National Centre for Radio Astrophysics

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Addition of central gear 'CG' to the existing Antenna simulation model in Matlab

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Objective: Analyzing stability of Antenna speed loop model (matlab simulation) after incorporating central gear mechanical parameters.

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Abstract

1. To design and fabricate a new test rig for carrying out/study the dynamic behavior of GMRT antenna to external disturbances like wind (High frequency noise) for arriving at an improved servo system with faster and higher tracking accuracy.
2. (A)

   As a part of the design of new test rig, to model the existing test rig with antenna (plant model) and plot its behavior to step input

   (B)

   Compare the above obtained response with already available Antenna plant model (referred from Simulations of GMRT servo-mechanical system by Ms Trupti Ranka)

3. Tune the difference between model in 2A and 2B if any using Matlab to arrive at the new dimensions of test rig.

Acknowledgements

I am thankful to Dr. Bhal Chandra Joshi and Shri. Suresh Sabhapathy for giving me an opportunity to work on this project.

Shri Suresh Sabhapathy also helped me understand Antenna plant model and guided me with step by step approach to problem statement assigned.

I would also like to thank mechanical group members for providing information related to mechanical system of GMRT antenna.

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1. Introduction

The project is being done so that one can test the dynamics of the antenna at the lab itself without actually going to antenna. The setup developed will also be used as a test bench to do various experiments before implementing in Antenna

Matlab simulation model for complete antenna servo mechanical system is given in report 'Simulations of GMRT servo-mechanical systems' by Ms. Trupti Ranka.

The project proposed here incorporates actual hardware in loop with part of antenna dynamics represented as simulation model in Matlab instead of representing complete servo-mechanical system as a simulation model. This approach helps obtain results much closer to actual/physical antenna system. Figure1 below gives block diagram of proposed solution.

Dynamic load which includes controlled and uncontrolled inertia is modeled in Matlab. This load is interfaced to the third motor of test rig setup which creates an affect of actual load on test rig setup. Output of controlled inertia which is position of antenna will be logged in as well reproduced as a real world signal to give position feedback signal to PC104 based servo station computer. SSC will again generate required speed demand to drive BLDC motors of test rig setup.

To understand dynamics incorporated by 'CG' simulations were carried out for,
Velocity loop + Current loop + Antenna dynamics + Central gear.
Block diagram for this setup is shown in figure2. It is validated in section 6 that incorporating CG does not make any significant change in output response.

Once simulation setup in figure2 is validated, controlled and uncontrolled inertia is separated from plant model and remaining setup which includes velocity loop, current loop, motors, gearbox and CG is represented as actual physical hardware. This is shown in figure3. It also shows how figure1 and figure2 are related to each other

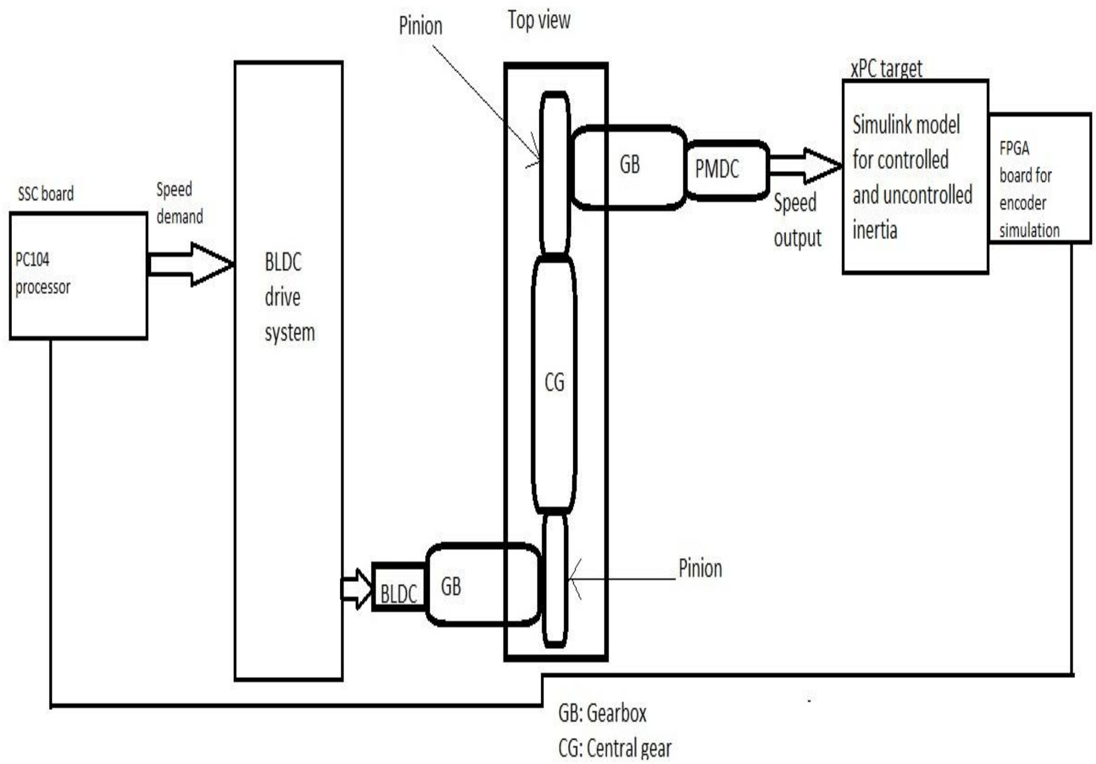


Figure1: Hardware in Loop setup

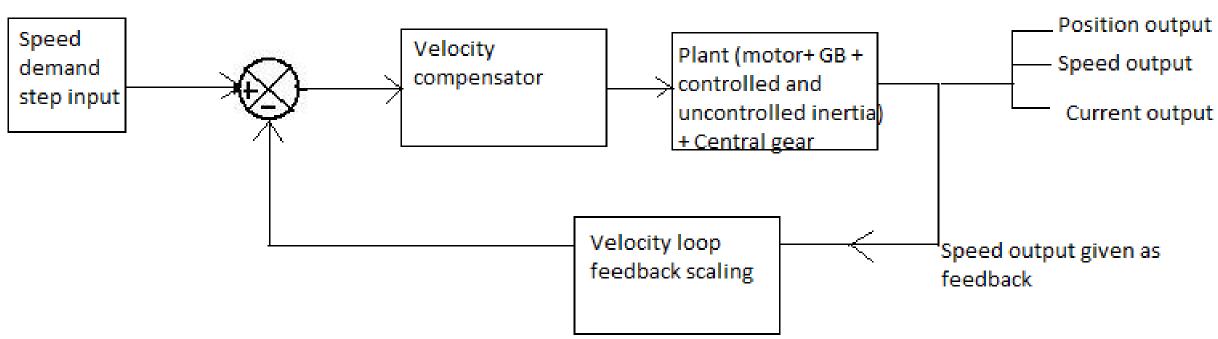


Figure2: Block diagram of velocity loop

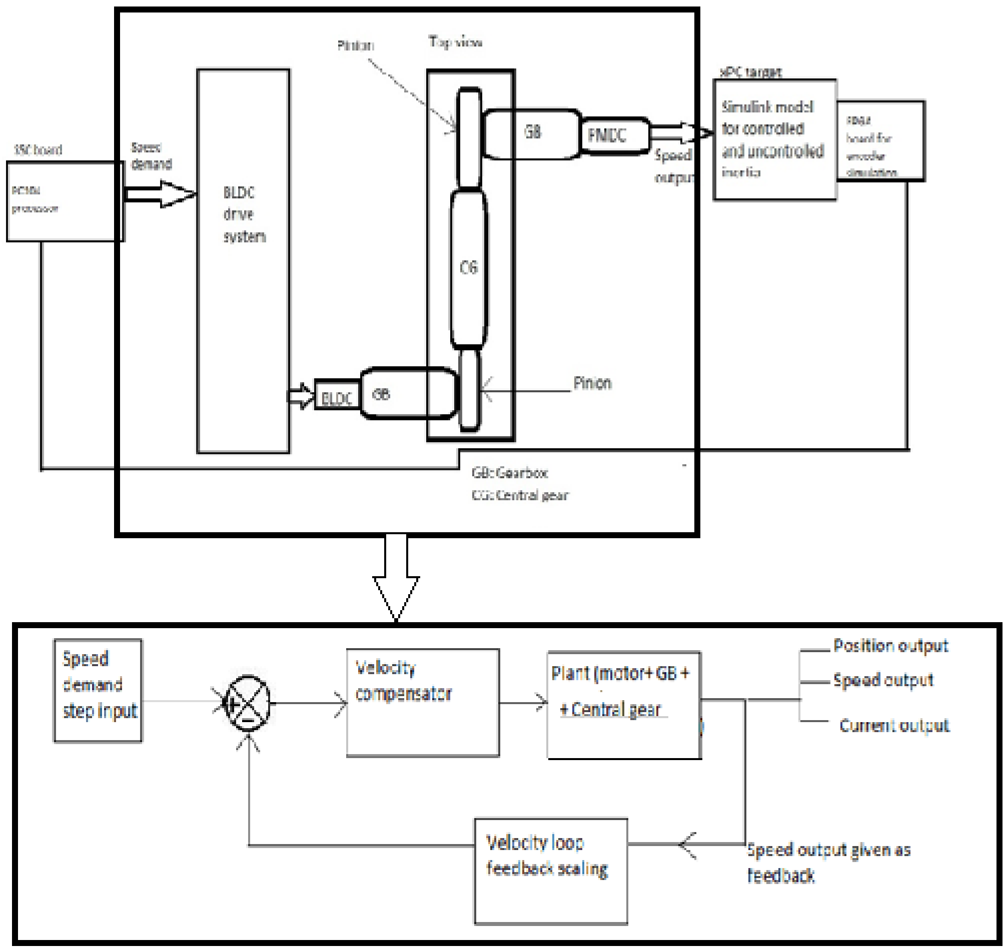


Figure3: Relation between Figure1 and Figure2

2. Structure of Report

The report is divided in different sections. Section 3 presents equations and model of servo-mechanical system of GMRT antenna. The equations are followed by section 4 which gives Matlab model for velocity loop and brief explanation of blocks therein. Section 5 show mechanical parameter values given in [1] and velocity loop output results obtained from them. Section 6 repeats simulation done in Section 5 with mechanical parameter values obtained from TCE.

Section 7 gives theory related to mechanical parameter such as Moment of Inertia, Stiffness and damping and presents ways to calculate these values for central gear of test rig setup. Values calculated are added to mechanical parameter values of Antenna and simulation results are obtained.

Section8 gives conclusion and further work to be done.

3. **Review of GMRT Antenna model**

3.1 Introduction

The present servo system at GMRT has brushed DC motors and associated control system to move each axis. From the control system perspective, the positioning of the antenna is done with the help of three nested loops. The final output of these loops is given to the servo amplifier which drives the motors and the antenna. These loops are current loop, velocity loop and position loop.

For analysis of test rig setup we will not consider position loop. We will consider transfer function of velocity loop and represent current loop by a gain factor.

The mechanical system of Antenna consists of gearbox, Pinion, slew ring and Antenna parabolic structure consisting of the hub (Rigid structure) and parabolic dish curve and quadripod (Non rigid structure). Complete mechanical system can be represented by model as shown in figure4 referred from 'simulation study of servcontrol system of 45 meter dish'.

3.2 Antenna servo mechanical model

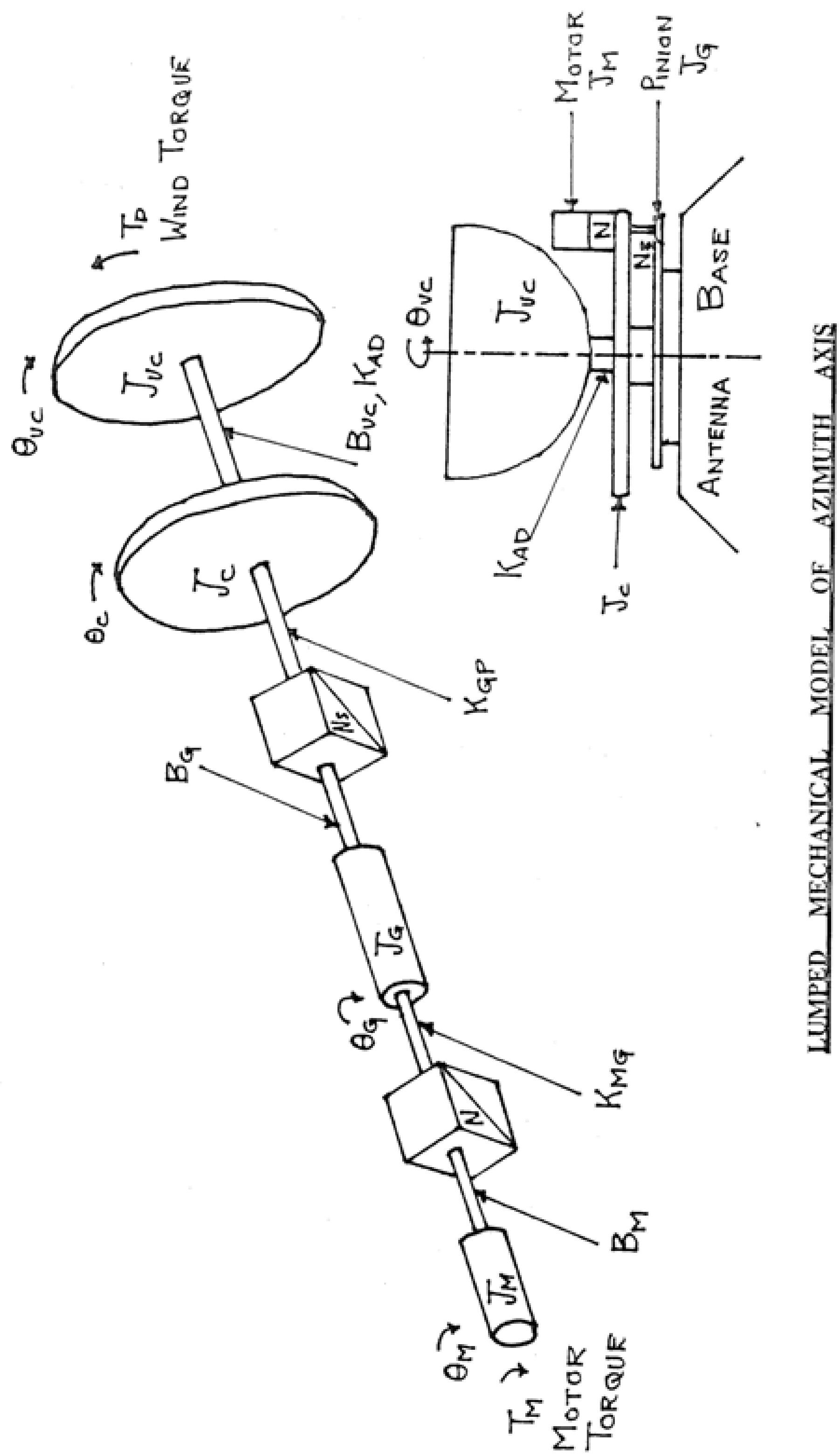


Figure4

3.2 Representation of model using state space equations

The Mechanical model shown in Figure 1 is represented by differential equations. Using these differential equations state space matrices can be formed as follows,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-K_{mg}}{J_m N^2} & \frac{-B_m}{J_m} & \frac{k_{mg}}{J_m N} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{K_{mg}}{J_g N} & 0 & \frac{-Kmg - K_{gp}/N_s^2}{J_g} & \frac{-B_g}{J_g} & \frac{K_{gp}}{J_g N_s} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{K_{gp}}{J_c N_s} & 0 & \frac{-(K_{gp}+K_{ad})}{J_c} & \frac{-(B_c+B_{uc})}{J_c} & \frac{K_{ad}}{J_c} & \frac{B_{uc}}{J_c} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{K_{ad}}{J_{uc}} & \frac{B_{uc}}{J_{uc}} & \frac{-K_{ad}}{J_{uc}} & \frac{-B_{uc}}{J_{uc}} & 0 \\ 0 & \frac{-K_b}{L_a} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-R_m}{L_a} \end{bmatrix} \quad \text{..... (1)}$$

$$B = \begin{bmatrix} 0 \\ \frac{K_t}{K_f J_m} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{E_a}{L_a} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

4. Matlab model of speed loop

Figure (5) shows velocity loop of servo system represented in Matlab. Standard Matlab blocks including state space block, transfer function block, gain block and terminator block is used. Brief description of each section is given below,

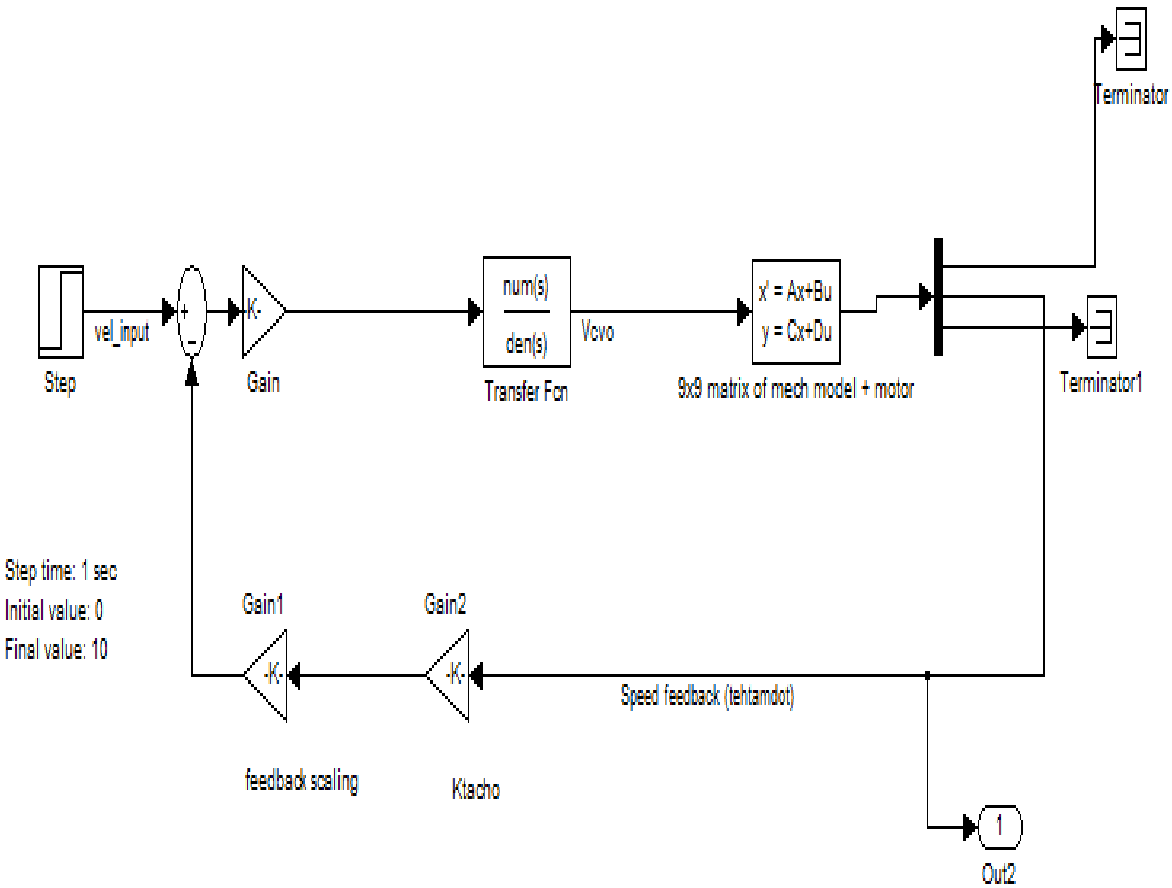


Figure 5

Description of each block in figure 5

Step block: Is used to give step signal to velocity loop. Following parameters are configured for step input,
Step time: 1
Initial value: 0
Final value: 10

Transfer function block: Is used to represent velocity loop lead lag compensator transfer function

Following is the transfer function:

$$\frac{V_{output}}{V_{input}} = \frac{R4[1 + S*C1(R1+R2)][1+S*C2*R3]}{R2[1+S*C1*R2]\,[1 + S*C2(R3+R4)]}$$

$$= \frac{5.3583*[S^2 + 111.531S + 1594.21]}{[S^2 + 200.215S+2049.611]} \quad \ldots\ldots (5)$$

Gain block: Is used represent gain term ‘5.3583’ in eqn (5). It is used adjoining with transfer function block in figure 5

State space block: Is used to incorporate state equations of mechanical model shown in figure (5). State space matrices A,B, C and D mentioned in section 2.2 are used to state space block.

Gain2 block: Is used to represent tacho scaling. For PMDC motor system tacho scale factor is calculated as,
17V = 1000rpm
34V = 2000rpm
Calculations:
1 revolution = 2*pi radians
Therefore,
1000rpm = (1000*2*pi) radianspermin
In one minute it rotates 2000*pi radians
In one second it rotates (2000*pi/60) radians
Therefore
Tacho constant (Ktacho) = 17/(2000*pi/60)

Ktacho = 0.162 volts/radians/sec

Gain1 block: Is used as scaling factor to match input speed demand signal with tacho feedback signal. Input speed demand signal varies from 0 to 10V. Feedback signal is scaled accordingly. When we have 34V at the output we require 10V as feedback signal. Therefore ratio becomes **10/34 = 0.29**

Terminator block: Is used to terminate signal. In state space block output matrix 'C' is gives three outputs namely position, speed and current. As we are using only speed feedback we terminate other two output signal.

Out2 block: Is used to log speed output signal which can be plotted later on.

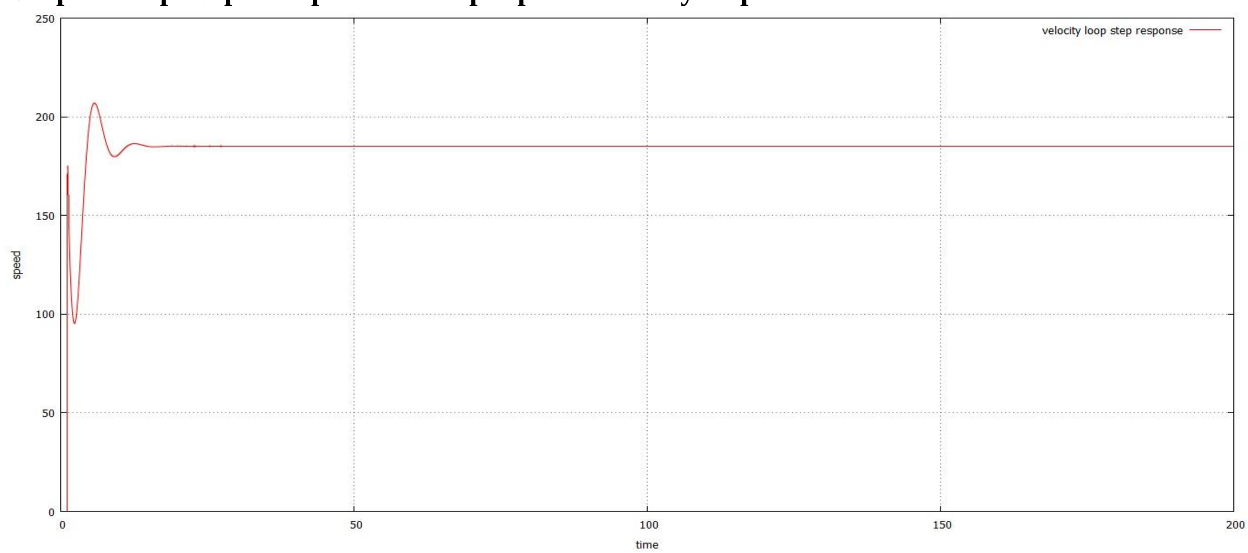
5. Simulation of model using parameter values given in 'Simulation study of servocontrol system of 45 meter GMRT' by Prof. Bhal Chandra Joshi

State space block consists of state space matrix A, B, C and D. Each Matrix have J,B and K parameters of different mechanical subsystem presented in Figure 4. Parameter values used and results obtained are shown in subsection below

5.1 Parameter values

```
jm=.066768;           % Motor and gearbox
jg = 79.288;          % Slew ring inertia
jc = (3.0804) * (10^6) % Yoke inertia
juc=6.4464 * (10^8);      % Dish, Cradle and bullgear inertia
bm=.005424;           % motor viscous friction
bg=4379.2;            % Slew ring viscous friction
bc=7.8336*(10^6);     % controlled inertia viscous friction
buc=1.34368*(10^7);    % Uncontrolled inertiaa viscous frcition
kmg=1e7;              % motor to pinion spring constant
kgp= 5.0846*(10^9);    % Pinion to controlled inertia spring constant
kad= 1.428*(10^9);    % Controlled inertia to uncontrolled inertia spring constant
n=1488;
ns=12.6;
kf=.055;
kt=0.56;              % torque constant
kbe = 0.5634;         % Tacho constant
ra = 0.045;
la = 0.33e-3;
```

5.2 speed loop output response for step input to velocity loop



Rise time (Tr)	3.0265 sec
Peak time (Tp)	5.68 sec
Settling time (Ts)	18 sec
% Peak overshoot	11.35%

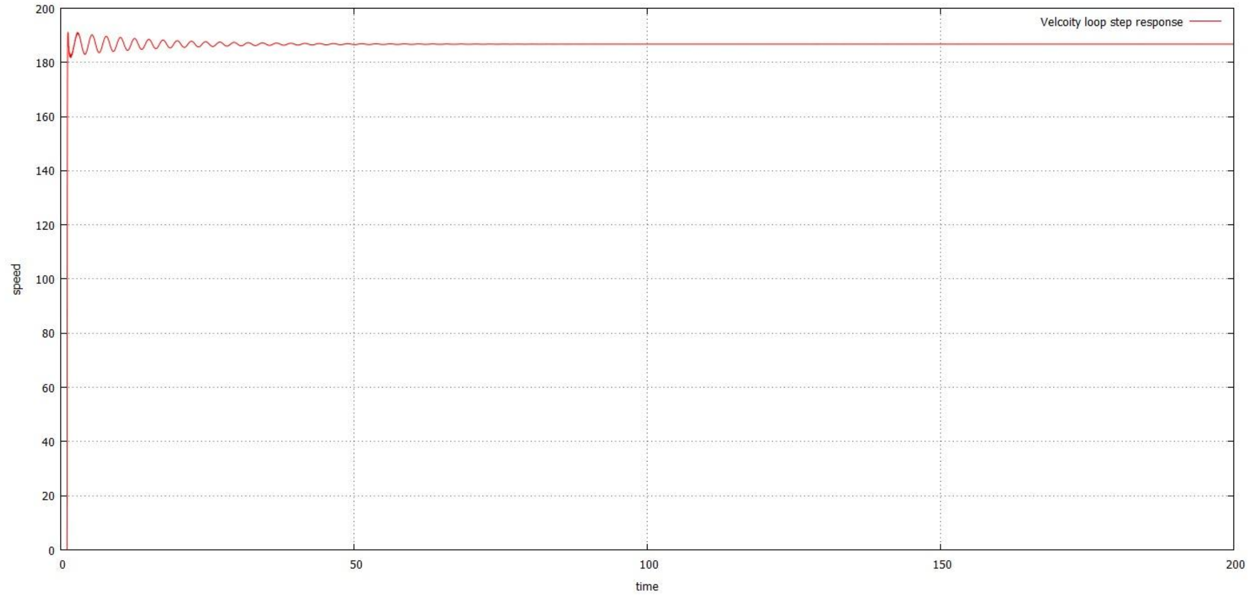
6. Simulation of model using parameter values given by TCE

Simulation results obtained in section 5 does not represent output response for actual GMRT antenna. To obtain simulation results representing actual GMRT antenna new mechanical parameter values obtained from TCE are used.

6.1 Parameter values

```
jm=.066768;          % Motor and gearbox
jg = 2908;           % Slew ring inertia
jc = (5.43734) * (10^5) % Yoke inertia
juc=6.8 * (10^6);    % Dish, Cradle and bullgear inertia
bm=.005424;          % motor viscous friction
bg=4379.2;           % Slew ring viscous friction
bc=5.9996*(10^5);    % controlled inertia viscous friction
buc=1.3854*(10^5);   % Uncontrolled inertiaa viscous frcition
kmg=1e7;             % motor to pinion spring constant
kgp= 5.43734*(10^7); % Pinion to controlled inertia spring constant
kad= 705.625*(10^6); % Controlled inertia to uncontrolled inertia spring constant
n=1488;
ns=12.6;
kf=.055;
kt=0.56;             % torque constant
kbe = 0.5634;        % Tacho constant
ra = 0.045;
la = 0.33e-3;
```

6.2 speed loop output response for step input to velocity loop



Rise time (Tr)	0.08 sec
Peak time (Tp)	0.21 sec
Settling time (Ts)	60 sec
% Peak overshoot	2.4%

7. Simulation with CG incorporated in model

To understand effect on output speed response due Central gear (Test rig central wheel),we need to incorporate values of Moment of Inertia (J), Stiffness (K) and Viscous friction in state space equations of GMRT model

7.1 Calculation of J, B and K parameters for Central gear

1. **Moment of Inertia (J)** (Refer Appendix A for MOI, Refer Appendix B for Central gear dimensions)

From above dimensional values and Moment of inertia formula for solid cylinder we can calculate 'I' for Central gear as follows,
$I = 1/2*M*r^2$
Where M is the mass of the object and r is the radius of solid cylinder
Now we need to calculate Mass (M)
Density = Mass/Volume
Therefore,
Mass = Density* Volume
Volume: $\Pi r^2 h$
$= \Pi(1168/2)^2*127$
$=136.075*(10^6)\ mm^3$
Volume of central gear = 136.075 *$(10^3)cm^3$
Density of material = 7.86 kg/cm^3 (Reference: Website)
Mass = Density * Volume
Mass = 1069.55kg
Now,
$I = 1/2*M*r^2$
$= ½*1069.55*(58.4)^2$

Moment of Inertia (J) = 1.8238 * $(10)^6$ kg.cm^2

2. **Stiffness**

Stiffness of material is defined as force per unit deflection.
$K = T/\theta$ (4)
$K = (G*J)/L$ (5)
Where,
T: Torque applied
θ: Unit deflection due torque applied
G: Modulus of Rigidity (Shear stress/Shear Strain) (Refer Appendix D)
J: Polar moment of Inertia. Where $J = \Pi d^4/32$ (Refer Appendix C)
L: Distance between rigid surface and point where force is applied for angular rotation of wheel.

To find values of parameters mentioned in equation 5,

G = 11.5 (Material dependent)

J = $\Pi d^4/32$

= $(\Pi*(116.8)^4/32)$

= $18.27*10^6$

L = 58.4 cm

Therefore,

K = $(11.5* 1.827*10^{11})/58.4$

Stiffness (K) = $3.597 * 10^6$

3. **Damping / Viscous Friction:**

Damping is an influence upon oscillatory system that has the effect of reducing or restricting its oscillations

Coulomb friction:

Two dry or lubricated surfaces rubbing together.

Viscous friction:

It results from shearing of fluid (lubricant) in gap between moving parts and is considered linear function of relative velocity.

Damping force **Fd = $C*V*|V|^{r-1}$**

Where

C: Constant damping coefficient

V: Relative velocity

r: constant defining type of damping

For viscous friction r =1

Therefore, **Fd = C*V**

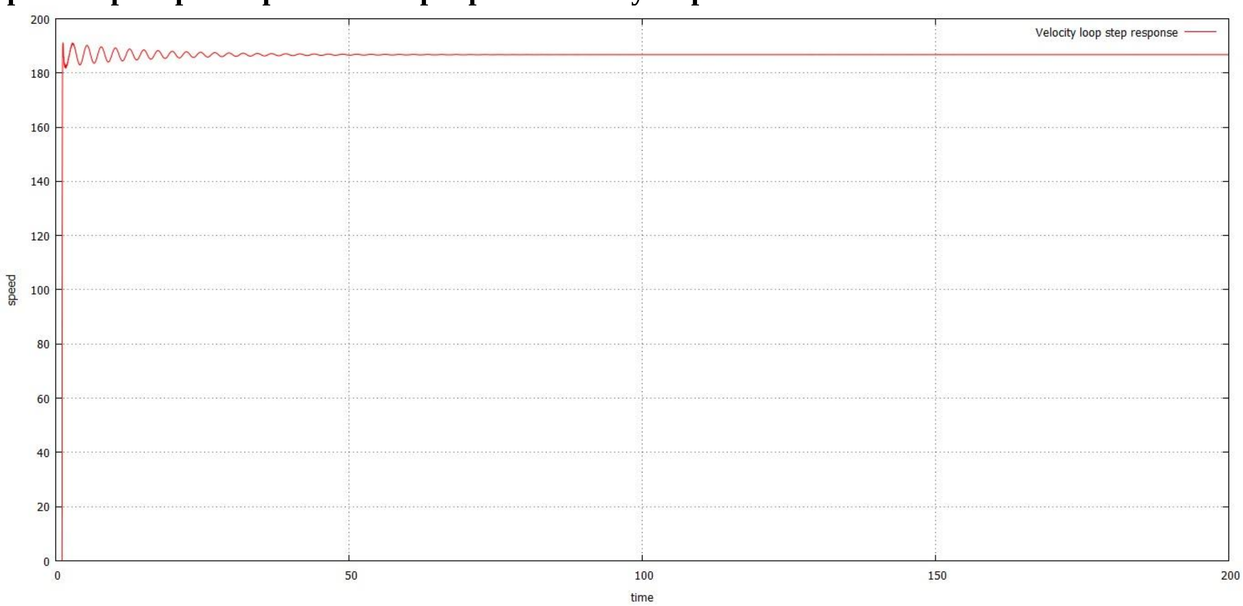
Note: Not able to find out way by which we can calculate viscous friction between Central gear and pinion

7.2 Parameter values used for simulation

```
jm=.066768;            % Motor and gearbox
jg = 2908;             % Slew ring inertia
jc = ((5.43734) * (10^5)) + (182.38)  % Yoke inertia
juc=6.8 * (10^6);      % Dish, Cradle and bullgear inertia

bm=.005424;            % motor viscous friction
bg=4379.2;             % Slew ring viscous friction
bc=(5.9996*(10^5));    % controlled inertia viscous friction
buc=1.3854*(10^5);     % Uncontrolled inertiaa viscous frcition
kmg=1e7;               % motor to pinion spring constant
kgp= (5.43734*(10^7));    % Pinion to controlled inertia spring constant
kad= (705.625*(10^6)) + (3.597 * (10^6));    % Controlled inertia to uncontrolled inertia spring constant
n=1488;
ns=12.6;
kf=.055;
kt=0.56;               % torque constant
kbe = 0.5634;          % Tacho constant
ra = 0.045;
la = 0.33e-3;
```

7.3 speed loop output response for step input to velocity loop



Rise time (Tr)	0.085 sec
Peak time (Tp)	0.2 sec
Settling time (Ts)	65 sec
% Peak overshoot	2.68%

8. Conclusion

Matlab simulations were carried out for GMRT servo system (Speed loop only). Simulation results with mechanical parameter values obtained from TCE is as shown in section 5.2. Results obtained with CG incorporated are as shown in section 6.3.
Comparing both the results we can see that there is no difference after incorporating CG. Step response output is also bounded and stable.

Further step is to do hardware in loop proposed in figure 3

References:

[1] Prof. B. C. Joshi, “Simulation study of servocontrol system of 45 meter giant metrewave radio telescope" Master's thesis, Indian Institute of Technology, Bombay, 1992.

Appendix A:

Moment of Inertia

Kinetic energy of a body having translational motion:

$KE = ½*M*V^2$

For rotating rigid body we need to consider collection of particles with different speeds.

Now,

$KE = ½*M_1*V_1^2 + ½*M_2*V_2^2 + ½*M_3*V_3^2 + ^{-----}$ (1)

$KE = \sum(½*M_i*V_i^2)$ (2)

Where Mi is the mass if the ith particle and Vi is its speed

For rotating body Vi is not same for all particles. Substituting v= ωr in eqn (2) we have,

$KE = \sum(½*M_i*r_i^2)* \omega^2$ (3)

(ω is same for all particles)

Quantity in parentheses in equation (3) tells how mass of the rotating body is distributed about its axis of rotation. This quantity if called rotational inertia or **Moment of Inertia** '**I'** with respect to axis of rotation.

Based on above equation (3) Moment of Inertia for different body shapes are formulated.

Central gear of test rig can be closely represented by a solid cylinder.

Moment of Inertia for solid cylinder is $I =1/2*M*r^2$)

Appendix B:

Dimension values of Central Gear (available with us)

Outer diameter: 1200mm

Pitch circle diameter (PCD): 1168mm

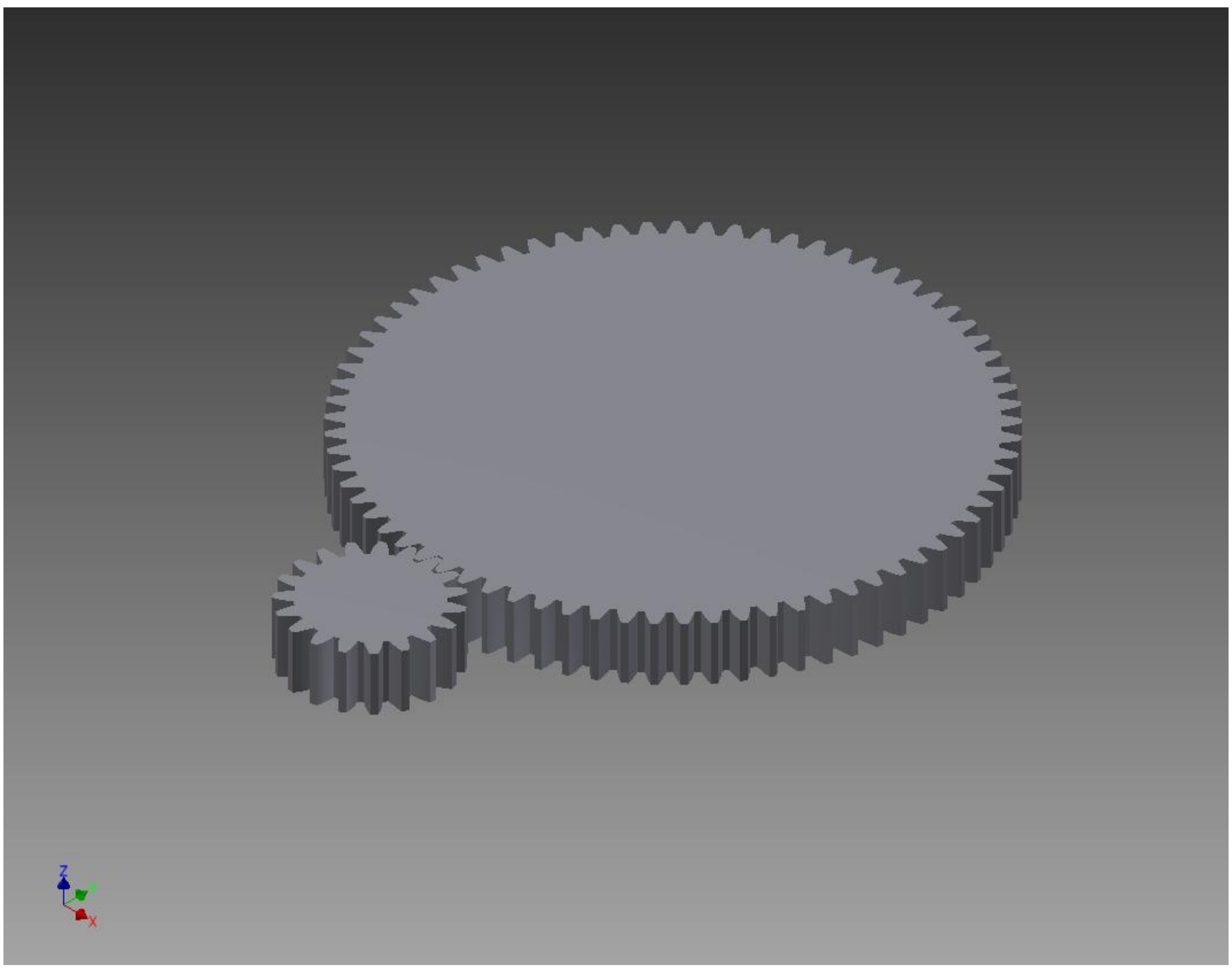
Module: 16mm (Module =PCD/No of teeths)

No of teeths: 73

Material : En-24 (High quality, High Tensile alloy steel)

Face width: 127mm (approx)

Image created by 3D CAD software (Inventor)



Appendix C:

Polar moment of Inertia

Moment of Inertia of Plane figure with respect to an axis in its plane:

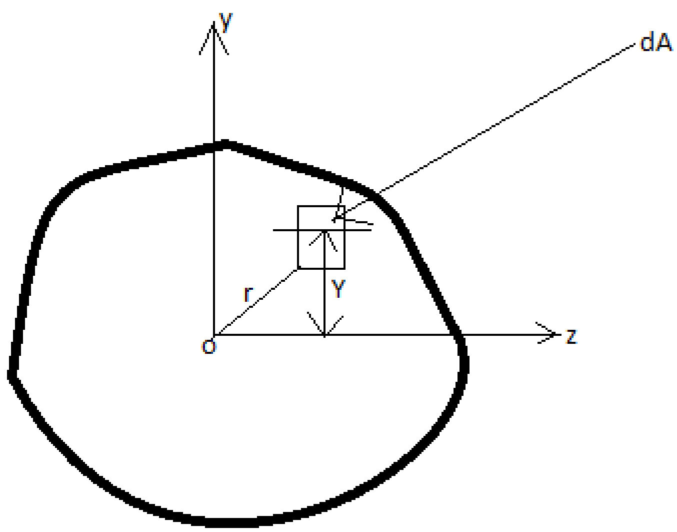


Figure 1

Each element of area dA is multiplied by square of its distance from the Z axis and integration is extended over cross sectional area A of the beam. Such integral is called moment of inertia of the area A with respect to Z axis.

$I_Z = \int_A Y^2 * dA$

Polar moment of Inertia

The moment of inertia of a plane area with respect to axis perpendicular to the plane of the figure is called polar moment of inertia w.r.t point where axis intersects the plane (POINT **o** in figure 1)

$I_P = \int_A r^2 * dA$

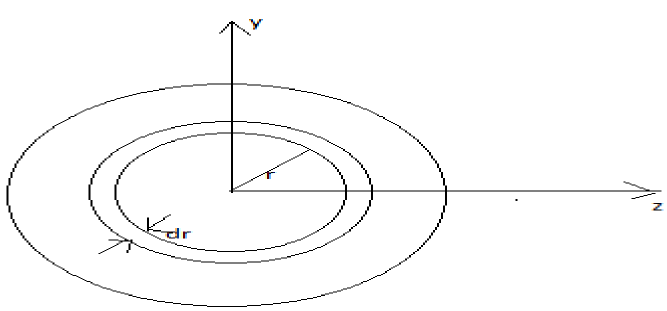


Figure 2

Let us consider a circular cross section shown in figure 2. If we divide area of circle into thin elemental rings as shown in fig 2 we have,

$dA = 2\Pi r.dr$

Therefore,

$I_{P} = \int_{A} r^{2} * (2\Pi r.dr)$

$I_p = \Pi d^4/32$

Appendix D:

Stress and Strain:

A stress or deforming force per unit area produces a strain or unit deformation.

Stress and Strain are proportional to each other.

Stress = modulus * Strain

Modulus of Rigidity = Stress/Strain

Types of Stress:

1. Tensile stress
2. Shearing stress (We consider this stress for Test rig model design)
3. Hydraulic stress