$

where the subscript • refers to values for a point source at the same elongation. The scale size on the ground is given by

$$\ell = V/2\pi \; f_{2_0} \tag{6.5}$$

and combining this with equation (6.4) we can express θ_0 in terms of f_2. If we define ψ as the half power diameter of the source we have

$$\psi = 2.34\,\theta_0 = \left(V/1 \cdot 2\pi f_2 z \right) \left[1 - \left(f_2/f_{2_0} \right)^2 \right]^{1/2} \tag{6.6}$$

Once the diameter is known we can estimate, using the equation (6.3), what the scintillation index would have been if all the flux of the scintillating component had been in a point source. If we find that even after correcting for the source size the scintillation index is less than that of a point source at the same elongation, we will regard it as indicating a core halo structure with the fraction of the flux in the core being equal to the ratio of the observed corrected scintillation index to that of a point source at the same elongation. The extended halo does not contribute to the intensity fluctuations and all we can say about it is that its diameter is greater than about 1". As the Sun moves with respect to the source, the position angle of the solar wind velocity across the source varies. If the estimated diameter of the source varies with position angle, we will interpret it as indicating an elliptical Gaussian structure for the source.

The main difficulty in implementing the procedure described above is that we do not know m_0 and f_{2_0} and their variation with elongation. We have to combine our observations with interferometric observations of the sources and make a model for the variation of m_0 and f_{2_0} with p and only then can we deduce the structure of any given source.

Even if we do not know the value of f_{2_0} we can set an upper limit on the diameter of a source by assuming that $f_{2_0} \gg f_2$ so that from equation (6.6)

$$\psi \leq V / 1.2\pi f_2 z \qquad (6.7)$$

The most stringent limit on ψ is got when f_2 is maximum, and this occurs closest to the Sun. From our observations at $p \sim 0.1$ A.U., f_2 for the unresolved sources is typically around 2 Hz and putting $z = 1$ A.U. and $V = 300$ km s^{-1} we get

$$\psi \leq 0\overset{''}{.}05$$

Using arguments similar to that in section 4.2 we can show that even if the observed sources have diameter equal to the upper limit, the effect of source size is negligible for $p > 0.25$ A.U. where the second moment is much smaller. So, for $p > 0.25$ A.U. we can safely set f_{2_0} equal to the f_2 observed for those sources whose second moments attain a value around 2 Hz close to the Sun. This is strictly true only if the position angle of the solar wind across the source does not vary with elongation, since only then can we ignore the possibility that the source structure varies with position angle. Of the sources we have studied the position angle varies by less than 30° for 3C2, PKS 0116+08 and CTA 21, and we will take the average $f_2 - p$ curves for these sources for $p > 0.25$ A.U. to represent the actual variation of f_{2_0} with p. The resulting $f_{2_0} - p$ curve is essentially the same as that derived in section 3.3 which was shown in figure 15. For $p < 0.25$ A.U. our arguments that the effect of source size is negligible is no longer valid and we have no way of estimating f_{2_0} from our observations alone. What we need are low frequency observations of the structure of the sources used in IPS with resolution better than 0.″05. However, since such observations are not available, we will assume that the $f_{2_0} - p$ curves for $p < 0.25$ A.U. is the same as the observed $f_2 - p$ curves for these sources. The only justification for this assumption is that we have never observed any source whose $f_2 - p$ curve is systematically higher than the curve shown in figure 15. However, we should always keep in mind that we could be underestimating f_{2_0} at small elongations, which would mean that the diameters of the resolved sources could be larger than what we estimate.

The main problem in determining the $m_0 - p$ curve is that even if the scintillating components are point sources, there could be extended components around them that will not show up in scintillation and so cause us to underestimate m_0. The ratio of the observed scintillation indices of two sources gives the relative values of μ , the fraction of flux in the scintillating components. To determine the absolute value of μ we must know m_0 or we must, by some other method know the value of μ for one of the scintillating sources. There are a number of high resolution observations of CTA 21 at low frequencies. These have been summarised below and they all indicate that there is no extended structure around CTA 21, with angular size greater than about 0."5. So we will, as have most workers, assume $\mu = 1$ for CTA 21 and thus by comparing the scintillation index of any source with that of CTA 21 at the same elongation we can estimate μ for the source. It is a bit surprising to find two sources - PKS 0115-01 and NRAO 91 having average m-p curves slightly higher than that of CTA 21. The excess, 10 and 20% respectively, is quite small and is well within the uncertainties in the measurements due to statistical fluctuation in the properties of the solar wind, uncertainties in estimating the mean m-p curve, and errors in estimating the flux of the source due to confusion. We feel that the excess is not significant and it is not necessary to reduce the value of μ for CTA 21 to less than 1. To get a reliable estimate of the m_0-p curve we have scaled the observed scintillation indices for the sources with unresolved scintillating components, to make them coincides with the m-p curve for CTA 21 and have calculated the average value of m over small ranges of elongation. The resulting m_0-p curve is the same as shown in figure 14. For this curve also the effect of source size is negligible for $p > 0.25$ A.U. Since μ can be determined without going very close to the Sun, we are not very much concerned here with the correctness of the m_0-p curve for $p > 0.25$ A.U.

We will use these f_{2o} - p and m_o-p curves to estimate the structure of the scintillating sources. Our procedure of determining μ from the scintillation index and the diameter from the second moment, is similar to that used by the Arecibo group. A different method has been adopted by the Cambridge group in which both μ and the diameter are determined by comparing the observed m-p curves with theoretical m-p curves for different source models. In determining the theoretical m-p curves the turnover in the scintillation index is attributed purely to the finite source size, which, as seen in section 4.7 does not agree with the behaviour of the second moment close to the Sun. Readhead (1971) has avoided the problem of the turnover and has estimated the diameter of the sources by comparing the observed ratio of the scintillation indices at two elongations with values computed for different source models. For the computation of the ratio for different models Readhead has assumed that the scale size increases with distance from the Sun as $r^{1.5}$, which is too fast a variation to be compatible with our observations. If the scale size did indeed vary as $r^{1.5}$, the correction to the scintillation index for the effect of the 0.1 s time constant that has been used can be quite large and this has not been taken into account in calculating the theoretical ratios. The diameters estimated by the Cambridge group on the basis of the m-p curve alone are generally larger than the diameters estimated using the second moments. While it cannot be ruled out that the difference is genuine and is due to the Cambridge observations being at lower frequencies, it is possible that a part of the difference is due to the differences in the analysis procedure. The study of the diameters of some of these sources at low frequencies using the power spectrum method would be helpful in settling this question.

Source	Present work 327 MHz			Cohen et al. (1967a) 195, 430, 610 MHz			Harris & Hardbeck (1969) 430 MHz			Little & Hewish (1968) 178 MHz		Readhead & Hewish (1974) 80 MHz	
	μ	ψ	P.A.	μ	ψ	P.A.	μ	ψ	P.A.	μ	ψ	μ	ψ
3C 2	0.6	<0".06	240°	0.6	~0".1	232°	0.6	<0".06		0.5	<0".1	0.9	~0".7
PKS 0115-01	1.0	<0.1	144	–	–	–	–	–		–	–	>0.8	<0.3
PKS 0116+08	0.75	<0.08	70	–	–	–	0.7±.2	<0.05	70°	–	–	>0.65	<0.45
3C 49	0.75	<0.06	30	0.6	<0.1	263	0.8±.2	<0.04	All	0.4 or 1.0	<0.2 or ~0.6	0.75	~0.6
NRAO 91	1.0	~0.1	60	0.6	<0.1	268	1.0	<0.2	56 259	–	–	>0.55	<0.6
CTA 21	1.0	<0.08	90	1.0	<0.04	263	1.0	<0.02	70 260	–	–	1.0	~0.2
3C 138	1.0	Elongated see text		0.8	~0.1	136	1.0	~0.19	255	1.0	~0.15	0.9	~0.5
PKS 2203-18	1.0	<0.1	200	–	–	–	–	–	–	–	–	–	–
3C 446	1.0	Elongated see text		–	–	–	–	–	–	–	–	0.35	~0.35

In table II we have summarised the estimated structures of the nine sources we have observed. For each source we have given μ , the fraction of the flux in the scintillating component, ψ , the half power diameter of the source, and the position angle for which the diameter estimate holds. For comparison we have given the structures estimated from IPS observations made by other workers. The diameters given by the Cambridge group have been corrected for the difference in the definition of the diameter. We give below comments on individual sources with details of diameter estimates by other techniques.

3C2

Except for the 80 MHz observations there is reasonably good agreement between the various IPS observations. The higher value of μ at 80 MHz is curious since for core halo sources we would expect the core to have a flatter spectrum than the halo and so μ should decrease at low frequencies. This source is peculiar in that in spite of its having a strong compact component, its radio spectrum remains a power law down to about 30 MHz (Veron et al. (1974)) and shows no signs of self absorption. From IPS observations alone we cannot rule out the possibility that the source instead of having a core halo structure, actually consists of two or three compact components separated by more than about a second of arc. In such a model, the scintillation index is reduced by $\sqrt{N}$, where N is the number of components (which are assumed to have equal flux). Such a structure would show up as a modulation in the power spectrum, but if the separation of the components is large the frequency resolution may not be adequate to observe this modulation. Even if the resolution is adequate, it is possible that we do not see this modulation because it gets smeared out by the spread in the velocity in the medium. In either case one could misinterpret the observations and conclude that the source has a core halo structure. However there exists independent confirmation of the core halo model from the lunar occultation observations of Lynne (1972) at 408 MHz which indicate that about 60% of the flux of the source is in an unresolved component which clearly is the scintillating component, and the rest of the flux is in an extended jet like structure with dimensions about 6".

3C49

For this source also there is good agreement between the various observations except for the fact that the 80 MHz diameter is much larger than the diameter estimates at higher frequencies. Here also we cannot rule out the possibility that the source instead of having a core halo structure actually consists of a number of point components. Occultation observations at 318 MHz reported by Lang (1970) indicate that the source is unresolved which goes against the core halo model. Clark et al. (1973) have made VLBI observations of this source at 111.5 MHz and 196.5 MHz with the NRAO Arecibo baseline. The system has a resolution of about 0."04 at 196.5 MHz and 0."07 at 111.5 MHz. They find that 35% of the 111.5 MHz flux and 20% of the 196.5 MHz flux to be unresolved. With the limited u-v coverage however, one cannot make an estimate of the actual structure of the source.

NRAO 91

From the observed fact that the second moment does not increase below p = 0.2 A.U., as it does for other sources, we have estimated the diameter to be equal to $\lesssim$ 0."1 at position angle 60° north to east. The scintillation index of this source, as pointed out earlier, is generally 20% higher than that of CTA 21, but we have not considered this particularly significant. The agreement between the various IPS observations is rather poor. VLBI observations by Clarke et al. (1969) at 448 MHz indicate a diameter around 0".04 if the source is simple Gaussian. Clark et al. (1973) find that at both 196.5 and 111.5 MHz, about half the flux of the source is unresolved. It seems likely that for this source the simple Gaussian model is inadequate and the source has considerable fine structure less than 0".1.

CTA 21

High resolution interferometric observations of this source have been made at a number of frequencies but here we will give only the results at low frequencies. Anderson and Donaldson (1967) at 408 MHz have found the source to be unresolved and they have set an upper limit on its diameter of < 0".3 x < 0".2. This observation which is in good agreement with high frequency VLBI observations is the main basis for assuming that $\mu = 1$ for this source. Clarke et al. (1969) using VLBI at 408 and 448 MHz have given a double structure for this source with two equal components of diameter less than 0".01 separated by 0".11 in position angle 70°. This model agrees with neither the high frequency VLBI observations nor the IPS observations since it predicts that close to the Sun the power spectrum should show a modulation with nulls appearing every 0.6 Hz and even though the width, stability and resolution of our spectra are all favourable, we do not see the modulation. Only by invoking large random velocities in the medium can we reconcile our observations with this model. At 111.5 MHz Clark et al. (1973) find about 35% of the flux to be unresolved.

3C138

All the flux of this source is in the scintillating component whose angular diameter, as estimated from the second moment varies with position angle. For the observations between June 28 and July 4, the position angle of the solar wind across the source is around 250° and for these observations the mean value of the second moment is around 0.45 Hz which is less than that for a point source. Using equation (6.6) we get $\psi \approx 0''.25$ in position angle 250°. If the source had a circularly symmetrical structure with this diameter, the second moment should not exceed about 0.5 Hz even when the source comes close to the Sun. However, the observations on July 7, 10 and 11 give second moments greater than 1 Hz. Since for these days the position angle of the solar wind across the source is about 150°, which is roughly perpendicular to that of the earlier observations, the simplest explanation of the data is that the source has an elongated structure with diameter $\sim 0''.25$ in position angle 250° and less than $0''.1$ in the perpendicular direction. Cohen (1969) also reports an elongated structure with major axis in position angle 40°. Anderson and Donaldson (1967), using interferometry at 410 MHz find the source to be elongated and they give its dimensions as $0''.47 \times < 0''.29$ with major axis in position angle 64°.

3C446

The structure of this source also varies with position angle. The entire flux of the source is in a component with dimensions $\sim 0''.25 \times < 0''.1$ with major axis in position angle 260°. Clarke et al. (1969) at 408 and 448 MHz estimate its diameter to be equal to $0''.03$ in position angle 95°. At 21 cm Miley et al. (1967) find the source to be elongated with size $\gtrsim 0''.4$ in position angle 270°.

CHAPTER 7

CONCLUSIONS

In this thesis we have discussed the theory of Interplanetary Scintillations and used the results to interpret our single station observations of IPS at 327 MHz.

We have discussed the use and range of validity of the thin screen model in interpreting the observations of IPS. Using a unified formalism we have derived expressions for the power spectrum and scintillation index in the Weak scattering, Geometrical optics and Far field approximations. The problem of a pattern that changes as it drifts has been considered and it is shown that in the strong scattering regime the apparent lifetime of the intensity fluctuations on the ground can be much smaller than the actual lifetime of the phase pattern on the screen. We have discussed the effect of the evolution of the pattern on the estimation of the various parameters of the medium. We have suggested that the enhancement of turbulence in the solar wind close to the Sun that has been estimated by Ekers and Little (1971) could be an artifact of strong scattering and that the increase in turbulence can be established only by making multistation observations of IPS at two or more frequencies and by showing that the estimated turbulence does not change with frequency.

From our observations at 327 MHz we have estimated ϕ_0, the r.m.s. phase fluctuation produced by the IP medium, and a, the scale size of the irregularities, for distances between 0.25 and 0.7 A.U. from the Sun. ϕ_0 was found to vary as a power law having the form

$$\phi_0 = 0.04\ p^{-1.6}$$

which is in good agreement with the measurements of other workers. The scale size of the irregularities was roughly constant in this range of distances and had a value of about 100 km. This disagrees with the earlier work of Hewish and Symonds (1969) and Little (1971) which indicated that the scale size increases roughly linearly with distance from the Sun. We have shown that the earlier observations are all compatible with the scale sizes being roughly constant for distances greater than 0.25 A.U. and that the discrepancy arises mainly because of corrections that have been made earlier for the effect of the finite diameter of the scintillating sources. From our measurements of the power spectrum observed when the sources were close to the Sun, we have shown that for our observations at $p > 0.25$ A.U., the correction for the source size is negligible. Since our estimates of the scale size are in general agreement with the uncorrected estimates of earlier workers, we feel that the source size corrections that have been made earlier are not necessary. By combining the estimates by different workers at different distances from the Sun, we have tried to form an overall picture of the variation of the scale size with distance from the Sun. We find that though the scale size is roughly independent of distance from the Sun for $p > 0.20$ A.U. the scale size decreases rapidly closer to the Sun. The agreement between observations of different workers is rather poor and it is difficult to give a quantitative description of the variation of the scale size with distance from the Sun. Since this variation is likely to give us information on the origin of the small scale irregularities in the interplanetary medium, systematic multi frequency observations of IPS should be made so that the exact variation of the scale size with distance from the Sun can be established.

The turnover in the scintillation index close to the Sun has been examined and it has been shown that the turnover cannot be explained by the simple diameter blurring hypothesis. In this model the turnover is accompanied by a saturation of the width of the power spectrum. However the observed second moment of the power spectrum increases sharply in the turnover region and shows no sign of saturating, which is incompatible with the diameter blurring hypothesis. While it may be possible to explain the observations by invoking complicated models for the structure of the source or by invoking an increase in the turbulence in the solar wind close to the Sun, we feel that the turnover is associated with the onset of strong scattering, for which, however, we do not have a fully satisfactory theory. For understanding the origin of the turnover, it is important to establish the exact variation of the scintillation index in the turnover region. In this connection, we have pointed out that in the turnover region where the power spectrum becomes broad, the time constant used for the observations plays an important role in determining the shape of the observed m-p curve. Before comparing the observed m-p curve with that of a model one must correct the observed curve for the attenuation of the high frequencies by the time constant. This correction has seldom been made before and this is the reason why the scintillation index observed by most workers decreases sharply in the turnover region.

We have estimated the Bessel transforms of the intensity fluctuations with the hope of observing the Fresnel modulation which would enable us to estimate the velocity of the solar wind from single station observation. The Bessel transforms, which were estimated by taking the Abel transforms of the power spectra of the intensity fluctuations, seldom showed the Fresnel modulation and it was not possible to estimate the solar wind velocity. We have attributed the absence of the modulation to the presence of an irregular component in the velocity of the solar wind.

For the nine sources that were observed in detail, we have made estimates of the angular diameter and the fraction of the flux in the scintillating component. Two of the sources, 3C138 and 3C446 showed variation in the structure with position angle, indicating that the sources have an elliptical structure.

Most of the uncertainties in the interpretation of the IPS observations arise from the fact that we do not have independent estimates of the angular structures of the scintillating sources, so that we have to make a self consistent picture for both the angular structure of the source and the scattering properties of the interplanetary medium. High resolution observations at low frequencies of some of the sources commonly used for scintillation studies would be of great assistance in resolving this difficulty. The resolution required would be about 0.″05, which can be achieved only by VLBI techniques. There have been a few VLBI observations of radio sources at metre wavelength but the baseline coverage is too small for us to make reliable models for the source. Further low frequency VLBI observations of the standard scintillation sources should be carried out since only then can we satisfactorily settle the question of the necessity of the source size correction to the observed scale size and the origin of the turnover in the scintillation index close to the Sun.

Appendix A

Reduction of the five fold integral occurring in the expression for the spatial power spectrum.

The spatial power spectrum of the intensity fluctuations on the ground is given by

$$M_z^2(\underline{q}) + \delta(\underline{q}) = \left(\frac{k}{2\pi z}\right)^4 \iiiint d\underline{\xi}_1\, d\underline{\xi}_2\, d\underline{\xi}_3\, d\underline{\xi}_4\, d\underline{r} \times$$

$$e^{i\underline{q}\cdot\underline{r}}\, F(\underline{\xi}_1, \underline{\xi}_2, \underline{\xi}_3, \underline{\xi}_4) \times$$

$$\exp\left[\frac{ik}{2z}\left\{(\underline{\xi}_1 - \underline{\xi})^2 - (\underline{\xi}_2 - \underline{\xi})^2 + (\underline{\xi}_3 - \underline{\xi} - \underline{r})^2 - (\underline{\xi}_4 - \underline{\xi} - \underline{r})^2\right\}\right] \qquad \text{(A1)}$$

where $F(\underline{\xi}_1, \underline{\xi}_2, \underline{\xi}_3, \underline{\xi}_4)$ is a function only of the differences of the ξ_is taken in pairs i.e.

$$F(\underline{\xi}_1, \underline{\xi}_2, \underline{\xi}_3, \underline{\xi}_4) = F(\underline{\xi}_1 - \underline{\xi}_2, \underline{\xi}_1 - \underline{\xi}_3, \cdots, \underline{\xi}_i - \underline{\xi}_j, \cdots)$$

where i and j go from 1 to 4.

Making the substitution

$$\underline{\eta}_1 = \underline{\xi}_1 - \underline{\xi} \quad , \quad \underline{\eta}_2 = \underline{\xi}_2 - \underline{\xi}$$

$$\underline{\eta}_3 = \underline{\xi}_3 - \underline{\xi} \qquad \underline{\eta}_4 = \underline{\xi}_4 - \underline{\xi}$$

which leaves F unchanged, since it depends only on the difference of the $\underline{\xi}$ is', equation (A1) becomes

$$M_z^2(q) + \delta(\underline{q}) = \left(\frac{k}{2\pi z}\right)^4 \iiiint d\underline{\eta}_1\, d\underline{\eta}_2\, d\underline{\eta}_3\, d\underline{\eta}_4\, d\underline{r}$$

$$e^{i\underline{q}\cdot\underline{r}} \; F(\underline{\eta}_1, \underline{\eta}_2, \underline{\eta}_3, \underline{\eta}_4) \times$$

$$\exp\left[i \frac{k}{2z} \left\{ \underline{\eta}_1^2 - \underline{\eta}_2^2 + (\underline{\eta}_3 - \underline{r})^2 - (\underline{\eta}_4 - \underline{r})^2 \right\} \right] \tag{A2}$$

If we expand

$$(\underline{\eta}_3 - \underline{r})^2 - (\underline{\eta}_4 - \underline{r})^2 = \underline{\eta}_3^2 - \underline{\eta}_4^2 + 2\underline{r}\cdot(\underline{\eta}_4 - \underline{\eta}_3)$$

and collect the terms containing $\underline{r}$, the $\underline{r}$ integral has the form

$$\int d\underline{r}\; e^{i\underline{q}\cdot\underline{r}} \; e^{i\frac{k}{z}\underline{r}\cdot(\underline{\eta}_4 - \underline{\eta}_3)}$$

$$= \int d\underline{r}\; \exp i\frac{k}{z}\,\underline{r}\cdot\left\{\underline{\eta}_4 - \underline{\eta}_3 + \frac{z}{k}\underline{q}\right\}$$

$$= \left(\frac{2\pi z}{k}\right)^2 \delta\left(\underline{\eta}_4 - \underline{\eta}_3 + \frac{z}{k}\underline{q}\right)$$

If we insert this in equation (A2) and perform the integral over $\underline{\eta}_3$ we get

$$M_z^2(\underline{q}) + \delta(\underline{q}) = \left(\frac{k}{2\pi z}\right)^2 \iiint d\underline{\eta}_1\, d\underline{\eta}_2\, d\underline{\eta}_3$$

$$F(\underline{\eta}_1, \underline{\eta}_2, \underline{\eta}_4 + \tfrac{z}{k}\underline{q}, \underline{\eta}_4) \exp \frac{ik}{2z}\left\{\underline{\eta}_1^2 - \underline{\eta}_2^2 + (\underline{\eta}_4 + \tfrac{z}{k}\underline{q})^2 - \underline{\eta}_4^2\right\}$$

(A3)

If we make the substitution

$$\underline{x}_1 = \underline{\eta}_1 - \underline{\eta}_4 \quad , \quad \underline{x}_2 = \underline{\eta}_2 - \underline{\eta}_4$$

$$\underline{x}_4 = \underline{\eta}_4$$

and expand and simplify the squares occurring in the exponential, equation (A3) becomes

$$M_z^2(\underline{q}) + \delta(\underline{q}) = \left(\frac{k}{2\pi z}\right)^2 e^{i\frac{z}{2k}\underline{q}^2} \iiint d\underline{x}_1\, d\underline{x}_2\, d\underline{x}_4 \times$$

$$\exp \frac{ik}{2z}\left\{x_1^2 - x_2^2\right\} \exp \frac{ik}{z}\, \underline{x}_4 \cdot (\underline{x}_1 - \underline{x}_2 + \tfrac{z}{k}\underline{q})$$

$$F(\underline{x}_1 + \underline{x}_4, \underline{x}_2 + \underline{x}_4, \tfrac{z}{k}\underline{q} + x_4, x_4)$$

(A4)

The coordinate $\underline{x}_4$ appears in all the four arguments of F and since F depends only on the difference of the four arguments taken in pairs, it is clear that $F(\underline{x}_1+\underline{x}_4, \underline{x}_2+\underline{x}_4, \underline{x}_3+\underline{x}_4, \underline{x}_4)$ is independent of $\underline{x}_4$. So we can write

$$F(\underline{x}_1+\underline{x}_4, \underline{x}_2+\underline{x}_4, \frac{z}{k}\underline{q}+\underline{x}_4, \underline{x}_4) = F(\underline{x}_1, \underline{x}_2, \frac{z}{k}\underline{q}, 0)$$

Since F is independent of $\underline{x}_4$, the integration over $\underline{x}_4$ can be performed giving a δ function. Equation (A4) now becomes

$$M_z^2(\underline{q}) + \delta(\underline{q}) = e^{i\frac{z}{2k}q^2} \iint d\underline{x}_1\, d\underline{x}_2 \times$$

$$\exp \frac{ik}{2z}\{x_1^2 - x_2^2\}\, \delta(\underline{x}_1 - \underline{x}_2 + \frac{z}{k}\underline{q})\, F(\underline{x}_1, \underline{x}_2, \frac{z}{k}\underline{q}, 0)$$

(A5)

Integrating over $\underline{x}_2$, equation (A5) becomes

$$M_z^2(\underline{q}) + \delta(\underline{q}) = e^{i\frac{z}{2k}q^2} \int d\underline{x}_1 \exp \frac{ik}{2z}\left\{x_1^2 - (\underline{x}_1 + \frac{z}{k}\underline{q})^2\right\}$$

$$F(\underline{x}_1, \underline{x}_1 + \frac{z}{k}\underline{q}, \frac{z}{k}\underline{q}, 0)$$

$$= \int d\underline{x}\; e^{-i\underline{q}\cdot\underline{x}}\, F(\underline{x}, \underline{x}+\frac{z}{k}\underline{q}, \frac{z}{k}\underline{q}, 0)$$

(A6)

Since the power spectrum is always real and symmetrical in $\underline{q}$ i.e. $M_z^2(\underline{q}) = M_z^2(-\underline{q})$ we can change the sign of $\underline{q}$ on the right hand side which gives

$$M_z^2(\underline{q}) + \delta(\underline{q}) = \int d\underline{x}\, e^{i\underline{q}\cdot\underline{x}}\, F(\underline{x}, \underline{x} - \tfrac{z}{k}\underline{q}, -\tfrac{z}{k}\underline{q}, 0) \tag{A7}$$

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