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Mechanical Pointing Model for GMRT antennas

Author:
Prakash Gaikwad

Supervisor:
Dr. Nimisha Kantharia

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1 INTRODUCTION

The basic function of radio telescope is to collect radio waves coming from cosmic source. These waves create fluctuating voltage at the antenna terminal. This voltage varies at the same frequency as the cosmic electromagnetic wave. This voltage is then subsequently processed to get output which is used to infer properties of source. Any mechanical structure suffers from mechanical imperfections e.g. misalignment of two axis or non orthogonality of two axis. Such errors would cause a systematic variations in apparent azimuth and elevation angle from true azimuth and elevation angle. e.g. If we command an antenna to point at $Az = x$ and $E = y$, due to pointing error it will point in direction $Az = x + \Delta x$ and $E = y + \Delta y$. In addition to mechanical imperfection, gravitational bending of support structure introduces elevation dependent offset. So why is it important to correct for pointing errors? If some source is discovered using radio telescope and one wants to observe the same source at some other wavelength say X-ray. Due to large pointing errors uncertainty in location of source will be high. In fact there is chance that more than one sources are present in that wavelength range. Moreover the flux received by antenna depends on pointing direction. Therefore it is important to point antenna in true direction where source is actually located. Pointing model helps to correct these errors and point antenna in source direction with reasonable accuracy.
A pointing model based on the model given by Greve et.al.(1996) and Ulich (1981), both of which use the derivation of Stumpf(1971), is determined from the observed GMRT data and applied to the data. GMRT observation was done at 1280 MHz to determine pointing offsets. The frequency, 1280 MHz, was chosen because beam is narrow at this frequency so pointing errors would be higher. Pointing offsets are found to vary by $4 - 6\,'$ with azimuth and $2 - 3\,'$ with elevation.

2 OBSERVATIONS[3]

Half Power Beam Width (HPBW) of an antenna is defined as the angle across the main lobe of an antenna pattern between the two direction at which the antenna's sensitivity is half its maximum value at the center of the lobe and is given as $HPBW \approx \frac{\lambda}{d}$. GMRT operates at frequencies around 153, 233, 325, 610 and 1200 MHz. Corresponding HPBW at all such frequencies is $2.88°$, $1.85°$, $1.35°$, $42.8'$, $19.43'$ respectively. So the 1200 MHz frequency will be most affected by pointing accuracy since its HPBW is lowest. In the range of 1000-1580 MHz there are 4 sub-bands each of bandwidth 140 MHz centered on 1060, 1170, 1280 and 1390 MHz. For our observation we choose 1280 MHz frequency.
To measure pointing offset, the method of grid pointing was used. In this method projected sky is divided into Elevation and Azimuth grids (see figure 1). The dimensions of grid depends on frequency of observation. This method uses uncalibrated cross-correlation data. Since one antenna (also called as refer-

Table 2.1: Source list for Oct 2006 and May 2007 observation

Source	Flux (Jy)	Declination	Source	Flux (Jy)	Declination
3C48	16.5	33.19°	0022+002	2.7	0.29°
3C286	15	30.47°	0059+001	2.5	0.15°
1445+099	2.6	9.94°	0025-260	8.3	25.99°
2214-385	1.89	−38.56°	0024-420	2.8	−41.99°
0217+738	2.27	73.86°	0204+152	4	15.27°
0110+565	1.9	56.58°	0116-208	3.6	−20.83°
0432+416	8.6	41.66°	0440-435	5	−43.54°
3C147	22.5	49.86°	0409-179	2.2	−17.93°

ence antenna) is constantly tracking source, pointing behavior of maximum 29 antennas can be modeled. In this method, a one dimensional grid of points is observed along the elevation to determine the pointing offsets along the elevation axis and a similar grid is observed along the azimuth axis for determining the azimuth offsets. While observing grid along the elevation, the source is constantly tracked in azimuth. Whereas while observing grid along azimuth axis, the source is constantly tracked in elevation direction. Pointing observation at each grid point will correspond to an amplitude of flux received. This will produce a point for each observation at each grid point in amplitude vs offset plot (see figure 2). Then Gaussian can be fitted to all data points. The peak of Gaussian curve will correspond to actual source position. The difference between reference position and peak of Gaussian curve is the offset.
To plot Gaussian at least 5 points are needed. For better fitting we chose 9 points in one dimensional grid. At 1280 MHz, HPBW $\approx 19'$ So we decided to observe the grid up to $20'$ on either side of the reference position with grid spacing of $5'$. The grid of points along the elevation axis were first observed followed by the grid of points along the azimuth axis. After observing each grid point, the source position was loaded into the antenna control system and then the offset broadcast to the antennas with respect to the source position. Thus in practice the antennas were moved along both the elevation and azimuth axis between two grid points.

2.1 Selection of Sources

In order to estimate a good pointing model one needs to observe well distributed sources in the entire sky. There are two methods to select sources for estimating a pointing model:
1) Observe several sources with different co-ordinates (right ascension and declination) so as to obtain good sky coverage.
2) Observe a few selected strong sources from rise to set. The sources are selected such that they have different declinations so as to result in extensive sky coverage in Az-El space. This method is easier and less prone to mistakes.
The sources were selected from VLA (Very Large Array) calibrator list. A total of 85 calibrators which had PPPP or one S flag for all the VLA arrays and

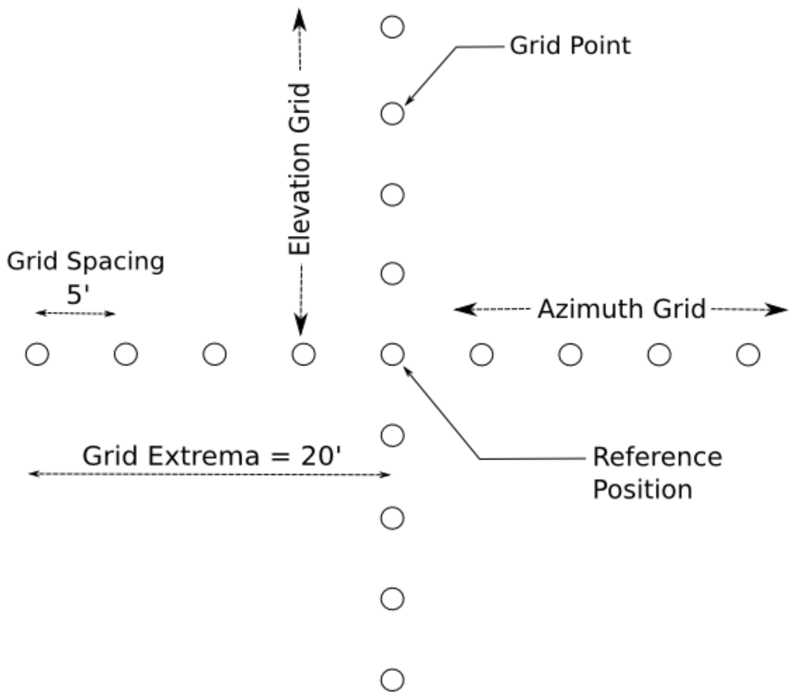

Figure 1: Grid pointing

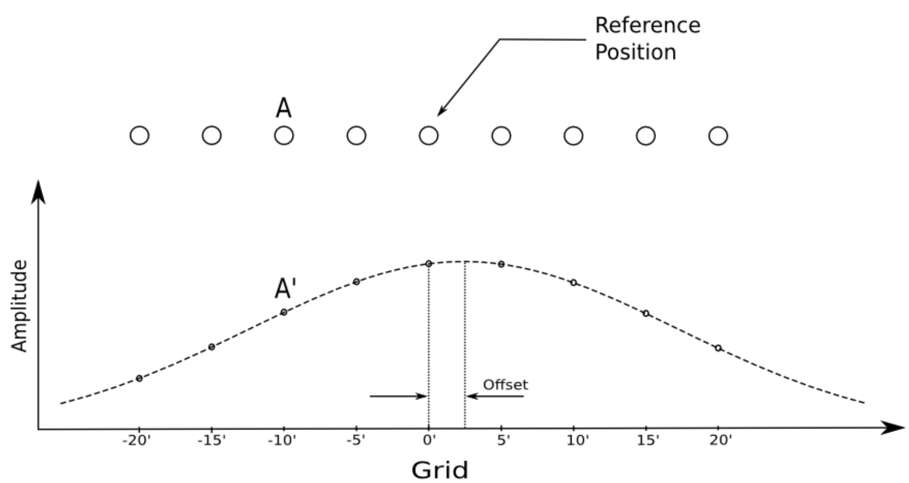

Figure 2: Offset Measurement

strength was greater than 1 Jy at 20 cm were selected. Sources observed on October 2006 and May 2007 are listed in Table 2.1. First 4 sources were observed in Oct-2006 using method 1. Whereas rest sources are observed in May-2007. These sources are observed from rise to set in order to cover sky as much as possible see Figure 3. Based on these measurements we estimated pointing models for two data sets one for Oct-2006 and other for May-2007.

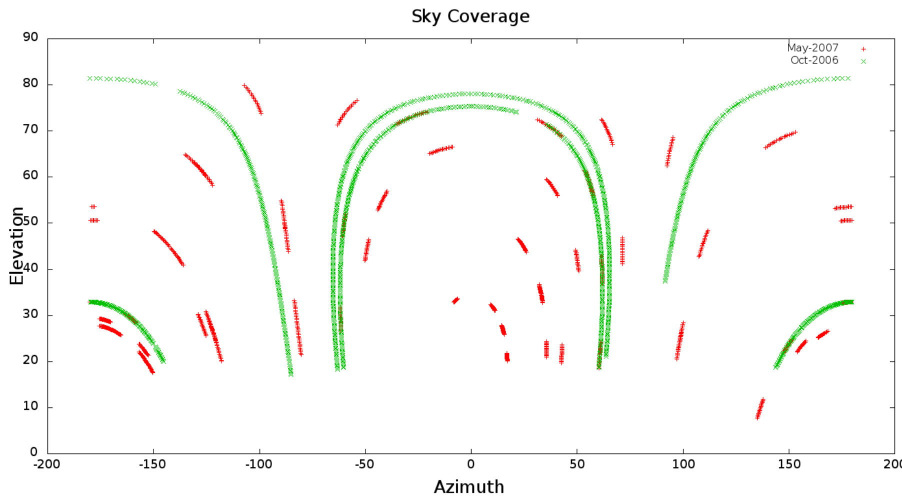

Figure 3: Sky coverage

3 POINTING MODEL[2,7]

Greve et al. (1996) [2] and Ulich (1981) [7] proposed a pointing model which was based on derivation by Stumpf (1971). The model is based on corrections due to mechanical imperfections of antenna. It depends on 9 parameters P_1 to P_9. For GMRT the relevant parameters are 8 (we are neglecting error in the declination of source). All GMRT antennas are Alt-Az mounted. So Azimuth (A) and Elevation (E) are required to locate a source in the sky. Because of pointing errors generally there exists a difference δA and δE between the commanded and the actual Azimuth (A) and Elevation (E). The aim of pointing model is to minimize errors δA and δE for a considerable time of operation. These mis pointing terms can be decomposed into horizontal $\delta h = \delta A \cos(E)$ and vertical components $\delta v = \delta E$. The errors are small and hence we can apply linear pointing model in which case the parameters are related to corresponding errors by,

$$\delta h_i = P_i H_i(A, E) \quad \delta v_i = P_i V_i(A, E)$$

Where $H_i(A, E)$ and $V_i(A, E)$ are functions of A and E as given in Table 3.1 . Significance of various parameters are as follows,

Table 3.1: Pointing[2] parameters P_i

Type of error		$H_i(A,E)$	$V_i(A,E)$
Collimation error $^{a)}$	P_1	1	0
Zero-offset Az-encoder	P_2	$\cos E$	0
Inclination El-axis $^{b)}$	P_3	$\sin E$	0
Inclination Az-axis $^{c)}$ N - S	P_4	$\cos A \sin E$	$-\sin A$
Inclination Az-axis $^{c)}$ E - W	P_5	$\sin A \sin E$	$\cos A$
Zero-offset El-encoder	P_6	0	1
Gravitational bending $^{d)}$	P_7	0	$\cos E$
Gravitational bending $^{d)}$	P_8	0	$\sin E$

Collimation error P_1: This error is due to non-orthogonality of the radio beam axis and the elevation axis.
Zero-offset Az-Encoder P_2: Azimuth encoder value (i.e. commanded value) of antenna zero but due to mis pointing Actual value of azimuth will be nonzero.
Inclination El-axis P_3: This error is due to non-orthogonality of the azimuth axis and the elevation axis.
Inclination Az-axis N-S P_4: This error arises due to inclination of azimuth axis towards N-S plane. For ideal case inclination should be zero.
Inclination Az-axis E-W P_5: This error arises due to inclination of azimuth axis towards E-W plane. For ideal case inclination should be zero.
Zero-offset El-Encoder P_6: Elevation encoder value of antenna zero but due to mis pointing Actual value of elevation will be nonzero.
Parameters P_7 and P_8 are necessary to take gravitational deformation (of the telescope) into account.

$$\delta h = P_1 + P_2 \cos E + P_3 \sin E + P_4 \sin E \cos A + P_5 \sin E \sin A \tag{3.1}$$

$$\delta v = -P_4 \sin A + P_5 \cos A + P_6 + P_7 \cos E + P_8 \sin E \tag{3.2}$$

Thus δh and δv are corrections in horizontal and vertical directions to account for the pointing errors. The above two equations are coupled together due to parameters P_4 and P_5 which makes it difficult to solve linearly. First we tried to minimize (3.1) and (3.2) separately. The values of parameters P_4 and P_5 obtained from two equations do not match. So one needs to minimize δh and δv simultaneously.

4 MODEL FITTING[4]

To fit a model to data one needs to choose or design a merit function which measures agreement between the data and the model with particular choice of parameters. Given a set of observations, the aim of model fitting is to find out the values of adjustable parameters which will minimize a merit function. A fitting procedure should provide

1. Best fit parameter values
2. Errors estimates on the parameters and
3. Statistical measure of goodness-of-fit.

As evident from equation (3.1) and (3.2) δh and δv are coupled together due to common parameters P_4 and P_5. Therefore we need to find parameters P_1 to P_8 so that both equations are minimized simultaneously. One obvious choice of merit function is,

$$\chi^2 = \sum \left[\sqrt{\delta {h_m}^2 + \delta {v_m}^2} - \sqrt{\delta {h_c}^2 + \delta {v_c}^2} \right]^2$$

We can imagine δh and δv as points in projected sky. Consider transformation from Cartesian to polar co-ordinate $(x, y) \rightarrow (r, \theta)$, so χ^2 in above equation is equivalent to r^2 in polar co-ordinate. When one tries to minimize χ^2 function with $\sqrt{\delta {h_m}^2 + \delta {v_m}^2}$ without using θ, δh_m may converge to δh_c or $-\delta h_c$ and similarly δv_m may converge to δv_c or $-\delta v_c$. So there is degeneracy in using above equation as merit function.
Instead if we try to minimize function

$$\chi^2 = \sum \{\delta h_m - \delta h_c\}^2 + \{\delta v_m - \delta v_c\}^2$$

we will be sure that for minimum value of χ^2, δh_m will converge to δh_c and similarly δv_m will converge to δv_c. Having decided which χ^2 function to use we can now think of different numerical methods. The above equation is nonlinear in parameters P_1 to P_8. So this problem can be solved using iterative optimization method. Before trying to minimize we may want to know qualitative behavior of function χ^2.

In general nonlinear functions can be thought of as a multidimensional hypersurface in parameter space and the space must be searched for the appropriate minimum value of χ^2. One of the difficulties of such a search is that for any arbitrary function there may be more than one local minimum for χ^2 within a reasonable range of values for the parameters. It may be advantageous to conduct a coarse grid mapping of the parameter space to locate the main minimum and identify the desired range of parameters over which to refine the search. In simplest brute force method,[1] permissible range for each parameter P_i is divided into n equal increments ΔP_i so that the parameter space is divided into hypercubes. The value of χ^2 is then calculated at each of the vertices of these hypercubes. This procedure yields a coarse map of the behavior of χ^2 as a function of all of the parameters P_i. Here we are dealing with 8 dimensional space. So for the equal increment of say 10, one needs to evaluate χ^2 10^8 times. This is computationally more time consuming if one wants more finer grids on hypersurface. (Finer grids corresponds to more number of increments of parameters). However there is alternative method which traverses only in the general direction of minimizing χ^2.

4.1 Ravine search: Qualitative method[5]

A more sophisticated method of locating various minima of the χ^2 hypersurface involves traversing the surface from minimum to minimum by the path of lowest value in χ^2, as a river follows a ravine in traversing from lake to lake. e.g. consider a hypersurface in 2 dimensions.

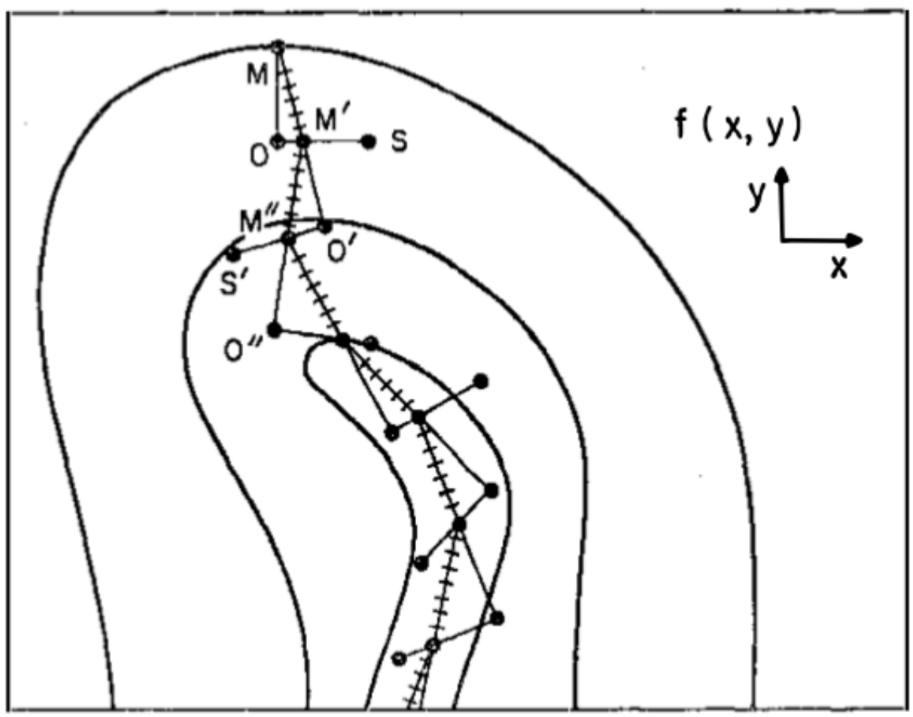

Figure 4: Diagrammatic illustration of the stepping procedure for a function of two variables. The starting point is M_i; the new minima at M', M'', $\cdots$ are represented by dots on the "railroad track" along the bottom of the ravine. The value of the function is calculated not only at the minima, but also at the "oversteps" O, $O' \cdots$ and the "sidesteps" S, S'. Derivatives are calculated at the overstep points only. See text for notation and stepping logic

Figure 4 shows "contour lines" for the function [i.e., all (x, y) on a line have the same value for the function] See reference.[5] The hunting procedure involves in sequence two different sorts of steps, an "overstep" to a point O and a "side step" to S, followed by calculation of a new minimum along the line OS; then the cycle repeats with a new overstep to point O' from the last minimum and through the new minimum. In more detail, the search starts at point M, where the starting direction is taken as being along the gradient. A step of fixed size is taken along this direction to a "reconnaissance" point O, where the function and its gradient are determined. A step is taken transverse to the line MO

from the point O to point S. At point S the function is evaluated. From the information available at points O and S, a minimum is predicted at point M' along line OS by assuming the function varies quadratically along the line OS. See equation (4.2) for reference. The function is calculated at point M' to verify the minimum at that point. (In case M' is not minimum relative to O or S, a smaller sidestep is taken) To complete the cycle, a step is now taken to a point O' along the line M M', and the operation repeats as described at point O. As described, this procedure does not stop at the minimum but continues right on along the "ravine" in the function. This is a useful mode of operation, for it allows a mapping of those sets of parameters which give rise to values of function near the minimum. This procedure tracks a ravine until about the spot where the minimum radius of the "contour lines" exceeds the step size, after which the search usually doubles back on itself. The actual position of the minimum is determined by altering the above procedure to the extent of reducing the step size and reversing direction each time steps lead to increased values of function.
Quadratic approximation of function: Assuming the variation of χ^2 near the minimum can be described in terms of a parabolic function of the parameter a_j, we can use the values of χ^2 for the last three values of a_j to determine the minimum of the parabola in figure (5)

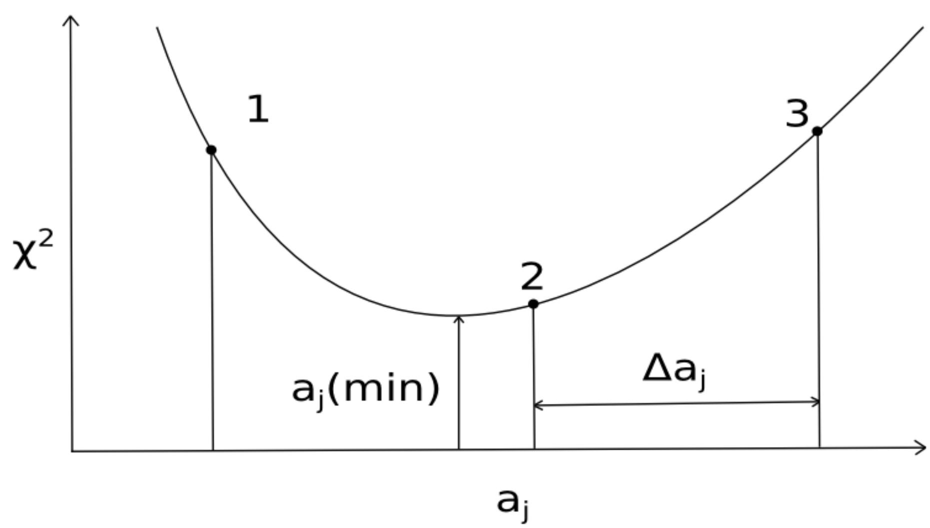

Figure 5: Parabolic approximation

$$\begin{aligned} a_j(3) = a_j(2) + \Delta a_j = a_j(1) + 2\Delta a_j \\ \chi^2(3) > \chi^2(2) \leq \chi^2(1) \end{aligned} \tag{4.1}$$

The minimum of parabola[1] is given by

$$a_j(min) = a_j(3) - \Delta a_j \left[\frac{\chi^2(3) - \chi^2(2)}{\chi^2(3) - 2\chi^2(2) + \chi^2(1)} + \frac{1}{2} \right] \tag{4.2}$$

4.2 Non-linear Numerical Methods[4]

The basic approach in model fitting is to define χ^2 merit function and determine best fit parameters by its minimization. With nonlinear dependences, however the minimization must proceed iteratively. Given trial values for parameters, we develop a procedure that improves a trial solution. The procedure is then repeated until χ^2 effectively stops decreasing.

Let $\chi^2(\mathbf{a})$ be a function to be minimized. Choosing some particular point $\mathbf{P}$ as the origin of co-ordinate system with co-ordinate $\mathbf{a}$. Expanding function $\chi^2(\mathbf{a})$ using Taylor's series[4]

$$\chi^2(\mathbf{a}) = \chi^2(\mathbf{P}) + \sum_i \frac{\partial \chi^2}{\partial a_i} a_i + \frac{1}{2} \sum_{i,j} \frac{\partial^2 \chi^2}{\partial a_i \partial a_j} a_i a_j + \cdots \tag{4.3}$$

which can be approximated to,

$$\chi^2(\mathbf{a}) \approx \gamma - \mathbf{d} \cdot \mathbf{a} + \frac{1}{2} \mathbf{a} \cdot \mathbf{D} \cdot \mathbf{a} \tag{4.4}$$

where,

$$\gamma \equiv \chi^2(\mathbf{P}) \qquad \mathbf{d} \equiv -\nabla \chi^2|_{\mathbf{P}} \qquad [D]_{ij} \equiv \frac{\partial^2 \chi^2}{\partial a_i \partial a_j} \tag{4.5}$$

The matrix $\mathbf{D}$ whose components are the second partial derivative matrix of the function is called the Hessian matrix of the function at $\mathbf{P}$. With such approximation one can adopt either Steepest Descent method or Quasi-Newton method for minimization.

Steepest Descent method: Start at a point $\mathbf{a}_0$. As many times as needed, move from point $\mathbf{a}_i$ to the point $\mathbf{a}_{i+1}$ by minimizing along the line from $\mathbf{a}_i$ in the direction of the local downhill gradient $-\nabla \chi^2(\mathbf{a}_i)$. So in above case, Next trial value of parameters can be obtained from current parameter value as

$$\mathbf{a}_{next} = \mathbf{a}_{cur} - constant \times \nabla \chi^2(\mathbf{a}_{cur}) \tag{4.6}$$

Where constant is small enough not to exhaust the downhill direction.

Quasi-Newton Method: Consider finding minimum by using Newton's method to search for a zero of the gradient of the function. Near the current point $\mathbf{a}_{cur}$ we have to expand

$$\chi^2(\mathbf{a}_{next}) \approx \chi^2(\mathbf{a}_{cur}) + (\mathbf{a}_{next} - \mathbf{a}_{cur}) \cdot \nabla \chi^2(\mathbf{a}_{cur}) + \frac{1}{2} (\mathbf{a}_{next} - \mathbf{a}_{cur}) \cdot \mathbf{D} \cdot (\mathbf{a}_{next} - \mathbf{a}_{cur})$$

$$\nabla \chi^2(\mathbf{a}_{next}) = \nabla \chi^2(\mathbf{a}_{cur}) + \mathbf{D} \cdot (\mathbf{a}_{next} - \mathbf{a}_{cur}) \tag{4.7}$$

So in this method we set $\nabla\chi^2(\mathbf{a}_{next}) = 0$ to determine next iteration point.

$$\mathbf{a}_{next} - \mathbf{a}_{cur} = -\mathbf{D}^{-1} \cdot \nabla\chi^2(\mathbf{a}_{cur}) \tag{4.8}$$

The left hand side in (4.8) is the finite step we need to take to get to the exact minimum, the right hand side is known once we have accumulated an accurate $\mathbf{H} \approx \mathbf{D}^{-1}$

To use (4.6) or (4.8) we must be able to compute the gradient of the χ^2 function at any set of parameters $\mathbf{a}$. In addition for equation (4.8) we also need the matrix $\mathbf{D}$ which is the second derivative matrix (Hessian Matrix) of the χ^2 merit function at any $\mathbf{a}$. We know the form of χ^2 from model therefore we can calculate gradient of the χ^2 and hessian matrix $\mathbf{D}$. The χ^2 merit function is,

$$\chi^2 = \sum_{i=0}^{N-1} \frac{1}{{\sigma_i}^2} \left\{ \left[\Delta h_i - S_1^{(i)}\right]^2 + \left[\Delta v_i - S_2^{(i)}\right]^2 \right\} \tag{4.9}$$

Where,

$$S_1^{(i)} \equiv S_1(E_i, A_i) = P_1 + P_2 \cos E_i + P_3 \sin E_i + P_4 \sin E_i \cos A_i + P_5 \sin E_i \sin A_i$$
$$S_2^{(i)} \equiv S_2(E_i, A_i) = -P_4 \sin A_i + P_5 \cos A_i + P_6 + P_7 \cos E_i + P_8 \sin E_i$$

σ_i is the measurement error (standard deviation) of the i^{th} data point.
From observation for each azimuth A_i and elevation E_i angle, we obtained horizontal offset Δh_i and vertical offset Δv_i. We would like to find values of parameters P_1 to P_8 for which function χ^2 given by (4.9) is minimum.
The gradient of χ^2 with respect to the parameter P_k $(k = 1, 2, \cdots, 8)$, which will be zero at the minimum χ^2, has components.

$$\frac{\partial\chi^2}{\partial P_k} = \sum_{i=0}^{N-1} \frac{2}{{\sigma_i}^2} \left[(\Delta h_i - S_1^{(i)}) \frac{\partial S_1^{(i)}}{\partial P_k} + (\Delta v_i - S_2^{(i)}) \frac{\partial S_2^{(i)}}{\partial P_k} \right] \tag{4.10}$$

Taking an additional partial derivative gives

$$\begin{aligned} \frac{\partial^2\chi^2}{\partial P_k \partial P_l} = \sum_{i=0}^{N-1} \frac{2}{{\sigma_i}^2} \Bigg[& - \frac{\partial S_1^{(i)}}{\partial P_k} \frac{\partial S_1^{(i)}}{\partial P_l} - \frac{\partial S_2^{(i)}}{\partial P_k} \frac{\partial S_2^{(i)}}{\partial P_l} \\ & + (\Delta h_i - S_1^{(i)}) \frac{\partial^2 S_1^{(i)}}{\partial P_k \partial P_l} + (\Delta v_i - S_2^{(i)}) \frac{\partial^2 S_2^{(i)}}{\partial P_k \partial P_l} \Bigg] \end{aligned}$$

The second derivative term can be dismissed when it is zero (e.g. in linear case) or small enough to be negligible when compared to term involving the first derivative. It also has an additional possibility of being small in practice: The terms multiplying second derivatives in above equation are $(\Delta h_i - S_1^{(i)})$ and $(\Delta v_i - S_2^{(i)})$. For a successful model, these terms should just be random measurement error of each point. This error can have either sign and should in general be uncorrelated with the model. Therefore, the second derivative terms tend to cancel out when summed over i. So now and onwards we will use,

$$\frac{\partial^2 \chi^2}{\partial P_k \partial P_l} = \sum_{i=0}^{N-1} \frac{2}{{\sigma_i}^2}\left[- \frac{\partial S_1}{\partial P_k}\frac{\partial S_1}{\partial P_l} - \frac{\partial S_2}{\partial P_k}\frac{\partial S_2}{\partial P_l}\right] \tag{4.11}$$

It is conventional to remove factors of 2 by defining

$$\beta_k \equiv -\frac{1}{2}\frac{\partial \chi^2}{\partial P_k} \qquad \alpha_{kl} \equiv \frac{1}{2}\frac{\partial^2 \chi^2}{\partial P_k \partial P_l}$$

Putting $[\alpha] = \frac{1}{2}\mathbf{D}$ in equation (4.8), in terms of which that equation can be rewritten as the set of linear equations.

$$\sum_{l=0}^{M-1} \alpha_{kl}\, \delta P_l = \beta_k \tag{4.12}$$

This set of equations is solved for the increments δP_l and added to the current approximation, give the next approximation. Equation (4.6), the steepest descent formula, translates to

$$\delta P_l = constant \times \beta_l \tag{4.13}$$

Levenberg-Marquardt Method: This is elegant method for varying smoothly between the extremes of the quasi-Newton method (also know as inverse Hessian method) (equation (4.12)) and the steepest descent method (equation (4.6)). The steepest descent method is used far from the minimum, switching continuously to the inverse Hessian method as the minimum is approached. This can be achieved by defining a new matrix α' by the following prescription

$$\begin{aligned} \alpha'_{jj} &\equiv \alpha_{jj}(1+\lambda) \\ \alpha'_{jk} &\equiv \alpha_{jk} \qquad (j \neq k) \end{aligned} \tag{4.14}$$

and then replace both equation (4.8) and (4.6) by

$$\sum_{l=0}^{M-1} \alpha'_{kl}\, \delta P_l = \beta_k \tag{4.15}$$

λ is called fudge factor. Its order of magnitude[4] is decided by the reciprocal of diagonal component of Hessian matrix $1/\alpha_{kk}$. Putting in equation (4.13)

$$\delta P_l = \frac{1}{\lambda \alpha_{ll}}\beta_l \qquad \text{or} \qquad \lambda\, \alpha_{ll}\, \delta P_l = \beta_l \tag{4.16}$$

When λ is very large, the matrix α' is forced into being diagonally dominant, so equation (4.15) goes over to be identical to equation(4.16). On the other hand as λ approaches zero equation(4.15) goes over to equation(4.12)

Given an initial guess for the set of fitted parameters $\mathbf{P}$, the Levenberg-Marquardt algorithm is as follows

1. Compute $\chi^2(\mathbf{P})$
2. Pick a modest value for λ e.g. $\lambda = 0.001$
3. Solve the linear equation (4.15) for $\delta\mathbf{P}$ and evaluate $\chi^2(\mathbf{P} + \delta\mathbf{P})$.
4. If $\chi^2(\mathbf{P} + \delta\mathbf{P}) \geq \chi^2(\mathbf{P})$, increase λ by a factor of 10 (or any other substantial factor) and go back to step 3.
5. If $\chi^2(\mathbf{P} + \delta\mathbf{P}) < \chi^2(\mathbf{P})$, decrease λ by a factor of 10, update the trial solution $\mathbf{P} \longleftarrow \mathbf{P} + \delta\mathbf{P}$ and go back to step 3.

Also necessary is condition for stopping. In practice one might stop iterating on the first or second occasion when χ^2 decreases by a negligible amount say less than 0.01 absolutely. Don't stop after a step where χ^2 increases: That only shows that λ has not adjusted itself optimally. Another criteria for stopping is evident from equation (4.15). Suppose all β_k are negligibly small so that RHS in that equation is zero. This will yield a trivial solution that all $\delta P_i = 0$. Thus algorithm needs to stop iterating if vector distance moved in parameter hyperspace is negligibly small. Once the acceptable minimum has been found, one wants to set $\lambda = 0$ and compute the matrix

$$[C] \equiv [\alpha]^{-1}$$

which is the estimated covariance matrix of the standard errors in the fitted parameters $\mathbf{P}$.
The inverse matrix $[C]$ is closely related to standard uncertainties of the estimated parameters $\mathbf{P}$. In other words, the diagonal elements of $[C]$ are the variances of the fitted parameters $\mathbf{P}$. The off diagonal elements C_{jk} are the covariances between P_j and P_k.

4.3 Linear Numerical Method[4]

In prevoius section we minimized equation (4.9) using non-linear Levenberg-Marquardt method. In this secion we will show for this particular case, we can minimize χ^2 using linear method. For simplicity consider function $S_1^{(i)}$ and $S_2^{(i)}$ of the form

$$\begin{aligned} S_1^{(i)} &= P_1\, f_1 + P_2\, f_2 + P_3\, f_3 + P_4\, f_4 + P_5\, f_5 \\ S_2^{(i)} &= P_4\, g_4 + P_5\, g_5 + P_6\, g_6 + P_7\, g_7 + P_8\, g_8 \end{aligned} \tag{4.17}$$

Where, $f_i \equiv f_i(E, A)$ and $g_i \equiv g_i(E, A)$ Condition for linear least square fit is

$$\frac{\partial \chi^2}{\partial P_k} = 0 \tag{4.18}$$

So setting LHS in equation 4.10 to 0,

$$\sum_{i=0}^{N-1} \frac{1}{\sigma_i^2} \left[S_1^{(i)}\, \frac{\partial S_1^{(i)}}{\partial P_k} + S_2^{(i)}\, \frac{\partial S_2^{(i)}}{\partial P_k} \right] = \sum_{i=0}^{N-1} \frac{1}{\sigma_i^2} \left[\Delta h_i\, \frac{\partial S_1^{(i)}}{\partial P_k} + \Delta v_i\, \frac{\partial S_2^{(i)}}{\partial P_k} \right] \tag{4.19}$$

Where,

$$\begin{aligned} \frac{\partial S_1^{(i)}}{\partial P_l} &= f_l \qquad l = 1, 2, \cdots, 5 \\ \frac{\partial S_2^{(i)}}{\partial P_m} &= g_m \qquad m = 4, 5, \cdots, 8 \end{aligned} \tag{4.20}$$

We can write equation (4.19) in the matrix form as follows,

$$\mathbf{A}\ \mathbf{P} = \mathbf{C} \tag{4.21}$$

$$\mathbf{A} = \begin{bmatrix} f_1^2 & f_1 f_2 & f_1 f_3 & f_1 f_4 & f_1 f_5 & 0 & 0 & 0 \\ f_2 f_1 & f_2^2 & f_2 f_3 & f_2 f_4 & f_2 f_5 & 0 & 0 & 0 \\ f_3 f_1 & f_3 f_2 & f_3^2 & f_3 f_4 & f_3 f_5 & 0 & 0 & 0 \\ f_4 f_1 & f_4 f_2 & f_4 f_3 & (f_4^2 + g_4^2) & (f_4 f_5 + g_4 g_5) & g_4 g_6 & g_4 g_7 & g_4 g_8 \\ f_5 f_1 & f_5 f_2 & f_5 f_3 & (f_5 f_4 + g_5 g_4) & (f_5^2 + g_5^2) & g_5 g_6 & g_5 g_7 & g_5 g_8 \\ 0 & 0 & 0 & g_6 g_4 & g_6 g_5 & g_6^2 & g_6 g_7 & g_6 g_8 \\ 0 & 0 & 0 & g_7 g_4 & g_7 g_5 & g_7 g_6 & g_7^2 & g_7 g_8 \\ 0 & 0 & 0 & g_8 g_4 & g_8 g_5 & g_8 g_6 & g_8 g_7 & g_8^2 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} \Delta h f_1 \\ \Delta h f_2 \\ \Delta h f_3 \\ \Delta h f_4 + \Delta v g_4 \\ \Delta h f_5 + \Delta v g_5 \\ \Delta v g_6 \\ \Delta v g_7 \\ \Delta v g_8 \end{bmatrix}$$

Where summation is implicit in matrix equation. e.g Explicitly writing third equation

$$\sum_{i=0}^{N-1} \frac{1}{\sigma_i^2} \left[f_3^{(i)} f_1^{(i)} P_1 + f_3^{(i)} f_2^{(i)} P_2 + f_3^{(i)} f_3^{(i)} P_3 + f_3^{(i)} f_4^{(i)} P_4 + f_3^{(i)} f_5^{(i)} P_5 \right] = \sum_{i=0}^{N-1} \frac{1}{\sigma_i^2} \left[\Delta h f_3^{(i)} \right]$$

To find $\mathbf{P}$ one needs to solve equation (4.21) by finding $\mathbf{A^{-1}}$.

$$\mathbf{P} = \mathbf{A^{-1}}\ \mathbf{C} \tag{4.22}$$

Note that $\mathbf{A^{-1}}$ is the estimated covariance matrix of the standard errors in the fitted parameters $\mathbf{P}$. Thus diagonal elements of covariance matrix gives variances of the fitted parameters.

Goodness of Fit: The typical rms error on pointing of most antennas without the pointing model is $\sigma = \sqrt{\sigma_E^2 + \sigma_A^2} \geq 2'$. Whereas accuracy of servo system of the GMRT is up to $1'$. After subtracting pointing model from measured data the data left is called Residuals. A fitting will be treated as good fitting if for rms error of residuals is less than $1'$ for elevation as well as azimuth offset. In the case where an variable takes random values from a finite data set $\{x_1, x_2, \cdots, x_N\}$ with each value having the same probability, the standard deviation is given by,

$$\sigma = \sqrt{\frac{1}{N}\left[(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots + (x_N-\mu)^2\right]} \tag{4.23}$$

With, $\mu = \frac{1}{N}(x_1 + x_2 + \cdots + x_N)$

5 Results and Discussion

In this section we will describe model fitting results using three methods described in previous section. All codes are written in C++. To handle files shell script is used. Shell script internally calls all codes. Further instruction on how to use codes and packages can be found in readme.txt file in CD.
We discussed two χ^2 expressions for Pointing model in previous section.

$$\chi^2 = \sum_{i=0}^{N-1} \frac{1}{\sigma_i^2}\left\{\sqrt{\Delta h_i^2 + \Delta v_i^2} - \sqrt{S_1^{(i)\,2} + S_2^{(i)\,2}}\right\}^2 \tag{5.1}$$

$$\chi^2 = \sum_{i=0}^{N-1} \frac{1}{\sigma_i^2}\left\{\left[\Delta h_i - S_1^{(i)}\right]^2 + \left[\Delta v_i - S_2^{(i)}\right]^2\right\} \tag{5.2}$$

$$\begin{aligned} \chi_h^2 &= \frac{1}{(N-8)} \sum_{i=0}^{N-1} \frac{1}{\sigma_i^2}\left\{\left[\Delta h_i - S_1^{(i)}\right]^2\right\} \\ \chi_v^2 &= \frac{1}{(N-8)} \sum_{i=0}^{N-1} \frac{1}{\sigma_i^2}\left\{\left[\Delta v_i - S_2^{(i)}\right]^2\right\} \end{aligned} \tag{5.3}$$

We will show in section 5.2 why it is not good idea to use equation (5.1) as merit function for pointing model. We shall use equation (5.2) as merit function unless specified.

5.1 Ravine Search Method

This method is used for qualitative understanding of χ^2 function. The method involves traversing along gradient direction from one minimum to another. The lowest value of χ^2 will correspond to global minimum. From Figure (4) one iteration is completed when one starts from point M and reaches point M'. So one iteration involves overstepping, sidestepping and finding minimum along side step. Figure (6) shows plot of χ^2 Vs iteration for data S02-175 (Oct-2006). As can be seen from plot method traverses from one minimum to another as iterations proceeds. The parameter values corresponding to lowest value of χ^2 are listed in Table (6.1). Factor $\lambda = 0.01$ is used as fixed step size.

Correlation among different parameter is evident from Figure (7) and Figure (8). Clearly from Figure (7) Parameters P_1 seems to be anticorrelated to P_2 and

P_3. And from Figure (8) Parameters P_6 seems to be anticorrelated to P_7 and P_8. Whereas parameter P_4 and P_5 are weakly correlated to both parameters P_1 and P_6.

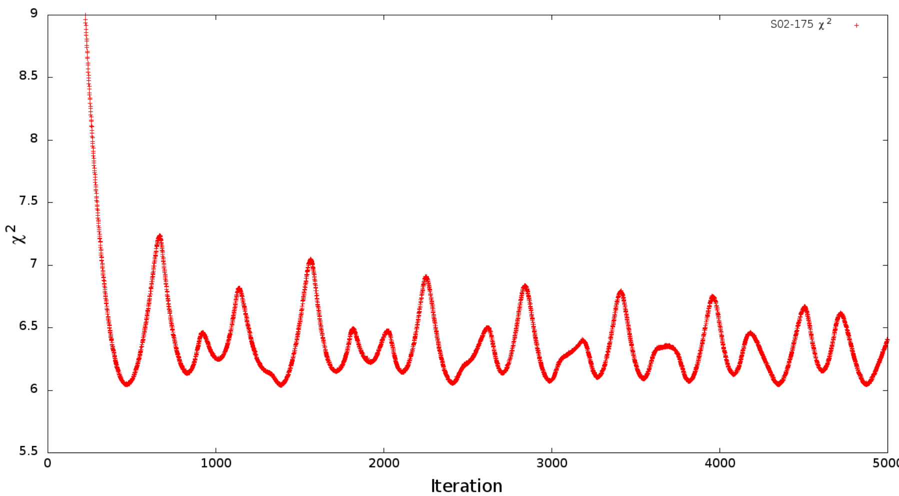

Figure 6: Variation of χ^2 with each Ravine search iteration for S02-175

5.2 Non-linear Numerical Method

We used Levenberg-Marquardt method to minimize equation(5.1) and (5.2). Figure (9) and (10) shows plots of Horizontal offset (Δh) and Vertical offset (Δv) as function of Elevation (E) and Azimuth (Az) for χ^2 given by equation (5.1) and (5.2) respectively. Red points indicate measured offsets while green points represents fitted values.

Clearly fitting in Figure (10) is better than fitting in (9). To understand why χ^2 merit function in equation (5.2) is better than that (5.1), assume model is ideal (i.e. $\chi^2 = 0$).

First consider equation (5.1), for ideal case each term must be zero.

$$\Delta h_i^2 + \Delta v_i^2 = S_1^{(i)\,2} + S_2^{(i)\,2} \tag{5.4}$$

There is inherent degeneracy in above equation. From above equation there is no garuntee that Δh_i, Δv_i will converge to $S_1^{(i)}$, $S_2^{(i)}$ respectively. Since Δh_i, Δv_i may converge to $-S_1^{(i)}$, $-S_2^{(i)}$ respectively and above equation will still hold good.

Now consider ideal model fitting for χ^2 given by equation (5.2). Since $\chi^2 = 0$, each term in equation (5.2) must be zero.

$$\Delta h_i - S_1^{(i)} = 0 \quad \text{and} \quad \Delta v^i - S_2^{(i)} = 0 \tag{5.5}$$

So for ideal case Δh_i, Δv_i will converge to $S_1^{(i)}$ and $S_2^{(i)}$ respectively.

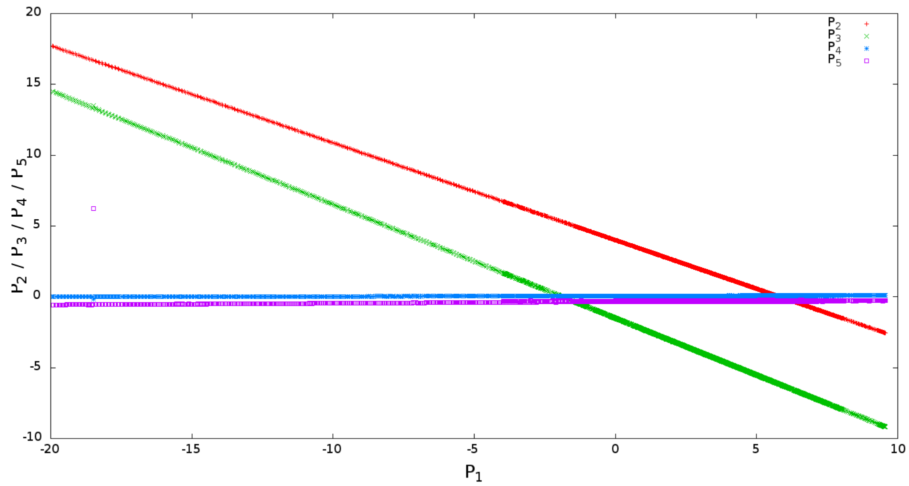

Figure 7: Correlation of parameters $P_2,\ P_3,\ P_4,\ P_5$ with P_1 for S02-175

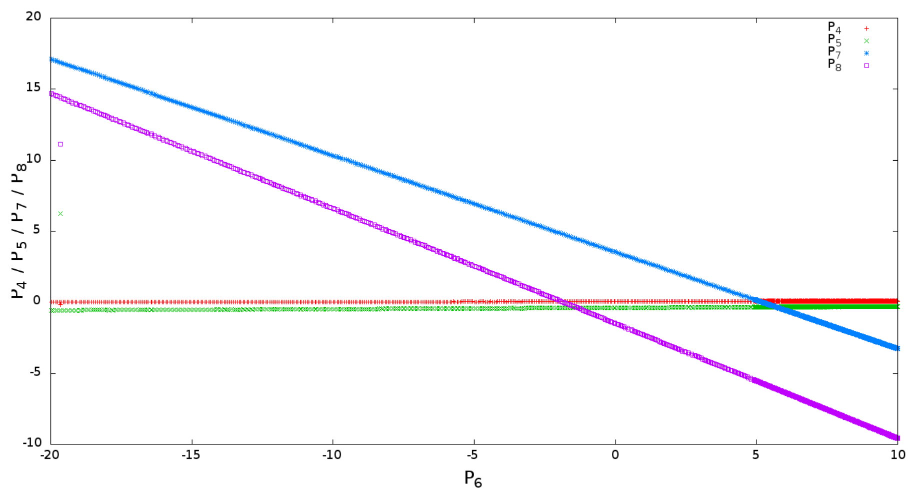

Figure 8: Correlation of parameters $P_4,\ P_5,\ P_7,\ P_8$ with P_6 for S02-175

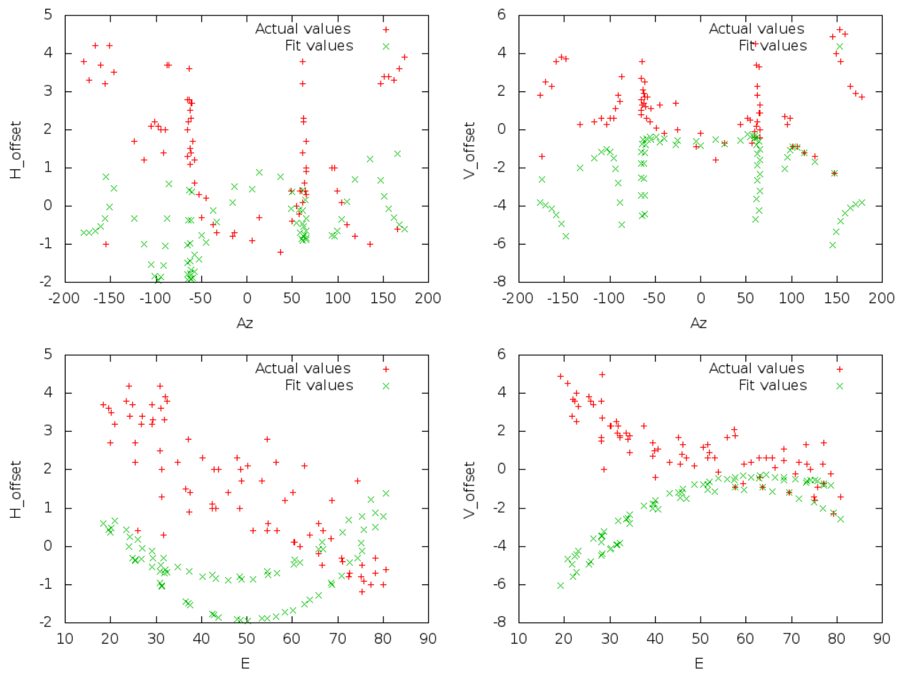

Figure 9: Model fitting for S02-175, χ^2 is given by Equation (5.1)

Figure (11) shows residual after subtracting model given in Figure (10). Note that Root Mean Square After (RMSA) is less than Root Mean Square Before (RMSB) and is less than $1'$. Furthermore systematics has been removed and resulting scatter seems to be random. Hence this can be treated as good fit. Figure (12) to (17) shows more examples of good fit.
Covariance matrix for S02-175 is,

$$\begin{bmatrix} 0.7889 & -0.5414 & -0.6324 & 0.0028 & 0.0015 & 0.0032 & -0.0017 & -0.0033 \\ -0.5414 & 0.3812 & 0.4268 & -0.0014 & -0.0008 & -0.0018 & 0.001 & 0.0019 \\ -0.6324 & 0.4268 & 0.5154 & -0.003 & -0.0012 & -0.0025 & 0.0014 & 0.0026 \\ 0.0028 & -0.0014 & -0.003 & 0.0021 & 0 & -0.0004 & 0.0002 & 0.0003 \\ 0.0015 & -0.0008 & -0.0012 & 0 & 0.0025 & 0.0061 & -0.0032 & -0.0062 \\ 0.0032 & -0.0018 & -0.0025 & -0.0004 & 0.0061 & 0.8401 & -0.5708 & -0.6781 \\ -0.0017 & 0.001 & 0.0014 & 0.0002 & -0.0032 & -0.5708 & 0.3978 & 0.4531 \\ -0.0033 & 0.0019 & 0.0026 & 0.0003 & -0.0062 & -0.6781 & 0.4531 & 0.5561 \end{bmatrix}$$

We can clearly see anticorrelation between parameter P_1 and P_2, P_3 and anticorrelation between P_6 and P_7, P_8

5.3 Linear Numerical Method

We used linear method described in previous section to minimize χ^2. Parameters obtained from the Non linear and linear method are very similar. These are tabulated in Table (6.1). There is striking similarity between Levenberg-Marquardt (L-M) and linear least square because L-M is nonlinear and so linear least square method is just a special case of L-M. But linear least square is easy to implement. Covariance matrix obtained using this method is,

$$\begin{bmatrix} 0.7889 & -0.5414 & -0.6324 & 0.0028 & 0.0015 & 0.0032 & -0.0017 & -0.0033 \\ -0.5414 & 0.3812 & 0.4268 & -0.0014 & -0.0008 & -0.0018 & 0.001 & 0.0019 \\ -0.6324 & 0.4268 & 0.5154 & -0.003 & -0.0012 & -0.0025 & 0.0014 & 0.0026 \\ 0.0028 & -0.0014 & -0.003 & 0.0021 & 0 & -0.0004 & 0.0001 & 0.0003 \\ 0.0015 & -0.0008 & -0.0012 & 0 & 0.0025 & 0.0061 & -0.0032 & -0.0062 \\ 0.0032 & -0.0018 & -0.0025 & -0.0004 & 0.0061 & 0.8398 & -0.5705 & -0.6778 \\ -0.0017 & 0.001 & 0.0014 & 0.0001 & -0.0032 & -0.5705 & 0.3977 & 0.4529 \\ -0.0033 & 0.0019 & 0.0026 & 0.0003 & -0.0062 & -0.6778 & 0.4529 & 0.5559 \end{bmatrix}$$

5.4 Comparison of Numerical methods

In this section we are going to compare results of model fitting from 3 Numerical methods. The parameters obtained from all methods are given in Table (6.1). Parameters match within error bars from 3 method. Error bars are not provided in Ravine search method since it was only used for qualitative behavior of χ^2. From covariance matrix one can obtain correlation matrix. Correlation matrix is composed of element say r_{ij}, which is correlation coefficient between parameter P_i and P_j. In simple term if $|r_{ij}| \sim 1$, then P_i and P_j are strongly correlated. On the other hand if $|r_{ij}| \sim 0$, then P_i and P_j are weakly correlated. Let σ_{xy} denotes covariance between quantities x,y.

$$\sigma_{xy} = cov(x, y) = \langle (x - \mu_x)(y - \mu_y) \rangle = \langle xy \rangle - \langle \mu_x \mu_y \rangle \tag{5.6}$$

σ_{xx}, σ_{yy} is then variance (σ_{xx}^2, σ_{yy}^2) of quantities x,y respectively. Correlation coefficient can be calculated as,

$$r_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \quad \text{with} \quad \sigma_x = \sqrt{\sigma_{xx}} \text{ and } \sigma_y = \sqrt{\sigma_{yy}} \tag{5.7}$$

Correlation matrix for S02-175 is,

$$\mathbf{R} = \begin{bmatrix} 1 & -0.987 & -0.992 & 0.069 & 0.034 & 0.004 & -0.003 & -0.005 \\ -0.987 & 1 & 0.963 & -0.05 & -0.027 & -0.003 & 0.002 & 0.004 \\ -0.992 & 0.963 & 1 & -0.093 & -0.034 & -0.004 & 0.003 & 0.005 \\ 0.069 & -0.05 & -0.093 & 1 & -0.006 & -0.009 & 0.005 & 0.009 \\ 0.034 & -0.027 & -0.034 & -0.006 & 1 & 0.133 & -0.1 & -0.166 \\ 0.004 & -0.003 & -0.004 & -0.009 & 0.133 & 1 & -0.987 & -0.992 \\ -0.003 & 0.002 & 0.003 & 0.005 & -0.1 & -0.987 & 1 & 0.963 \\ -0.005 & 0.004 & 0.005 & 0.009 & -0.166 & -0.992 & 0.963 & 1 \end{bmatrix}$$

Table 5.1: Slopes of Regression line

Slope	Ravine Search	Linear least square
P_2 Vs P_1	-0.6972	-0.6862
P_3 Vs P_1	-0.792	-0.8016
P_4 Vs P_1	0.0033	0.0035
P_5 Vs P_1	0.009	0.0019
P_4 Vs P_6	0.0018	-0.0005
P_5 Vs P_6	0.0085	0.0073
P_7 Vs P_6	-0.6912	-0.6794
P_8 Vs P_6	-0.7949	-0.8071

Clearly P_2 and P_3 are strongly anti-correlated with P_1, whereas P_7 and P_8 are strongly anti-correlated with P_6.

If one plots a quantity y vs x and fit a straight line, slope of this regression line can be obtained from correlation coefficients as,

$$\beta_{yx} = r_{xy} \frac{\sigma_y}{\sigma_x} \tag{5.8}$$

In ravine search method we found correlation between various parameters see figure(7) and (8). We can fit straight lines for various correlation and can obtain slopes. From correlation matrix (which is obtained from linear least square method) we can find slope. Table (5.1) shows slopes obtained from two methods. They are in well agreement.

6 Pointing model: Estimation and Implementation

Pointing model estimation and implementation is never ending process because parameters may vary over some period. So it is better to check after some period if model is degraded. In general,the pointing model procedure is as follows,

1. Make observation: Use grid pointing method for observation. Measure pointing offsets Δh and Δv for corresponding E and A of source.
2. Estimate pointing model parameters from above data set.
3. Apply this model to each subsequent observation.
4. Do pointing observation after some period (e.g 3 months).
   Check if model degrades ($RMSA > 1'$)
5. If No go to step (3).
6. If Yes go back to step (1).

Table (8.1) shows parameters and χ^2 values obtained for various antennas using linear least square fits for Oct-2006 data. Residuals are shown in figure (18) to (21). This model is applied to Dec-2006 data to check. It was found that for many antennas model was not degraded figure (22) to (25). The model is applied to May-2007 data. It was found that for many antennas model was degraded figure (26) to (29). This is also evident from table (8.3), since in May-2007 for many antennas $RMSA > 1'$. So we found out the new set of parameter from May-2007 data. The values of parameters for various antennas are given in table (8).

7 Future Work

- The model seems to be applicable for a period of about 3 months. We would like to extend this window. So one needs to analyze temporal variation of parameters.
- Some of the parameters may not be dominant for GMRT. We would like to find out such parameters. These parameters may be different for different antennas. Correlation matrix information may be helpful to find out such parameters.
- The model was based on mechanical imperfections in antenna. We would like to take other effects such as ionospheric effects into account.[6]

8 Summary

- To measure pointing offsets method of grid pointing can be used. Using this method pointing offsets were measured on Oct-2006, Dec-2006 and May-2007.
- We discussed several pointing parameters,leading to pointing error, that arises due to mechanical imperfections in antenna.
- Numerical methods: 3 different numerical methods were tried on data sets. Out of this Ravine search method was used for understanding qualitative behavior of χ^2 function. Non-linear and linear methods gives identical results because χ^2 function chosen in such a way that it will be linear in parameters.
- Linear model is applied to Oct-2006. Parameters were found from this data set. The pointing model is applied to each subsequent observation. Model was applied to Dec-2006 data and it was not degraded. But model when applied to May-2007, found to be degraded.
- So in future we would like to study temporal variation of parameters.

Table 6.1: Comparison of Numerical methods: Pointing Model Parameters

S02-130			
Parameters	**Ravine Search**	**Levenberg- Marquardt**	**Linear Least square**
$\mathbf{P_1}$	0.27	-0.38 ± 0.89	-0.38 ± 0.89
$\mathbf{P_2}$	3.51	3.97 ± 0.62	3.97 ± 0.62
$\mathbf{P_3}$	-1.93	-1.41 ± 0.72	-1.41 ± 0.72
$\mathbf{P_4}$	0.05	0.06 ± 0.05	0.06 ± 0.05
$\mathbf{P_5}$	-0.34	-0.32 ± 0.05	-0.32 ± 0.05
$\mathbf{P_6}$	4.59	4.77 ± 0.92	4.77 ± 0.92
$\mathbf{P_7}$	0.57	0.48 ± 0.63	0.48 ± 0.63
$\mathbf{P_8}$	-5.26	-5.41 ± 0.75	-5.41 ± 0.75
S02-175			
Parameters	**Ravine Search**	**Levenberg- Marquardt**	**Linear Least square**
$\mathbf{P_1}$	0.88	0.41 ± 0.89	0.41 ± 0.89
$\mathbf{P_2}$	3.41	3.75 ± 0.62	3.75 ± 0.62
$\mathbf{P_3}$	-2.2	-1.83 ± 0.72	-1.83 ± 0.72
$\mathbf{P_4}$	0.07	0.08 ± 0.05	0.08 ± 0.05
$\mathbf{P_5}$	-0.32	-0.31 ± 0.05	-0.31 ± 0.05
$\mathbf{P_6}$	7.3	7.38 ± 0.92	7.38 ± 0.92
$\mathbf{P_7}$	-1.43	-1.45 ± 0.63	-1.45 ± 0.63
$\mathbf{P_8}$	-7.38	-7.46 ± 0.75	-7.46 ± 0.75
C11-130			
Parameters	**Ravine Search**	**Levenberg- Marquardt**	**Linear Least square**
$\mathbf{P_1}$	-5.36	-6.24 ± 1.8	-6.24 ± 1.8
$\mathbf{P_2}$	6.78	7.4 ± 1.24	7.4 ± 1.24
$\mathbf{P_3}$	1.75	2.45 ± 1.46	2.45 ± 1.46
$\mathbf{P_4}$	-0.41	-0.4 ± 0.09	-0.4 ± 0.09
$\mathbf{P_5}$	-0.23	-0.2 ± 0.1	-0.2 ± 0.1
$\mathbf{P_6}$	2.2	2.97 ± 1.89	2.97 ± 1.89
$\mathbf{P_7}$	2.87	2.37 ± 1.29	2.37 ± 1.29
$\mathbf{P_8}$	-5.27	-5.91 ± 1.54	-5.91 ± 1.54
C11-175			
Parameters	**Ravine Search**	**Levenberg- Marquardt**	**Linear Least square**
$\mathbf{P_1}$	-5.95	-5.66 ± 1.81	-5.66 ± 1.81
$\mathbf{P_2}$	7.09	6.9 ± 1.25	6.9 ± 1.25
$\mathbf{P_3}$	1.98	1.74 ± 1.46	1.74 ± 1.46
$\mathbf{P_4}$	-0.4	-0.38 ± 0.09	-0.38 ± 0.09
$\mathbf{P_5}$	-0.08	-0.06 ± 0.1	-0.06 ± 0.1
$\mathbf{P_6}$	5.13	5.22 ± 1.85	5.22 ± 1.85
$\mathbf{P_7}$	1.12	1.08 ± 1.27	1.08 ± 1.27
$\mathbf{P_8}$	-7.54	-7.62 ± 1.51	-7.62 ± 1.51

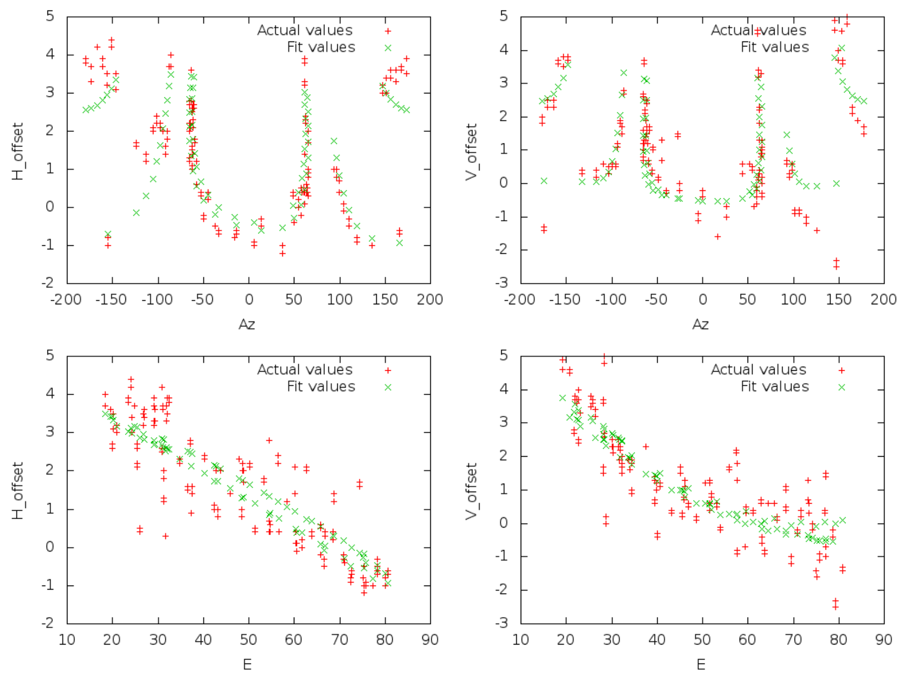

Figure 10: Model fitting for S02-175, χ^2 is given by Equation (5.2)

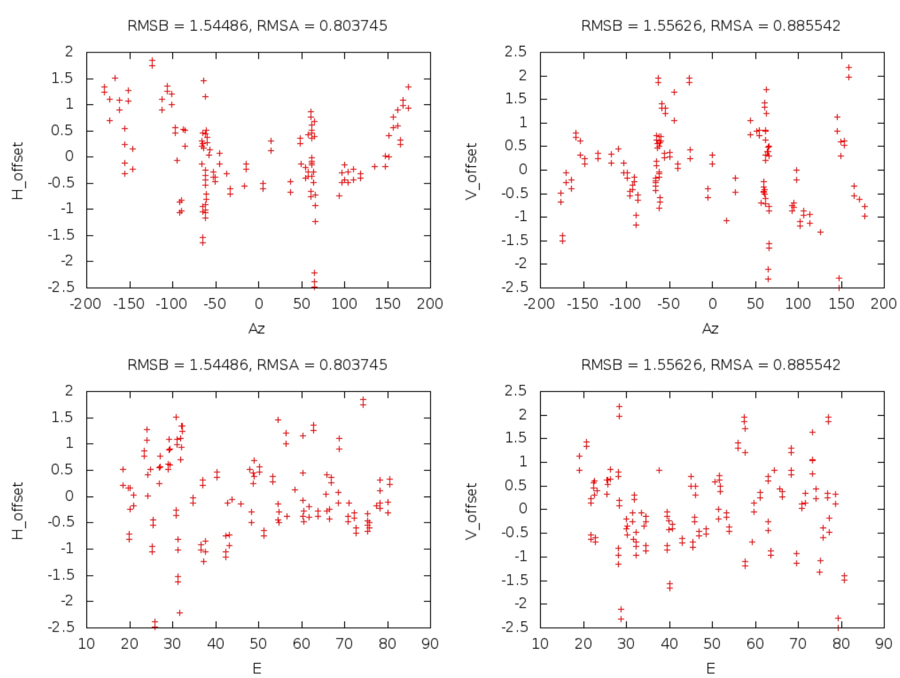

Figure 11: Residual after subtracting model for S02-175

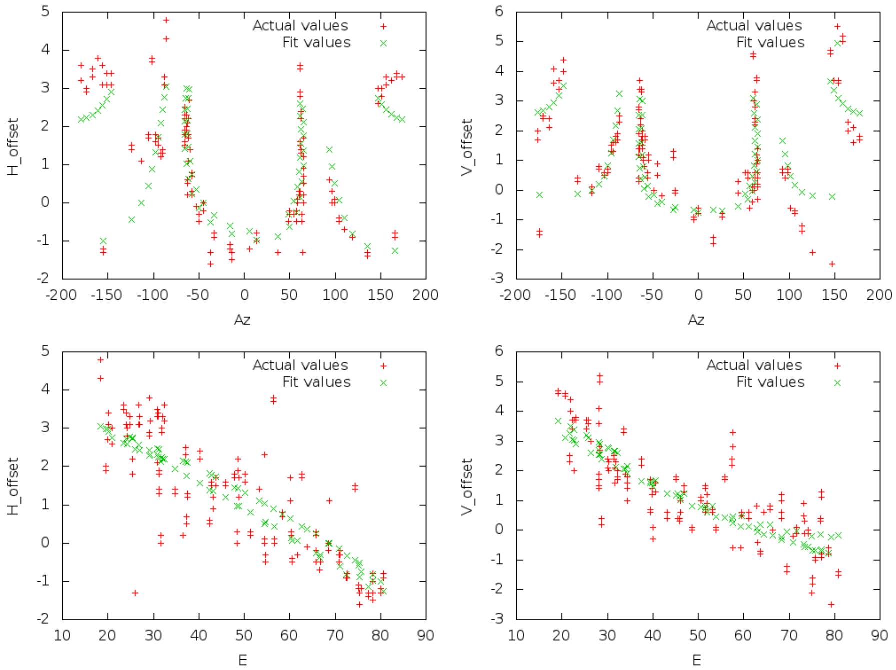

Figure 12: Model fitting for S02-130

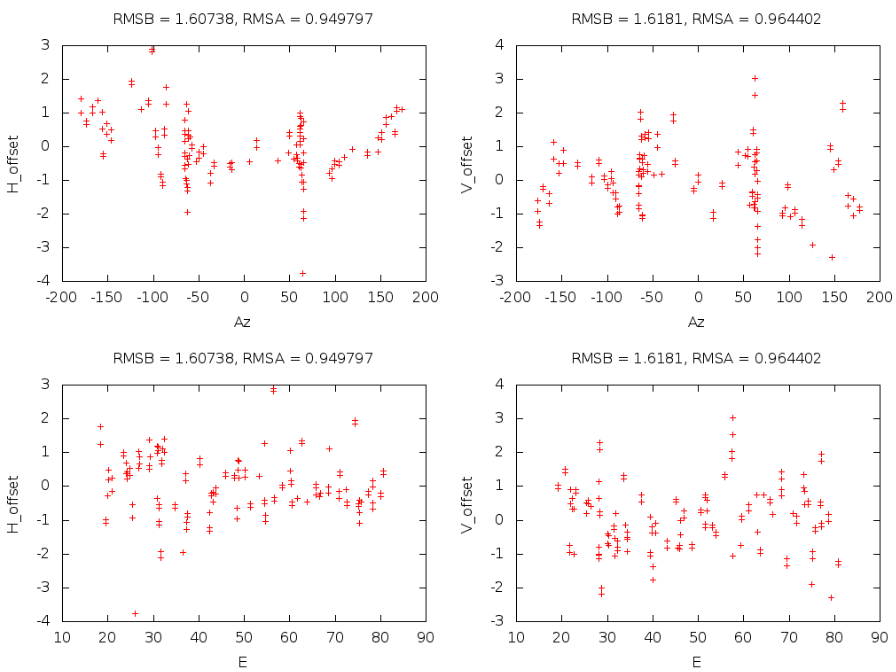

Figure 13: Residual after subtracting model for S02-130

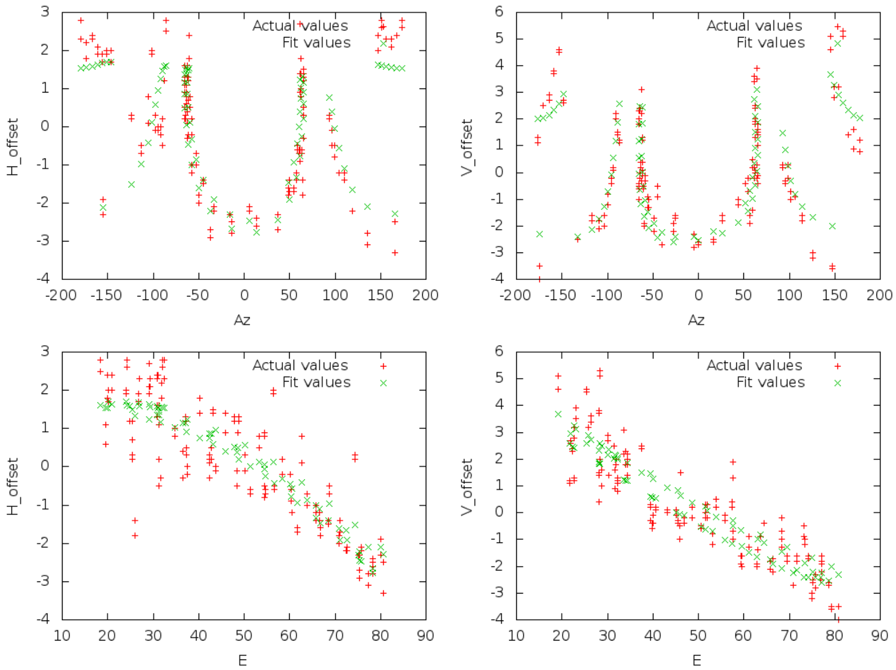

Figure 14: Model fitting for C11-130

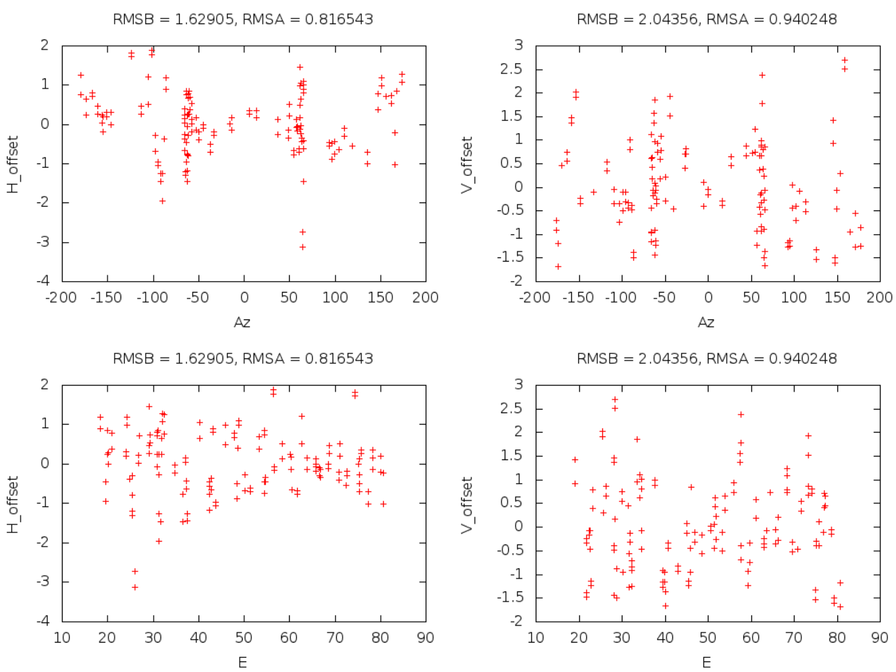

Figure 15: Residual after subtracting model for C11-130

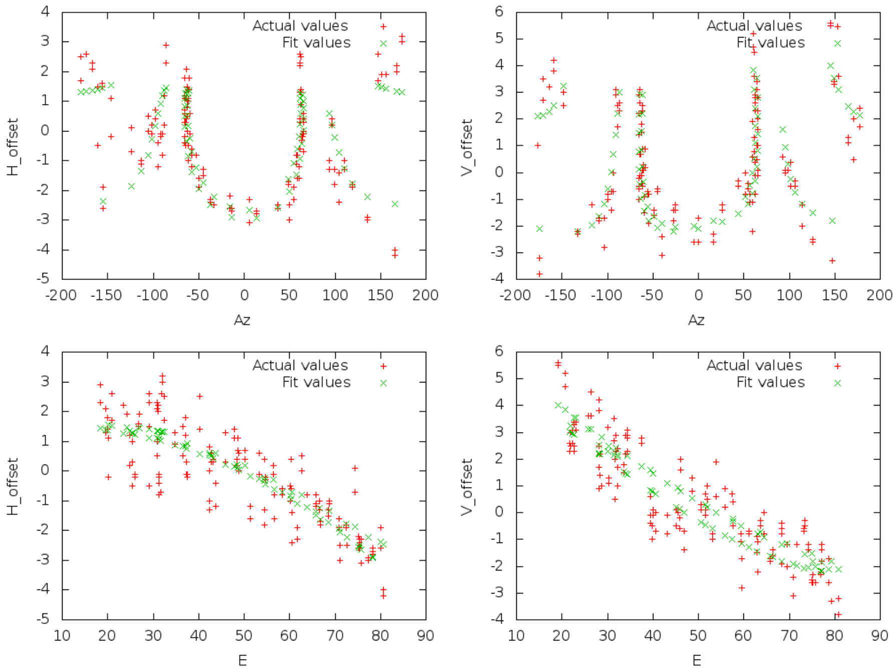

Figure 16: Model fitting for C11-175

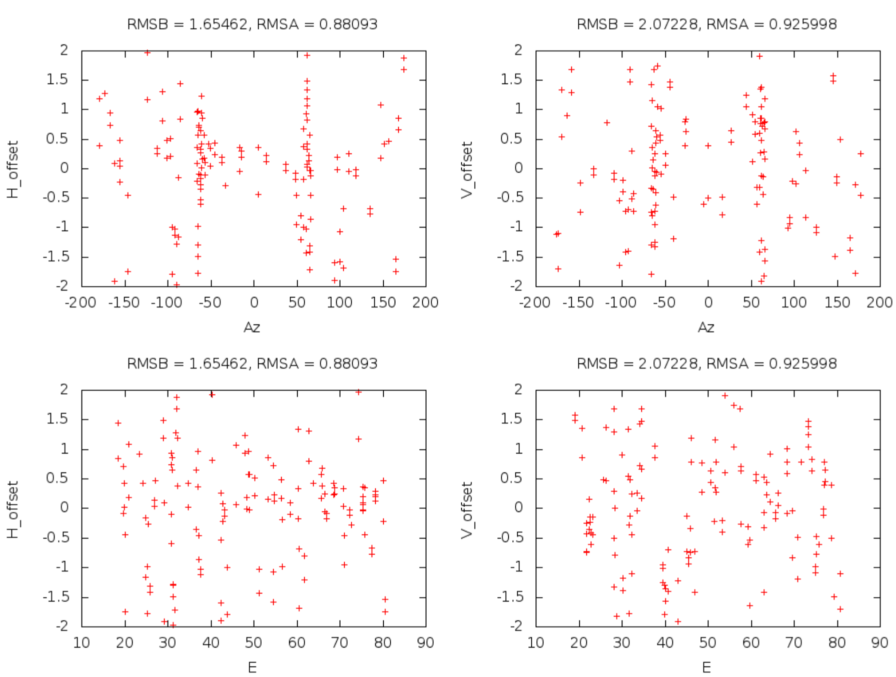

Figure 17: Residual after subtracting model for C11-175

Table 8.1: Parameters from Oct-2006 Data

Antenna	$\mathbf{P_1}$	$\mathbf{P_2}$	$\mathbf{P_3}$	$\mathbf{P_4}$	$\mathbf{P_5}$	$\mathbf{P_6}$	$\mathbf{P_7}$	$\mathbf{P_8}$	χ^2_h	χ^2_v
C03-130	4.03±1.89	0.48±1.32	-0.08±1.54	0.17±0.1	-0.55±0.11	-0.61±1.97	3.32±1.35	-3.98±1.62	0.97	0.82
C03-175	0.84±1.9	1.6±1.33	2.46±1.54	0.25±0.1	-0.46±0.12	0.71±1.98	3.17±1.37	-5.23±1.61	0.94	0.67
C08-130	3.52±1.81	0±1.25	-1.92±1.47	0.79±0.09	-0.15±0.1	4.16±1.88	1.97±1.28	-5.99±1.53	0.88	0.89
C08-175	2.61±1.78	0.17±1.23	-1.18±1.43	0.85±0.09	-0.18±0.1	6.44±1.83	0.43±1.26	-7.6±1.49	0.69	0.77
C09-130	-2.28±1.86	7.29±1.29	-5.05±1.51	0.22±0.1	0±0.11	-3.42±1.96	5.51±1.34	0.39±1.6	0.83	0.89
C09-175	-4.59±1.92	8.59±1.32	-2.82±1.55	0.28±0.1	-0.22±0.11	-0.76±2.06	3.57±1.4	-1.28±1.68	0.73	0.78
C11-130	-6.24±1.8	7.4±1.24	2.45±1.46	-0.4±0.09	-0.2±0.1	2.97±1.89	2.37±1.29	-5.91±1.54	0.7	0.93
C11-175	-5.66±1.81	6.9±1.25	1.74±1.46	-0.38±0.09	-0.06±0.1	5.22±1.85	1.08±1.27	-7.62±1.51	0.82	0.91
C12-130	-2.12±2.02	5.63±1.39	-1.51±1.64	-0.42±0.1	-0.21±0.12	4.47±2.09	0.37±1.41	-5.42±1.71	0.83	0.59
C12-175	-2.78±1.85	6.13±1.27	-0.75±1.51	-0.35±0.09	-0.02±0.11	4.63±1.95	0.11±1.31	-5.93±1.6	0.66	0.8
C13-130	-4.58±1.78	7.1±1.23	0.78±1.43	0.14±0.09	-0.17±0.1	8.61±1.83	-2.83±1.26	-10.86±1.49	0.83	0.81
C13-175	-3.78±1.78	6.66±1.23	0.24±1.43	0.13±0.09	-0.19±0.1	10.99±1.83	-3.98±1.26	-12.37±1.49	0.64	0.67
C14-130	1.09±1.8	1.74±1.25	-1.79±1.46	0.77±0.09	-0.01±0.1	6.57±1.88	-0.77±1.29	-7.43±1.53	0.92	0.86
C14-175	-0.23±1.78	2.57±1.23	-0.66±1.43	0.72±0.09	-0.11±0.1	8.49±1.83	-2.18±1.26	-8.72±1.49	0.85	0.78
E06-130	8.74±1.97	-3.88±1.38	-5.49±1.58	0.82±0.1	0.02±0.11	4.21±2.06	0.98±1.43	-6.02±1.67	1.04	0.77
E06-175	7.09±1.99	-2.81±1.41	-4.19±1.57	0.74±0.1	0.03±0.11	6.06±2.09	-0.42±1.46	-7.31±1.67	1.04	0.62
S01-130	-1.09±1.94	4.79±1.34	-1.84±1.57	0.15±0.1	-0.45±0.11	-1.24±2.01	7.85±1.37	-6.37±1.64	0.69	0.73
S01-175	-0.85±1.85	4.57±1.28	-1.88±1.5	0.16±0.09	-0.5±0.11	-0.96±1.94	7.65±1.32	-6.4±1.59	0.62	0.75
S02-130	-0.38±1.78	3.97±1.23	-1.41±1.43	0.06±0.09	-0.32±0.1	4.77±1.83	0.48±1.26	-5.41±1.49	0.95	0.98
S02-175	0.41±1.78	3.75±1.23	-1.83±1.43	0.08±0.09	-0.31±0.1	7.38±1.83	-1.45±1.26	-7.46±1.49	0.68	0.83
S03-130	7.18±1.77	-3.67±1.23	-0.24±1.43	-1.21±0.09	-0.22±0.1	6.97±1.83	-0.73±1.26	-8.5±1.49	1.03	0.87
S03-175	7.72±1.78	-4.3±1.23	-0.97±1.43	-1.17±0.09	-0.23±0.1	8.03±1.83	-1.64±1.26	-9.31±1.49	0.7	0.65
S06-130	5.52±2.04	2.7±1.38	-7.7±1.68	0.81±0.11	0.76±0.12	15.56±2.16	-5.29±1.44	-13.63±1.79	0.85	0.71
S06-175	-3.28±1.96	7.03±1.32	0.6±1.6	0.86±0.1	0.46±0.12	8.74±2.11	-1.89±1.4	-7.71±1.74	0.86	0.81
W02-130	-5.33±2.32	6.79±1.53	2.61±1.92	0.59±0.1	-0.39±0.12	2.76±2.3	4.73±1.51	-9.3±1.9	0.84	0.99
W02-175	-6.16±2.41	7.18±1.57	3.67±2	0.61±0.1	-0.36±0.12	2.84±2.46	4.39±1.59	-8.92±2.05	0.56	0.77
W03-130	1.23±1.78	1.73±1.23	-1.3±1.43	-0.3±0.09	-0.63±0.1	4.76±1.83	0.36±1.26	-4.21±1.49	0.88	0.84
W03-175	1.81±1.78	1.21±1.23	-1.89±1.43	-0.25±0.09	-0.66±0.1	7.12±1.83	-1.32±1.26	-5.88±1.49	0.7	0.69
W04-130	3.59±1.81	-0.09±1.25	-2.56±1.47	-0.75±0.09	-0.41±0.1	4.82±1.88	-0.15±1.28	-5.63±1.54	0.7	0.9
W04-175	3.76±1.79	-0.16±1.24	-2.54±1.45	-0.76±0.09	-0.45±0.1	6.74±1.86	-1.74±1.27	-7.17±1.52	0.66	0.74
W05-130	8.81±1.91	-4.34±1.33	-4.48±1.54	0.08±0.1	-0.42±0.11	5.09±1.97	2.42±1.35	-4±1.6	0.94	1.03
W05-175	9.19±1.89	-4.94±1.32	-4.78±1.52	0.06±0.1	-0.4±0.11	6.89±1.94	0.89±1.33	-5.65±1.58	0.84	0.9

Table 8.2: Parameters from May-2007 Data

Anetnna	$\mathbf{P_1}$	$\mathbf{P_2}$	$\mathbf{P_3}$	$\mathbf{P_4}$	$\mathbf{P_5}$	$\mathbf{P_6}$	$\mathbf{P_7}$	$\mathbf{P_8}$	χ^2_h	χ^2_v
C03-130	1.73±6.83	-0.87±5.44	3.22±4.5	-0.81±0.22	-1.18±0.2	15.08±7.01	-9.44±5.61	-11.75±4.57	0.34	0.79
C03-175	1.75±4.35	-1.11±3.36	2.44±3.07	-0.6±0.19	-1.5±0.17	6.71±4.17	-1.8±3.22	-7.22±2.96	1.15	0.94
C08-130	-11.13±35.59	12.62±30.31	7.22±19.4	0.69±0.33	-0.49±0.3	1.86±22.65	4.28±19.29	-4.24±12.55	0.07	0.36
C08-175	2.16±5.22	1.59±3.96	-1.12±3.76	0.77±0.24	-0.85±0.23	11.67±4.87	-3.78±3.72	-9.68±3.51	0.59	0.62
C09-130	-7.28±3.63	12.14±2.74	0.28±2.64	-0.07±0.17	-0.49±0.16	3.05±3.47	0.05±2.62	-5.28±2.55	1	0.78
C09-175	-2.02±3.68	8.56±2.78	-3.82±2.69	-0.17±0.17	-0.56±0.16	9.03±3.54	-4.32±2.67	-9.22±2.61	1.34	0.88
C10-130	0.57±4.47	0.98±3.42	0.05±3.18	0.89±0.19	-1.57±0.17	19.72±4.55	-10.99±3.5	-13.34±3.21	0.41	0.6
C10-175	1.55±3.89	0.37±2.94	-0.91±2.82	0.8±0.18	-1.75±0.16	16.3±3.68	-8.41±2.8	-11.27±2.67	1.18	0.8
C11-130	-7.08±3.56	9.01±2.68	3.8±2.61	-0.25±0.16	-0.68±0.15	8.5±3.46	-1.42±2.61	-8.61±2.54	1.34	0.74
C11-175	-1.46±3.68	5.12±2.75	-0.87±2.71	-0.58±0.16	-0.79±0.15	10.62±3.55	-3.59±2.67	-9.16±2.62	1.04	0.83
C13-130	3±6.45	0.73±4.25	-4.09±5.14	0.16±0.25	-0.26±0.24	0.36±6.19	2.76±4.05	-1.38±4.97	0.16	0.35
C13-175	-1.2±3.81	4.14±2.88	-1.47±2.75	0.06±0.17	-0.86±0.15	18.01±3.5	-7.66±2.63	-14.35±2.57	0.69	0.77
E04-130	5.86±4.28	-2.76±3.15	-3±3.18	0.14±0.21	-0.77±0.23	3.17±4.05	0.79±2.96	-1.99±3.03	0.28	0.3
E04-175	6.55±4.01	-2.98±2.99	-4.02±2.94	-0.02±0.18	-1.31±0.17	8.26±3.67	-3.13±2.74	-5.51±2.7	0.89	0.48
S01-130	2.82±5.69	1.11±4.47	-3.43±3.82	0.12±0.18	-1.51±0.16	15.37±5.82	-4.17±4.59	-15.42±3.89	0.64	0.56
S01-175	0.47±3.85	2.95±2.93	-1.94±2.77	0.04±0.17	-1.48±0.15	8.51±3.63	1.33±2.76	-10.75±2.62	1.43	0.95
S02-130	1.27±3.89	0.8±2.91	-1.4±2.86	0.24±0.18	-1.21±0.18	11.84±3.66	-4.02±2.74	-9.9±2.71	0.67	0.89
S02-175	-1.93±5.15	3.03±3.66	0.9±3.92	0.35±0.24	-1.27±0.23	12.97±4.61	-5.71±3.27	-9.86±3.54	0.61	0.54
S03-130	4.28±5.09	-1.41±3.97	2.78±3.49	-1.09±0.18	-1.18±0.17	16.55±5.46	-7.95±4.3	-12.63±3.7	1.23	0.64
S03-175	7.14±5.13	-3.39±4.02	-0.4±3.5	-1.31±0.18	-1.31±0.17	15.19±5.69	-7.15±4.49	-11.53±3.83	1.1	0.82
W02-130	-3.65±6.15	5.46±4.91	1.72±4.03	0.38±0.19	-1.35±0.17	20.18±6.27	-9.5±4.99	-18.41±4.14	1.08	0.74
W02-175	8.21±6.3	-3.23±5.02	-6.72±4.16	0.55±0.2	-1.39±0.18	21.9±6.38	-10.56±5.07	-19.7±4.24	1.08	0.72
W03-130	2.07±3.67	-0.14±2.75	-1.06±2.7	-0.19±0.16	-1.53±0.15	7.66±3.52	-2.4±2.64	-5.62±2.59	0.92	0.6
W03-175	3.06±3.59	-0.81±2.71	-2.24±2.63	-0.3±0.16	-1.69±0.15	10.81±3.48	-4.94±2.63	-7.12±2.56	1.08	0.75
W04-130	-0.41±3.56	0.11±2.68	1.66±2.61	-0.93±0.16	-1.43±0.15	6.26±3.46	-1.05±2.61	-5.91±2.54	1.53	0.59
W04-175	4.42±3.9	-3.01±2.9	-2.49±2.89	-0.81±0.18	-1.37±0.18	10.22±3.69	-4.57±2.74	-8.58±2.74	0.96	0.6
W05-130	2.08±4.45	0.34±3.4	1.07±3.18	-0.01±0.18	-1.24±0.17	13.49±4.41	-5.43±3.37	-9.21±3.15	0.52	0.92
W05-175	3.11±3.92	-0.79±2.99	0.43±2.81	-0.19±0.17	-1.58±0.16	17.26±3.64	-8.66±2.77	-11.79±2.62	1.29	0.9
W06-130	3.79±10.25	-1.4±8.14	-2.53±6.64	0.78±0.27	-1.66±0.28	5.99±8.08	0.58±6.45	-6.27±5.3	0.24	0.23
W06-175	-0.03±5.4	2.14±4.13	-0.96±3.77	0.35±0.23	-1.83±0.2	5.61±4.46	0.39±3.45	-6.11±3.16	0.66	0.53

Table 8.3: Implementation of Pointing model: Parameters obtained from Oct-2006

	October-2006				December-2006				May-2007			
Antenna	Δh		Δv		Δh		Δv		Δh		Δv	
	RMSB	RMSA	RMSB	RMSA	RMSB	RMSA	RMSB	RMSA	RMSB	RMSA	RMSB	RMSA
C03-130	1.03	0.96	2.01	0.88	0.79	0.71	2.18	1.45	1.27	1.21	1.26	1.16
C03-175	0.99	0.94	2.17	0.79	0.83	0.77	2.39	1.67	1.43	1.36	1.8	1.28
C08-130	0.93	0.91	2.1	0.92	0.58	0.55	1.34	0.62	0.6	0.51	1.24	0.88
C08-175	0.83	0.81	2.08	0.86	0.61	0.58	1.48	0.73	0.94	0.86	1.77	0.95
C09-130	2.95	0.88	1.61	0.91	0.64	0.87	1.11	0.58	2.34	0.92	1.44	0.93
C09-175	2.69	0.83	1.47	0.86	0.5	0.94	1.2	0.7	2.56	1.1	1.48	1.02
C11-130	1.63	0.82	2.04	0.94	0.87	0.41	1.16	0.42	1.32	0.85	1.88	1.08
C11-175	1.65	0.88	2.07	0.93	0.96	0.65	1.26	0.62	1.42	0.82	1.82	1.25
C12-130	1.96	0.88	1.38	0.74	0.77	0.43	1.16	0.85	3.52	3.79	3.78	3.68
C12-175	1.86	0.79	1.47	0.87	0.78	0.5	1.21	0.94	5.45	5.26	3.2	3.21
C13-130	1.81	0.89	1.87	0.88	0.86	0.43	0.96	0.35	0.87	0.63	1.81	0.97
C13-175	1.76	0.78	1.88	0.8	0.93	0.56	1.1	0.38	1.2	0.68	1.93	1.1
C14-130	1.06	0.93	1.85	0.9	0.53	0.47	1.38	1.01	0.89	1	8.69	8.16
C14-175	1.02	0.9	1.78	0.86	0.61	0.54	1.63	1.12	1.02	1.07	6.49	5.89
E06-130	0.95	0.99	1.91	0.85	0.59	0.63	1.22	0.53	0.79	0.95	1.15	1.1
E06-175	0.93	0.99	1.79	0.76	0.64	0.68	1.33	0.62	0.97	1.17	1.12	1.07
S01-130	1.76	0.81	3.33	0.83	72.6	72.39	59.8	59.16	1.18	0.85	2.63	1.15
S01-175	1.74	0.76	3.3	0.84	4.79	4.76	47.53	46.86	1.42	1.13	2.8	1.27
S02-130	1.61	0.95	1.62	0.96	Bad Antenna				0.97	1.07	1.87	1.2
S02-175	1.54	0.8	1.56	0.89	Bad Antenna				1.21	1.21	1.89	1.33
S03-130	1.41	0.99	2.03	0.91	0.56	0.5	1.01	0.33	1.37	0.94	1.65	1.27
S03-175	1.26	0.81	1.96	0.79	0.66	0.59	0.94	0.37	1.47	1	1.79	1.32
S06-130	2.21	0.89	1.77	0.81	1.69	1.58	1.22	1.1	1.52	1.58	2.24	1.86
S06-175	1.66	0.89	1.65	0.87	1.26	1.02	1.32	0.69	1.57	1.54	1.96	1.68
W02-130	1.38	0.88	2.98	0.96	0.43	0.32	2.15	0.93	1.12	1.08	2.38	1.29
W02-175	1.22	0.72	2.77	0.84	0.42	0.38	2.27	1.07	1.11	1.07	2.48	1.31
W03-130	1.33	0.91	1.4	0.89	0.52	0.46	0.67	0.36	0.96	0.89	1.61	1.1
W03-175	1.25	0.81	1.33	0.81	0.59	0.55	0.67	0.38	1.12	0.94	1.71	1.27
W04-130	1.17	0.81	1.59	0.92	0.57	0.46	0.91	0.31	1.21	1.28	1.81	1.1
W04-175	1.15	0.79	1.52	0.83	0.66	0.58	0.8	0.22	1.3	1.14	1.87	1.28
W05-130	1.01	0.94	1.83	0.99	0.55	0.6	1.02	0.6	0.8	0.76	1.63	1.21
W05-175	0.98	0.89	1.74	0.92	0.6	0.64	1.16	0.74	1.15	1.1	1.84	1.39

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Figure 18: Residual plots for Oct-2006 Data: Δh offset

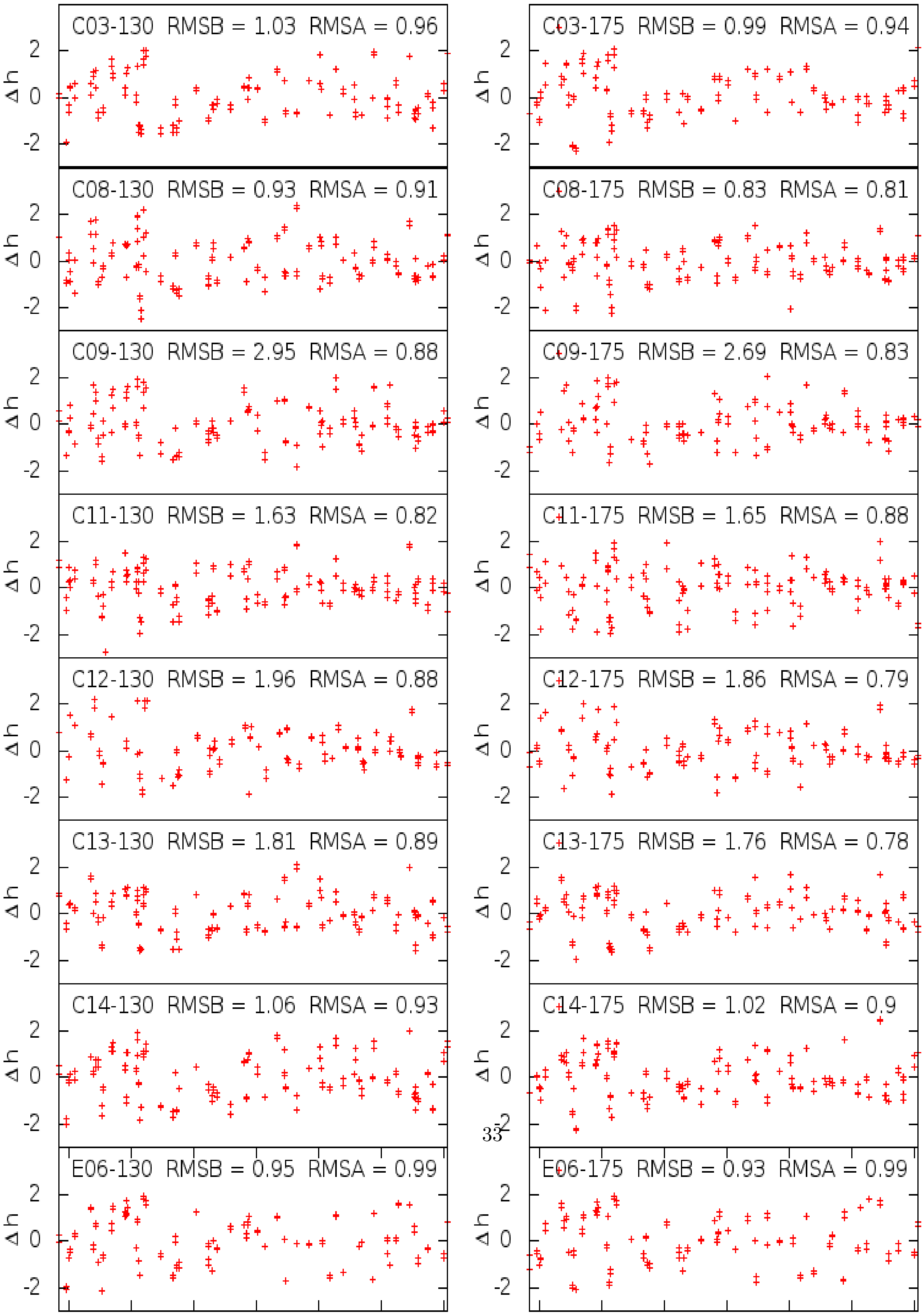

Figure 19: Residual plots for Oct-2006 Data: Δh offset

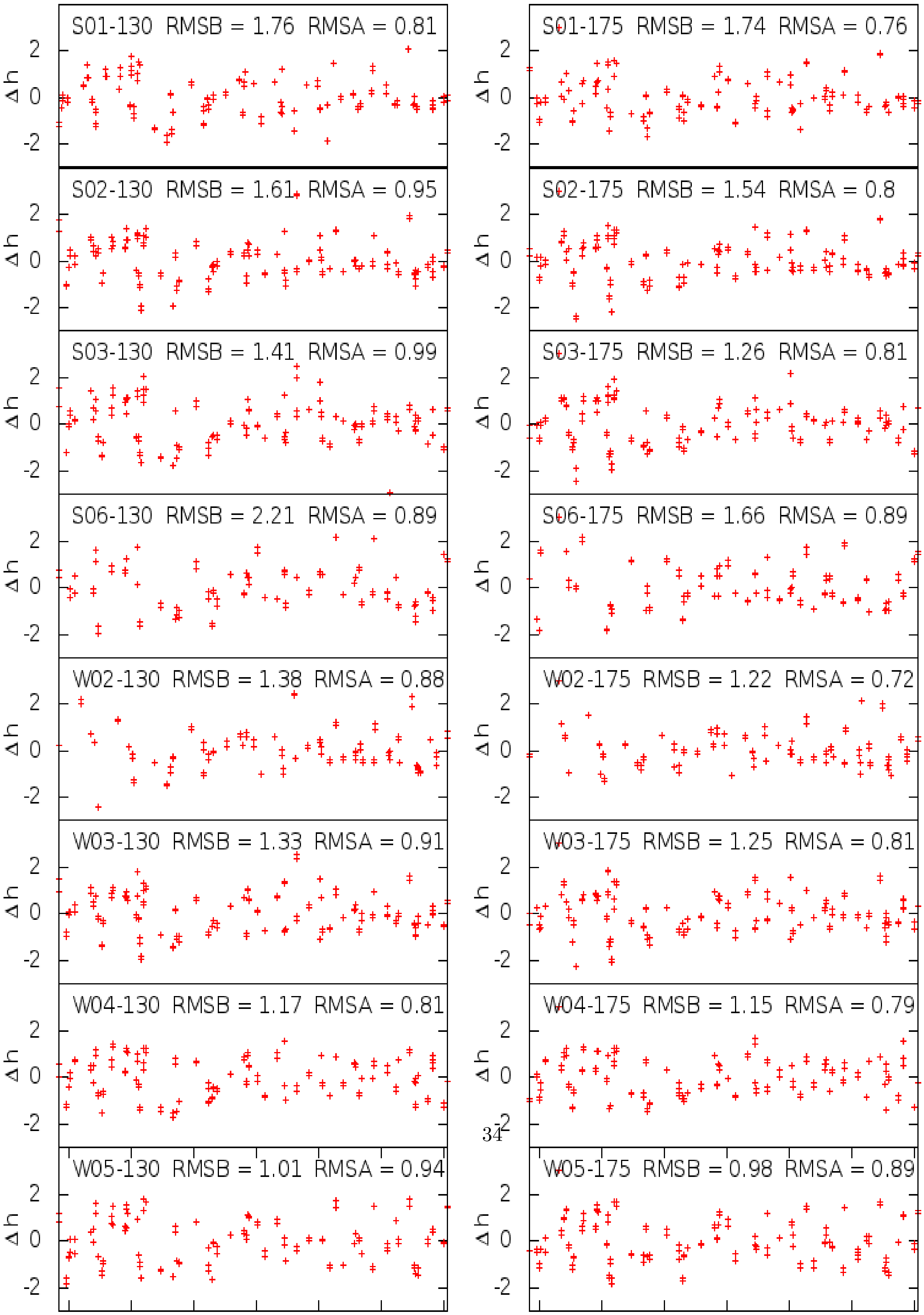

Figure 20: Residual plots for Oct-2006 Data: Δv offset

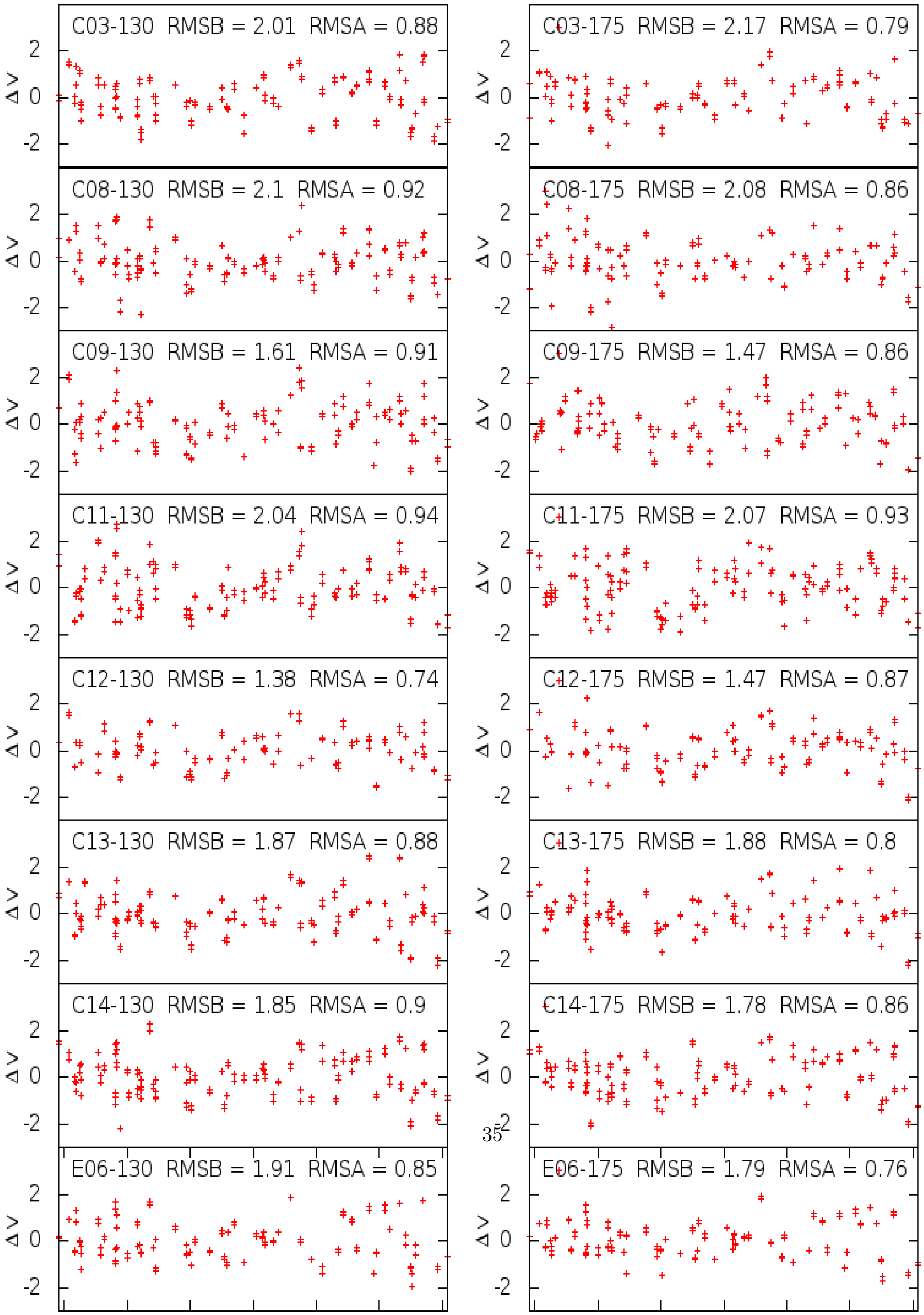

Figure 21: Residual plots for Oct-2006 Data: Δv offset

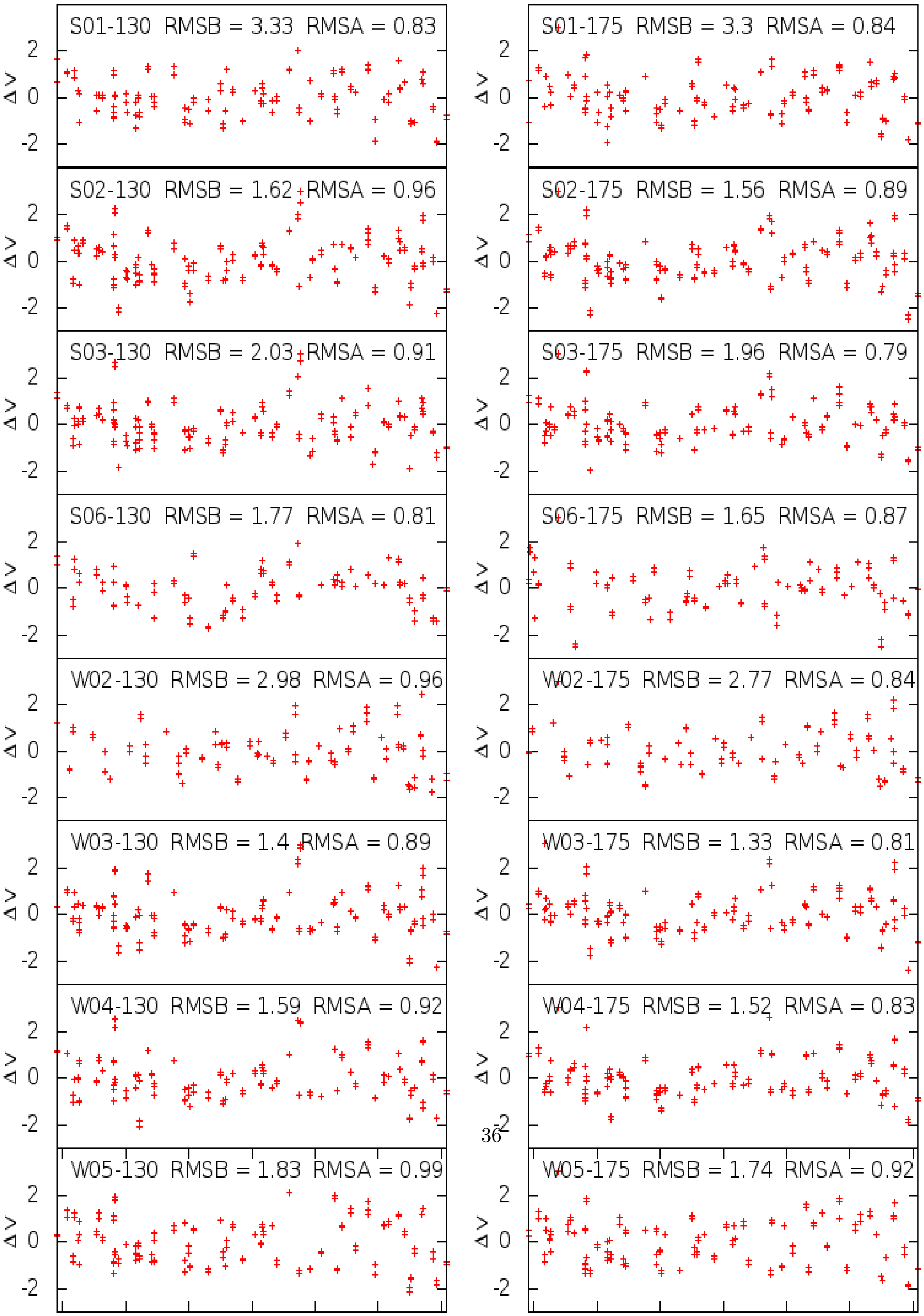

Figure 22: Residual plots for Dec-2006 Data: Δh offset

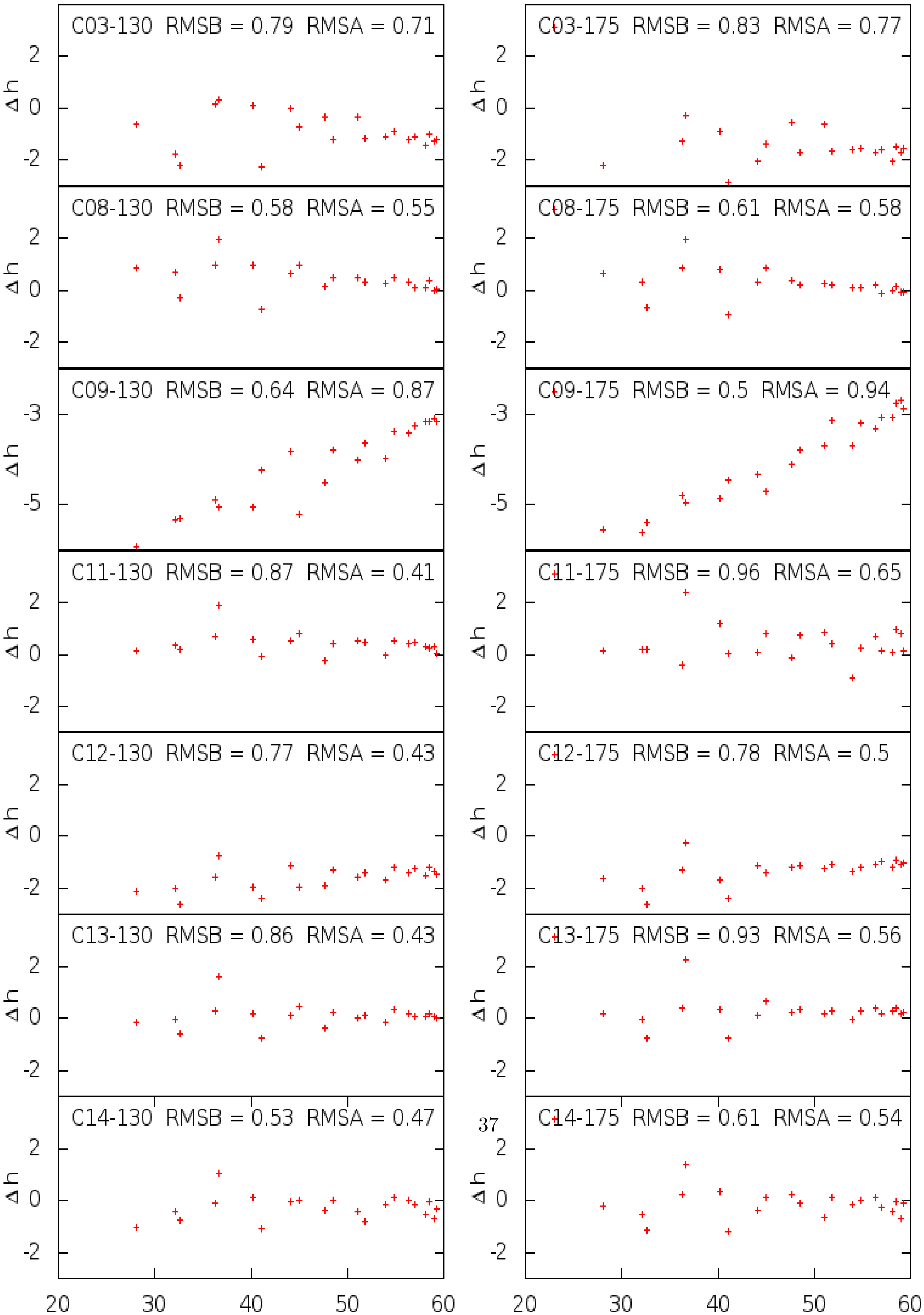

Figure 23: Residual plots for Dec-2006 Data: Δh offset

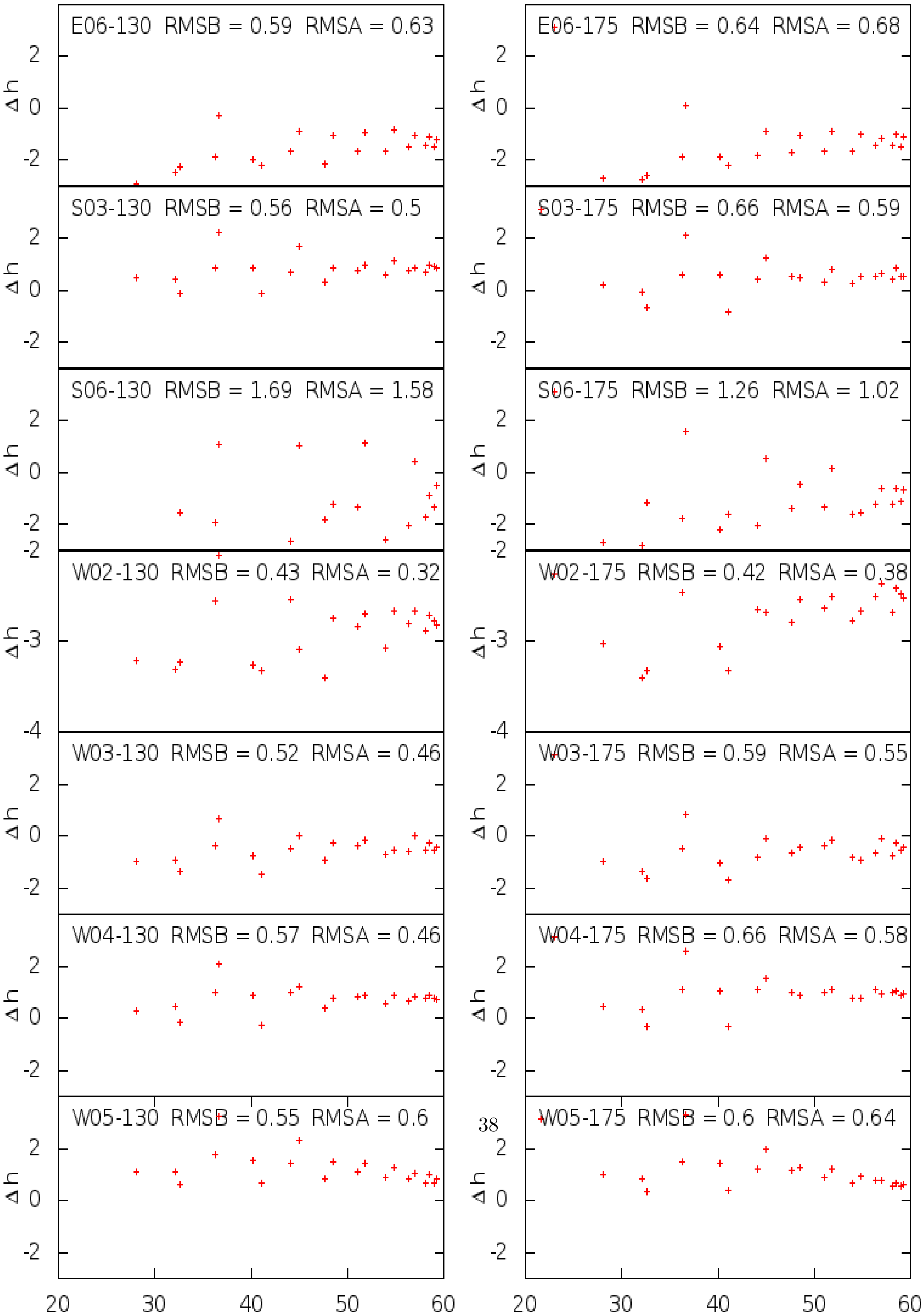

Figure 24: Residual plots for Dec-2006 Data: Δv offset

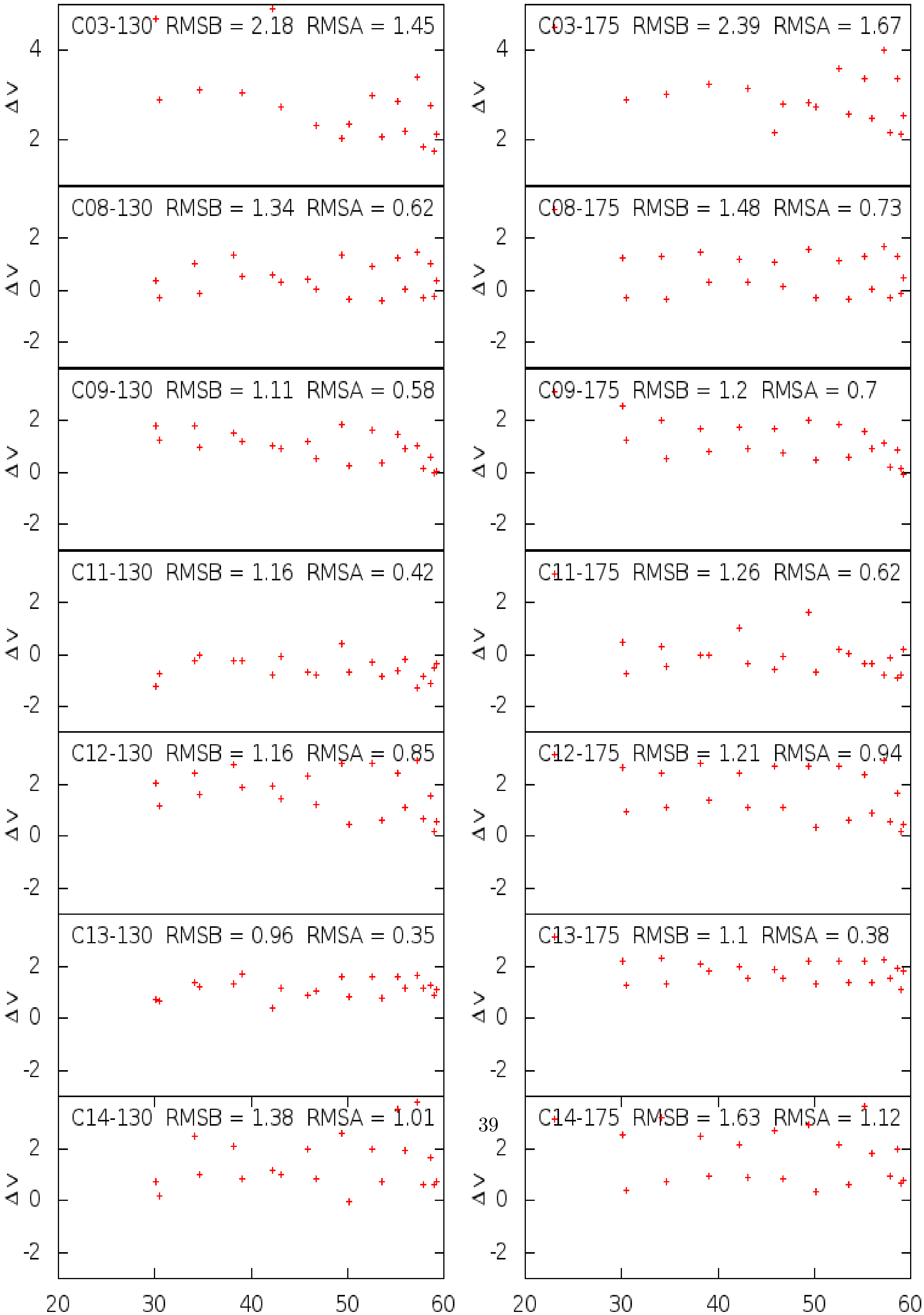

Figure 25: Residual plots for Dec-2006 Data: Δv offset

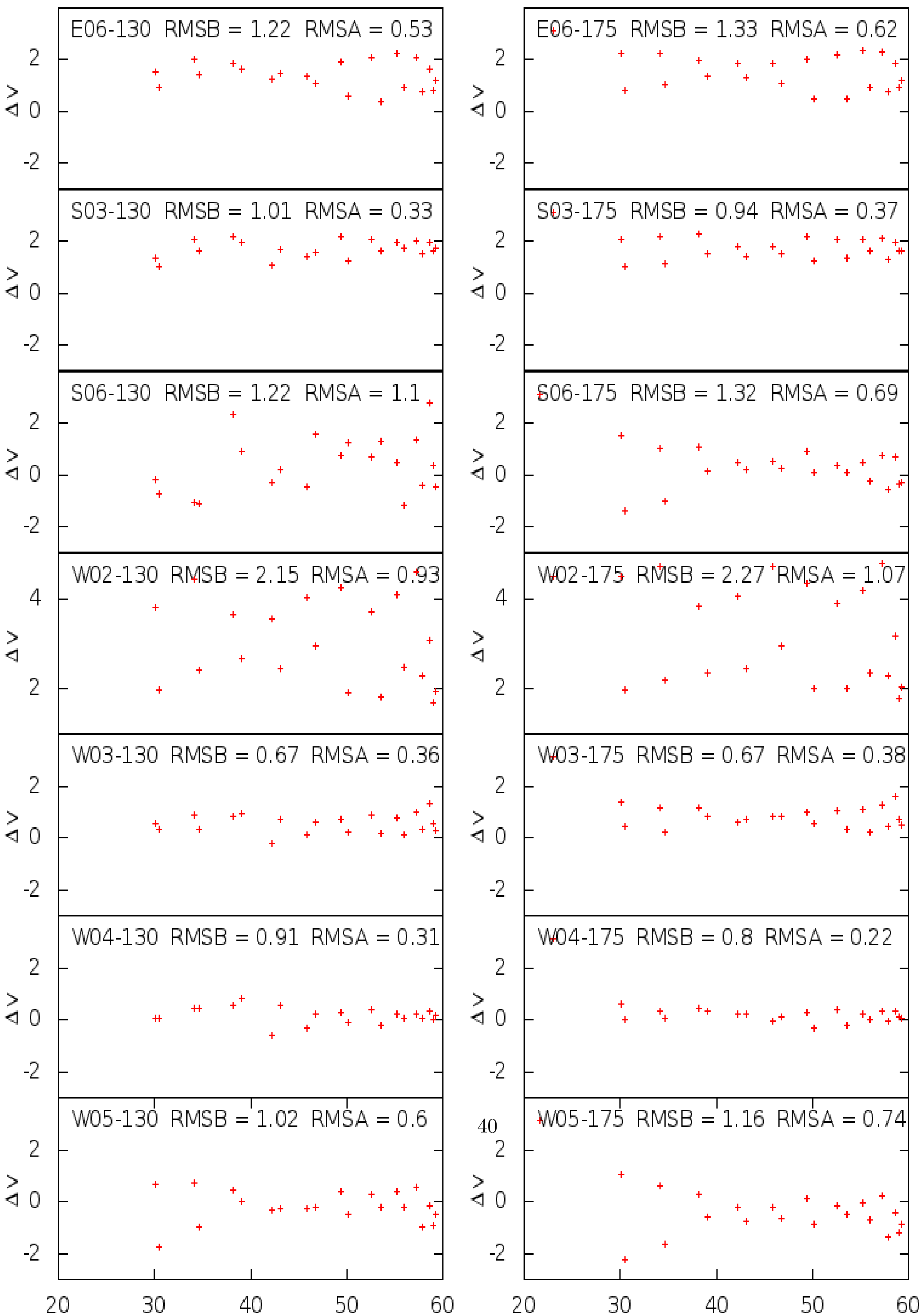

Figure 26: Residual plots for May-2007 Data: Δh offset

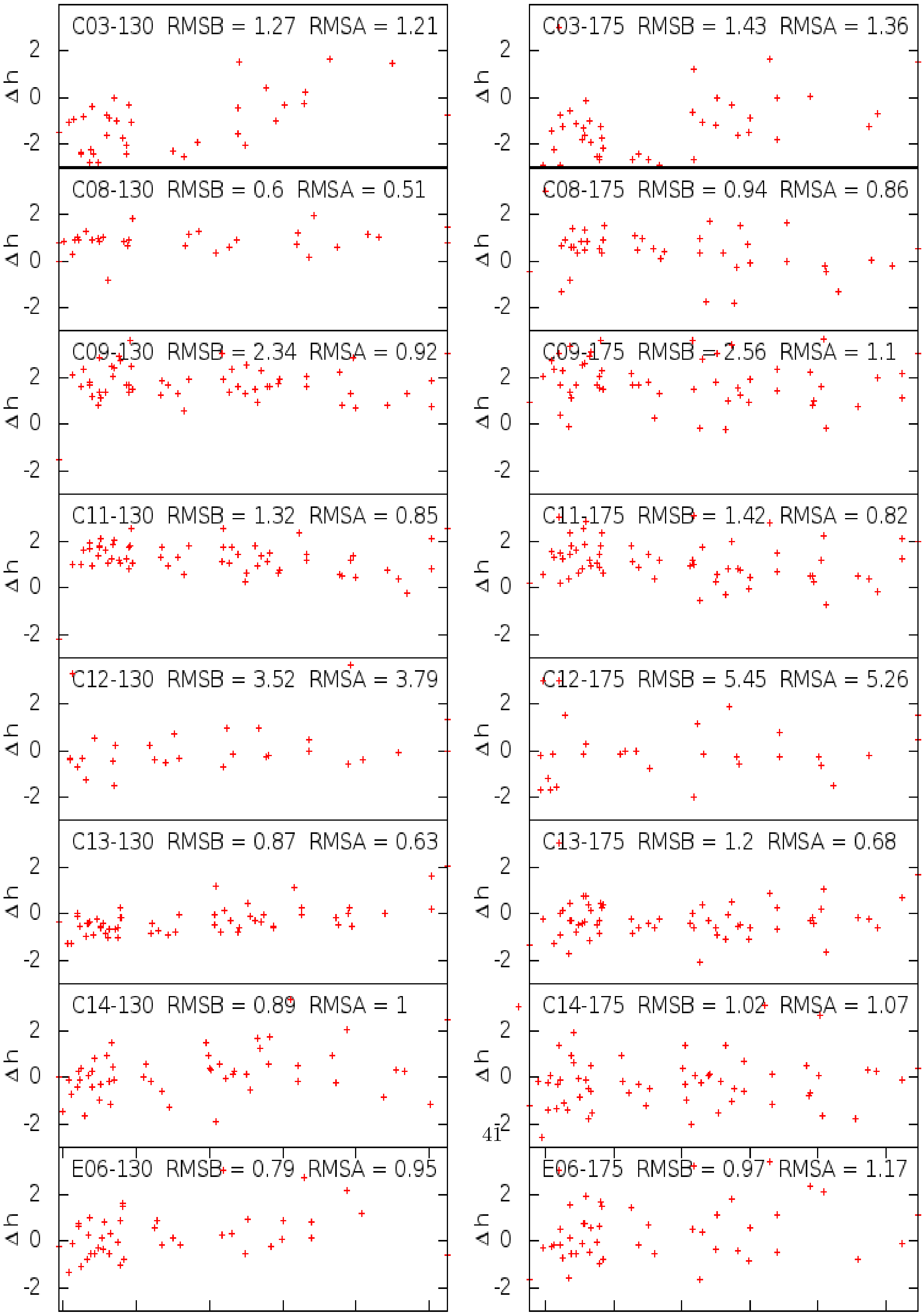

Figure 27: Residual plots for May-2007 Data: Δh offset

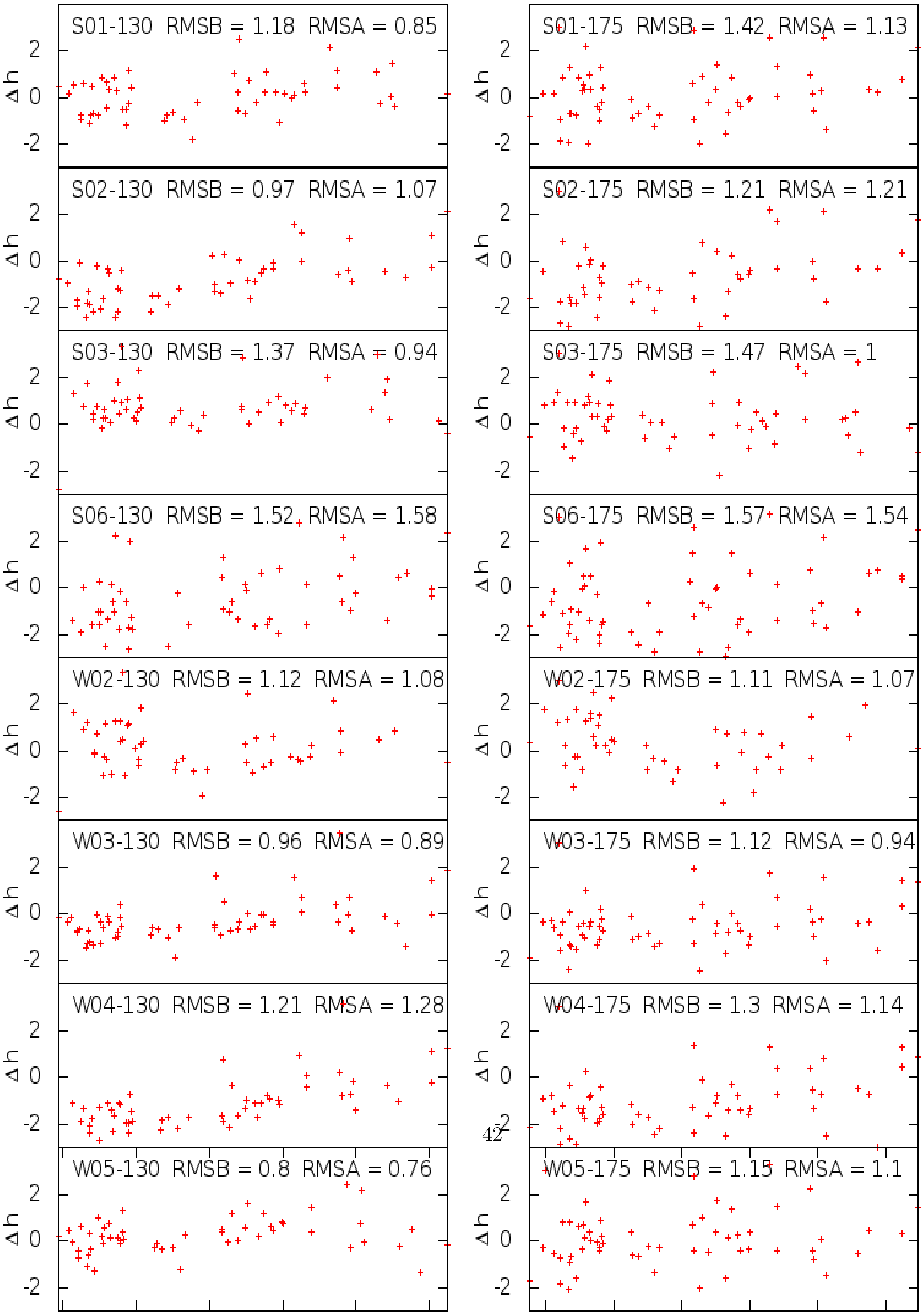

Figure 28: Residual plots for May-2007 Data: Δv offset

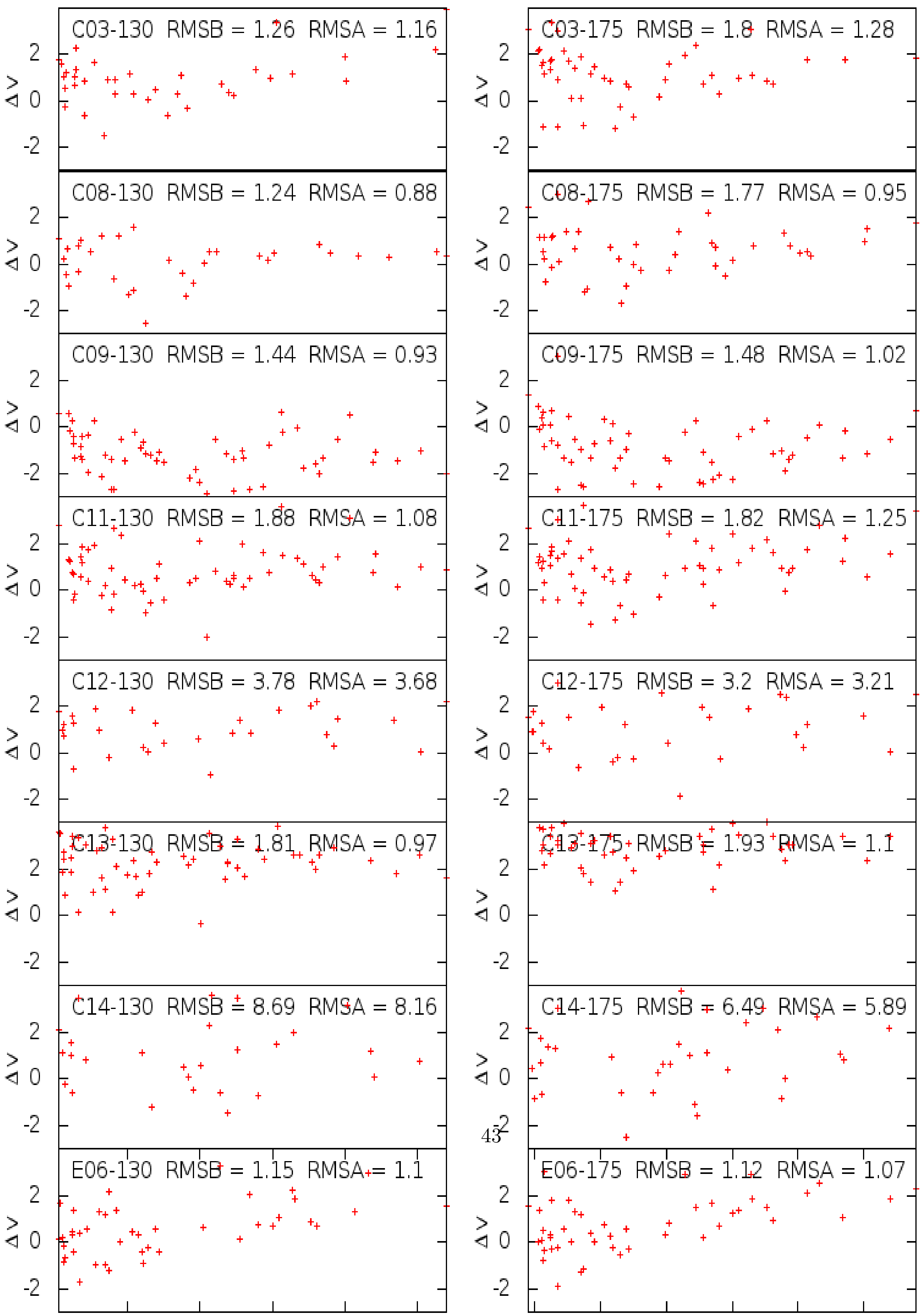

Figure 29: Residual plots for May-2007 Data: Δv offset