Technical Note: A discussion on the cross polar Fixed Delay behaviour across the UGMRT Band 3 and 4 for phased array Beam mode.

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1 Statement of the Problem

Assume the signal from the right and left hand circular feed to the sampler at a frequency ω is given by $\mathbf{R}(\omega)$ and $\mathbf{L}(\omega)$, where

$\mathbf{R}(\omega) = \mid \mathbf{R}(\omega) \mid \mathbf{e^{-i\omega\phi_R(\omega)}}$
$\mathbf{L}(\omega) = \mid \mathbf{L}(\omega) \mid \mathbf{e^{-i\omega\phi_L(\omega)}}$

And let the response function of the sampler for the two channels be,

$\mathbf{T_r}(\omega) = \mid \mathbf{T_r}(\omega) \mid \mathbf{e^{-i\omega\theta_R(\omega)}}$
$\mathbf{T_l}(\omega) = \mid \mathbf{T_l}(\omega) \mid \mathbf{e^{-i\omega\theta_L(\omega)}}$

The cross product of the two channels is hence

$\mid \mathbf{T_r}(\omega) \mid\mid \mathbf{T_l}(\omega) \mid \mathbf{e^{-i\omega(\theta_l(\omega)-\theta_r(\omega))}} \mid \mathbf{R}(\omega) \mid\mid \mathbf{L}(\omega) \mid \mathbf{e^{-i\omega(\phi_l(\omega)-\phi_r(\omega))}}$

The phase $\phi(\omega)$ consists of two parts, the rotation measure for the source due to ISM and ionosphere and the fixed delay due to propagation time difference between two channels. So,

$\phi(\omega) = \phi_{\mathbf{RM}}(\omega) + \phi_{\mathbf{FD}}(\omega)$

Assuming the system is linear[1], if one observes a highly polarized source and the faraday rotation of the source is known, then using the model

$\phi(\omega) = \mathbf{4\pi^2c^2RM}/\omega^\mathbf{2} + \mathbf{B2\pi c}/\omega + \mathbf{A}$

where A, B are constants, RM is the rotation measure and c is the velocity of light. If we also assume the phase of the response function $\theta(\omega)$ to be linearly depending on frequency, then those phases can be absorbed in the above equation. The left hand side of the equation is a measured quantity, $\phi = \mathbf{tan^{-1}\frac{Re}{Im}\frac{RL^\star}{RL^\star}} = \mathbf{tan^{-1}\frac{U}{Q}}$

and for a source with known RM the first term in right side of the equation is known. So the constants A and B can be found. This can be used to calibrate the instumental phase gradiant (so called Fixed Delay) across the band.

To do this we observe a bright and highly linearly polarized pulsar like PSR B1929+10, which has well know RM value (preferably choose a pulsar with low RM value). Data in full polar beam mode is recorded in the phased array. The raw data is in filterbank format with four channels as, $\mathbf{RR^\star}$,

$\mathbf{2Re(RL^\star)}$, $\mathbf{LL^\star}$, and $\mathbf{2Im(RL^\star)}$.

The data is used to produce the average bandshape for $\mathbf{RR^\star}$, and $\mathbf{LL^\star}$ across the band (i.e. frequency channels), and we define the gain $\mathbf{G(i)}$ for a frequency channel i as $\mathbf{G(i)} = \mathbf{RR^\star(i)/LL^\star(i)}$. The following operation is done to now obtain the measured stokes parameters for every sample

[1] The linear assumption include low cross coupling, linear response of feed and sampler, low spurious polarization due to pickup and reflection from antenna surface.

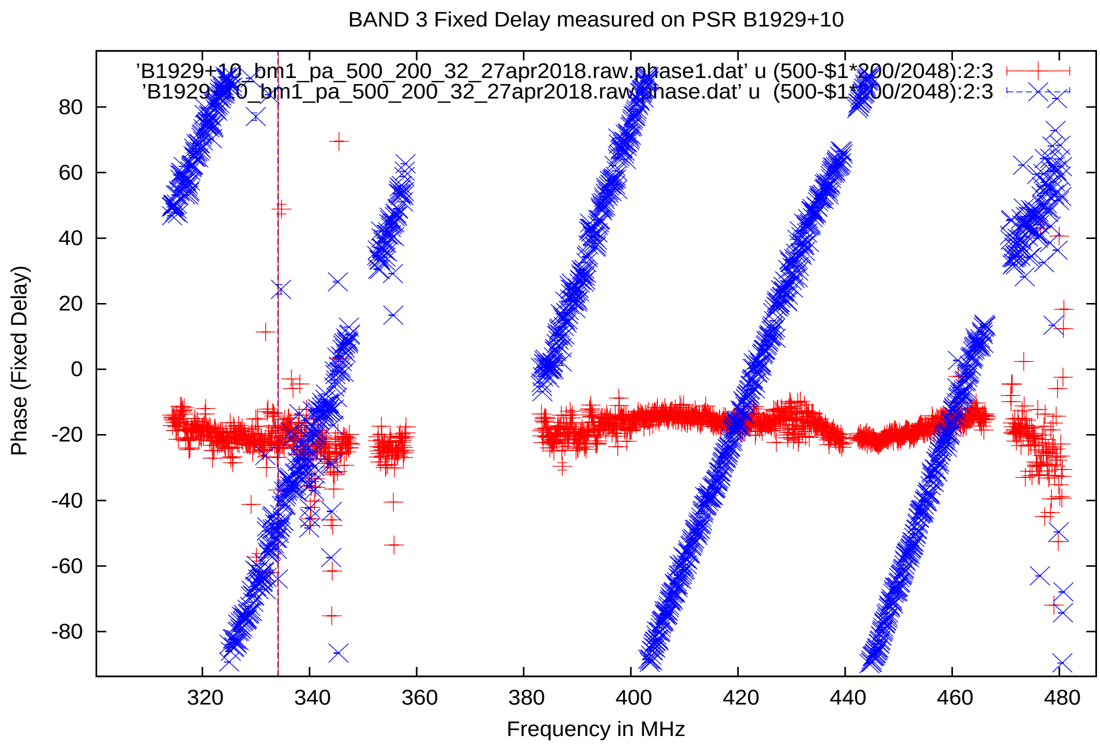

Figure 1: Fixed Delay measurement for Band 3, 300 to 500 MHz.

Convert measured parameters to measured stokes for every time sample:

$$\mathbf{I_m(i) = RR^\star(i) + G(i)(LL^\star(i))}$$

$$\mathbf{V_m(i) = RR^\star(i) - G(i)(LL^\star(i))}$$

$$\mathbf{Q_m(i) = 2\sqrt{G(i)}\ \ Im(RL^\star(i))}$$

$$\mathbf{U_m(i) = 2\sqrt{G(i)}\ \ Re(RL^\star(i))}$$

Subsequently one can flatten the bandpass by making $\mathbf{I_m = 1}$ for all channels. Once tha bandpass is done, the data is folded and the measured average full stoke profile as a function of channel is obtained. A high signal to noise ration point in the pulse is chosen to obtain $\phi(\mathbf{i}) = \mathbf{atan}\frac{\mathbf{U_m(i)}}{\mathbf{Q_m(i)}}$. The phase is corrected for the RM of the source and the resultant phase across the band is the Fixed Delay (FD). Note that at this stage the convention of $\mathbf{U_m, Q_m and V_m}$ is arbitrary and needs to be fixed appropriately defined using the calibrator.

In Fig. 1 the blue points are the $\phi_{\mathbf{FD}}$ FD phase obtained for Band 3. A straight line fit to $\phi_{\mathbf{FD}}$ is done and then the subtracted curve is shown as red points in Fig. 1.

In Fig. 2 zoomed in version of $\phi_{\mathbf{FD}}$ measured for two observations are shown. It is seen that there are some oscillatory features as well as the curve is nonlinear. The overall spread in the curve across the band is about 30°. The edge of the bands, below 300 MHz and above frequencies 470 MHzi, variation in $\phi_{\mathbf{FD}}$ of roughly 20° is seen.

A similar exercise can be done for BAND 4, i.e. 550 to 750 MHz and the result is shown in Fig. 3. Very similar conclusion, as was seen in BAND 3 seems to hold here. The oscillatory feature is better seen, and is measured to be around 5 MHz.

<u>Inference:</u>

- The $\phi_{\mathbf{FD}}$ across frequency for both BAND3 and BAND4 is nonlinear, and oscillatory. A simple linear fit to the fixed delay is not a good model. The oscillatory pattern has a frequency of about 5 MHz and the spread in $\phi_{\mathbf{FD}}$ is about 30°.

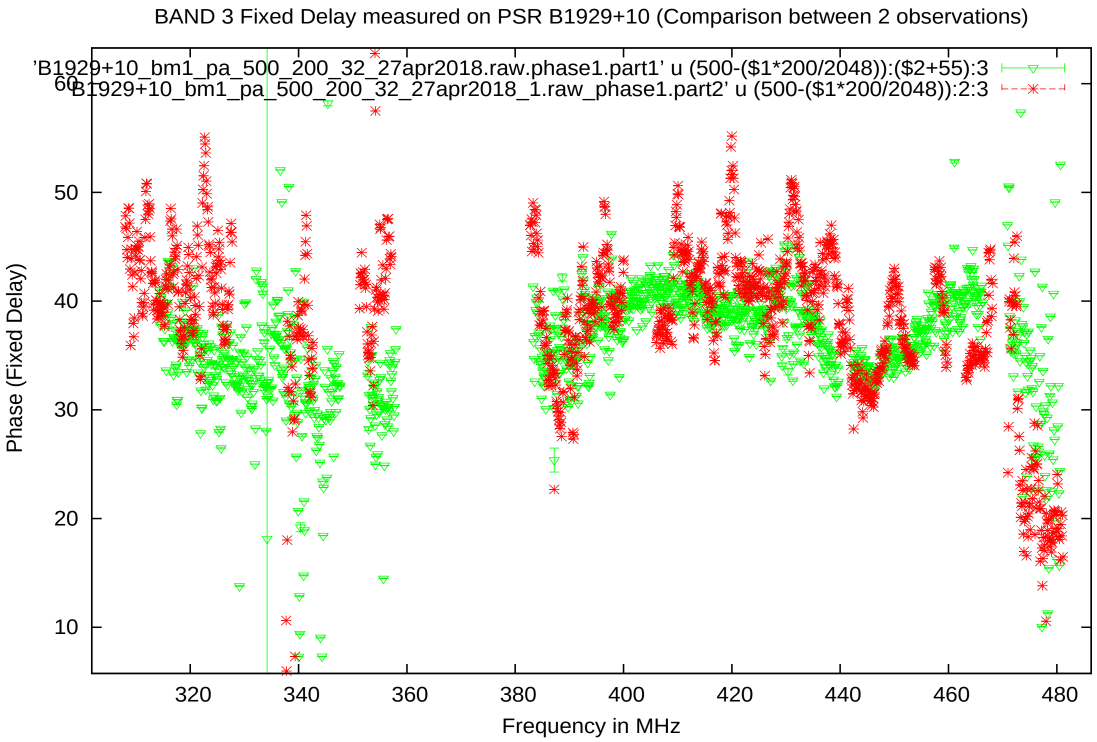

Figure 2: Fixed Delay measurement for Band 3, 300 to 500 MHz.

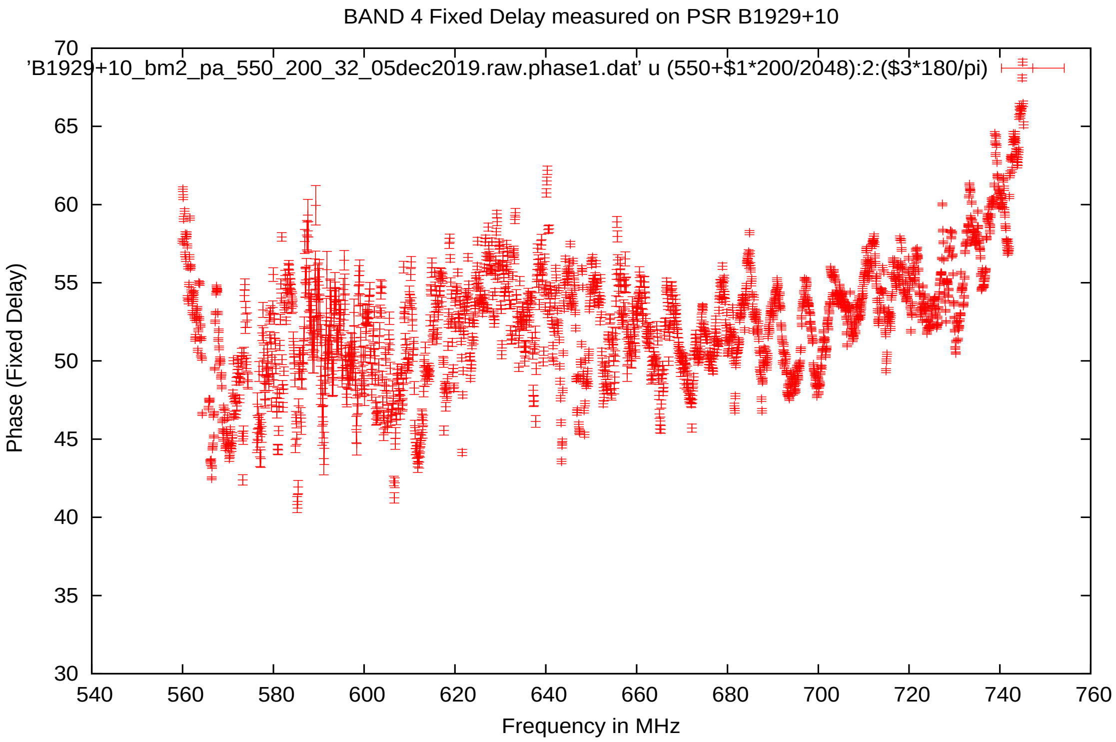

Figure 3: Fixed Delay measurement for Band 4, 550 to 750 MHz.

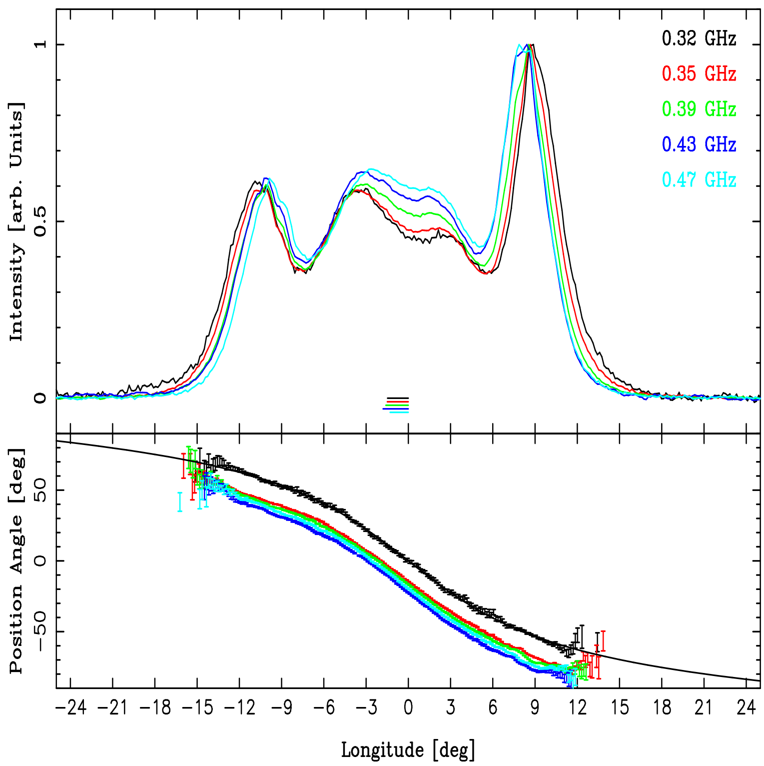

Figure 4: The top plot shows the position angle traverse for BAND 3, where the band is divided into 5 parts.

- The difficulty in modeling the $\phi_{\mathbf{FD}}$ across frequency impacts observations. It can lead to depolarization of the signal while collapsing the channels to produce average profiles. Secondly it can impact finding parameters like rotation measure. One such example of how this impacts the rotation measure is shown in Fig. 4. In this case PSR B2319+60 having rotation measure of -232.6 rad m^{-2} has been observed and the band is divided into five sub-bands with frequency range 300-330, 330-360, 379-409, 418-448 and 455-485 MHz respectively. Our primary interest here is to access how the observed ϕ changes across the band. As a wide-band science case, the relative change of ϕ across the band (once corrected for ISM faraday rotation) can be modeled to investigate if the pulsar magnetosphere introduces effects of generalized faraday rotation. In Fig. 4 we show how the polarization position angle ($\phi/2$) across the calibrated band across the pulse profile behaves for each of the five sub-bands. Clearly there is a spread in ϕ i.e. along the Y-axis, which arises due to our inability to correct the fixed delay. In Fig. 5 top plot ϕ corresponding to the zero longitude in Fig. 4 is shown with respect to the wavelength and in the second plot with square of the wavelenght. To quantify the variation we fit a similar functional form like rotation measure to this data and find the RM to be 0.7$\pm$0.1 rad m^2. This *proxy* rotation measure however arises due to the nonlinearity in the band.

- More analysis of pulsars in band 4 and the behaviour of ϕ with a variety of RM and fractional polarization across the band is shown in Fig. 6. Fig. 7 shows examples of the behaviour of fractional linear polarization across band for various sources. Fig. 8 shows the bandshape with front end terminated situations. The nonlinear and oscillatory patterns are clearly seen in these plots, and since the patterns are not repeatable they cannot be used to correct the fixed delay in the data.

- The nonlinear pattern of ϕ can arise due to response at the feed, path from feed to baseband, baseband to sampler etc. Given that the pattern of ϕ versus frequency is similar between BAND 3 and BAND 4. Futher careful tests at each subsystem (for e.g. standing wave due to cable lengths etc.) can be useful to identify the source of the nonlinearity.

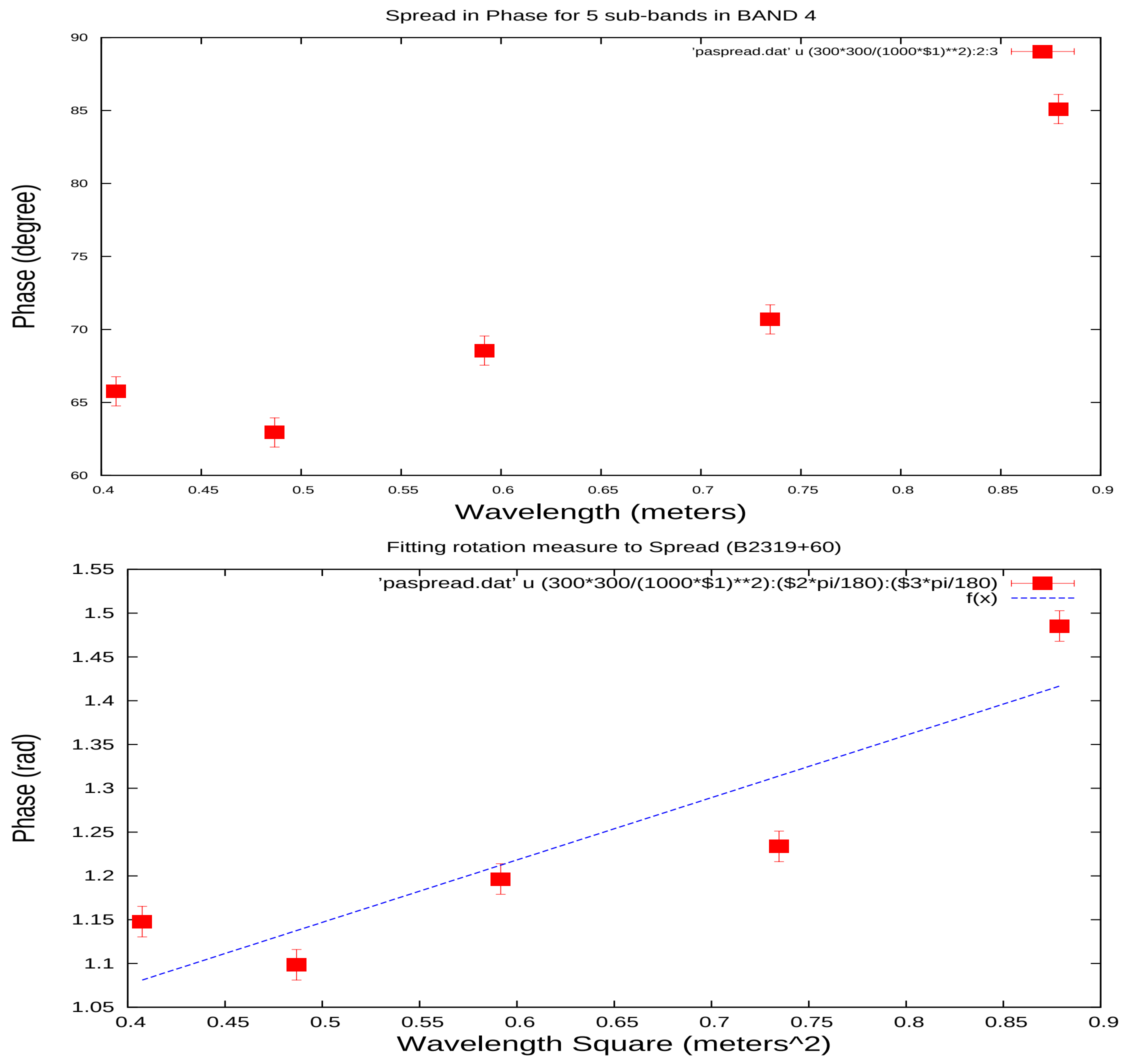

Figure 5: The phase ϕ for the zero pulse longitude in Fig. 4 plotted as a function of wavelength and square of wavelength.

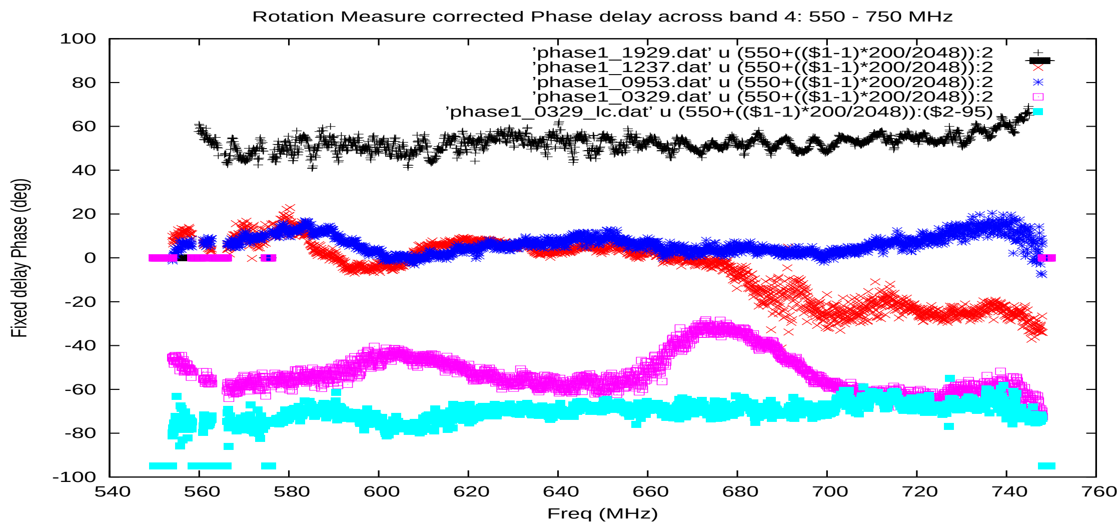

Figure 6: The fixed delay phase ϕ plotted for pulsars with different rotation measures and varying percentage polarization.

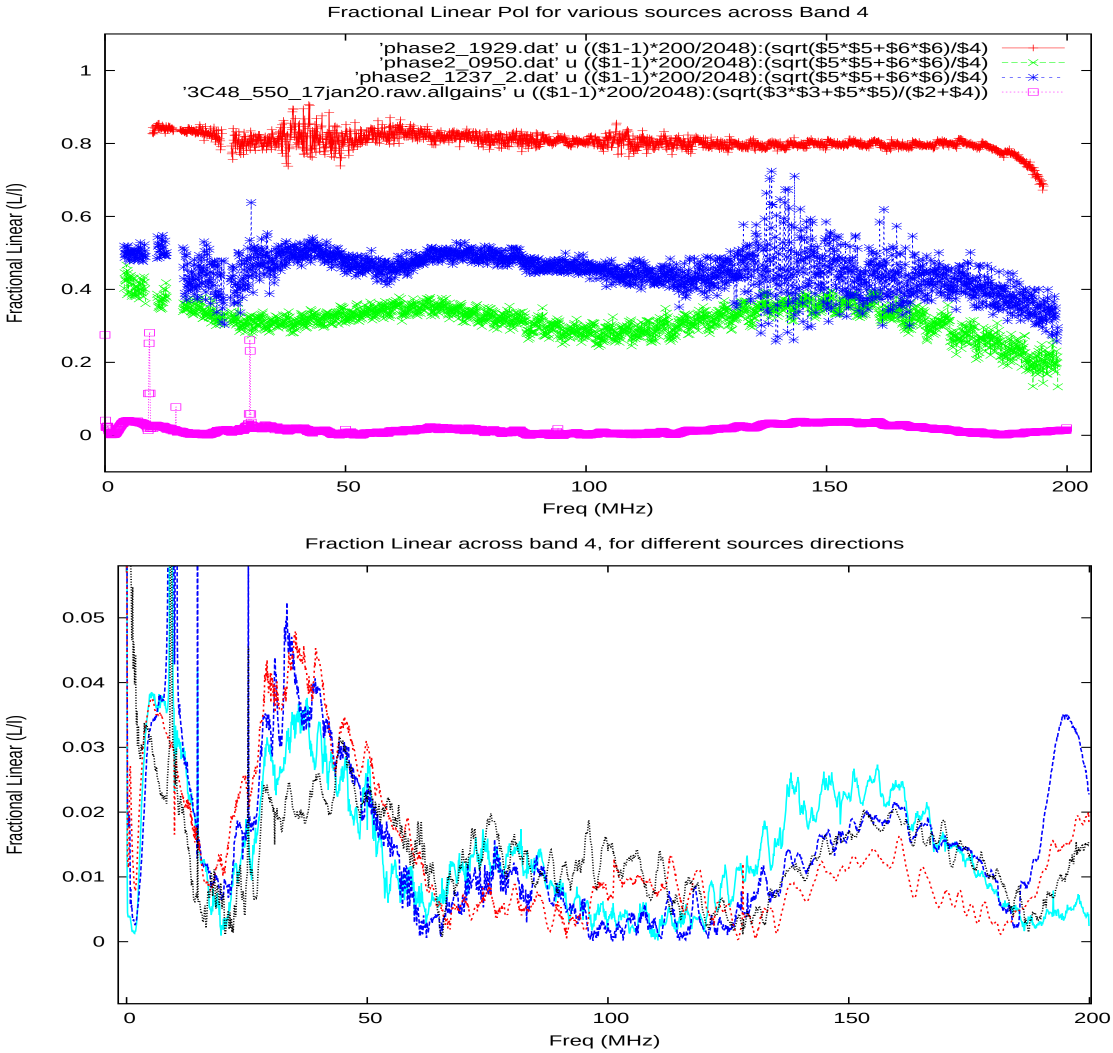

Figure 7: The fractional linear polarization across band4 for various cases.

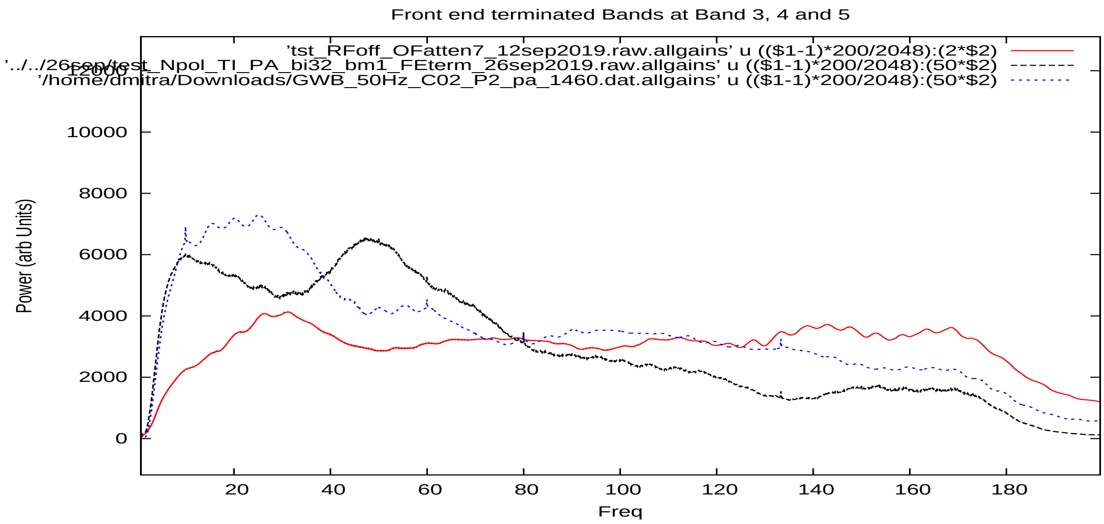

Figure 8: The front end terminated bands showing wiggles and oscillations.

Acknowledgement

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