Phase Stability and GMRT's Frequency Standard

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Chapter 1

Introduction

The number and variety of applications using precise timing are astounding and increasing. The applications range from ones, which touch our daily life such as synchronization of power system networks to more esoteric ones such as testing theory of relativity in astrophysics. Similarly, the research into atomic clocks has made remarkable progress in recent years. High quality time and frequency standards are a necessity for a large observatory such as Giant Meterwave Radio Telescope (GMRT). This report presents the results of experiments conducted to identify low-cost alternatives for GMRT and a proposal for suitable time and frequency electronics subsystem for GMRT.

1.1 Need for a good Time-Frequency reference in Observatory

Phase stability of the reference frequency standards over wide time scales is one of the most crucial requirements for the interferometric instruments such as GMRT. Stability on short time scale is related to the building up of phase noise when a signal is multiplied in frequency as is the case for Local Oscillator (LO) synthesis at GMRT(Ambrosini et al. 2000). The short term stability is also important for accurate time-stamps required by pulsar timing. New pulsar receivers employing coherent dedispersion can provide very high time resolution data (Joshi et. al. 2003) and this allows pulsar timing solutions with RMS residuals of the order of 35 ns (van Straaten et al. 2001). Such high precision timing is necessary for a wide range of astrophysical applications ranging from general relativity to the detection of primordial gravitational wave background. The discovery of a micro-second pulse width pulsar, PSR J1909-3744 (pulse width $\sim 43\mu s$), at Parkes radio telescope is another example where such precision is needed (Jacoby et al. 2003). The Global Positioning System (GPS) receivers usually provide timing edges with a jitter of about 300 ns and a more accurate time stamp is needed for the above studies. A local frequency standard with good short term stability is therefore needed for such observations to be possible at GMRT.

The long time scale stability is required for effectively integrating the signal obtained from the cross correlation of individual antennas for a usual synthesis of observations of 8 hours or more. Radio astronomical observatories such as GMRT require a good phase stability from the receiver electronics, in order to preserve the coherence of the radio astronomical signals spread over the array. This requires a stable frequency reference source for the observatory.

1.2 Time-Frequency Standards

Practical and precise timing came with the invention of the quartz crystal oscillator. The arrival of atomic clocks provided even greater accuracy. In recent times, the accuracy of atomic clocks has improved, on average by a factor of two every year. It is interesting that this geometric progression rate of improvement is the same for computer memory density.

A quartz crystal vibrates when electric charge is applied and vice-versa due to its piezoelectric nature and most modern wrist-watches make use of this property. For precise time and frequency standards, a variant of this oscillator known as oven controlled crystal oscillator (OCXO), is enclosed in temperature controlled chamber to reduce the sensitivity to environmental variations. Consequently, OCXO has good short term stability.

On the other hand, an Atomic clock uses as its reference the oscillation of an electromagnetic signal associated with a quantum transition between two energy levels in an atom. For a given quantum transitions, the photons emitted or absorbed have a unique frequency proportional to the energy difference.

The figure 1.1 shows the predictability of a wide variety of clocks. The different slopes are indicative of different kinds of random processes perturbing the timing data coming from that particular clock.

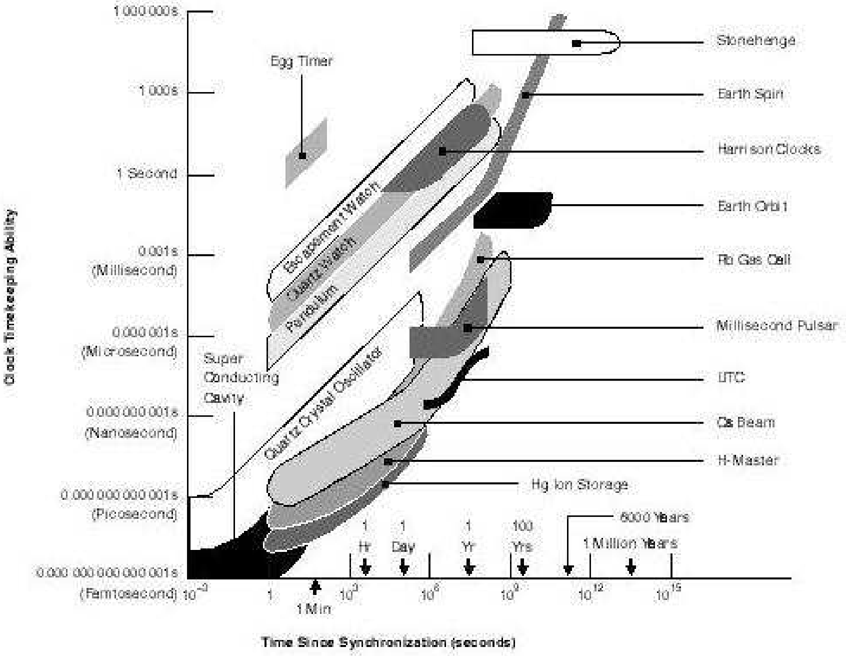

Figure 1.1: A comparison of the timekeeping ability of different sources as a function of time since being synchronized(HP application note)

1.3 Objectives

As atomic timekeeping has come of age, it has become increasingly important to identify the quality of a given frequency standard. With the advent of GPS, it has also become possible to achieve a high degree of stability with low cost frequency standards. However, such systems need to be characterized and evaluated with a reference to the needs of the observatory. It is also necessary to organize the standard and the associated electronics as compact, easily maintainable and fault tolerant subsystem with built in redundancy in the form of hot spares and/or hot swappable spares. The main objective of the experiments described in this report was to evaluate a GPS disciplined system of rubidium (Rb) frequency standards from the point of view of a frequency standard for GMRT and suggest a possible design of such subsystem.

Some of the most important quality factors are directly related to the inherent noise of a quantum device and its associated electronics. Objectives of these extensive time domain stability experiments on the Rb frequency source is a concise, yet complete, quantitative and standardized description of phase and frequency of the source including their nominal values,the fluctuations of these values and their dependence on time and environmental conditions. A stability analysis may be concerned with both the stochastic(noise) and the deterministic properties of the device under test. It is often best to characterize and remove deterministic factor(e.g. frequency drift, temperature sensitivity) before analyzing the noise.

Towards this aim we have used some practical techniques for time domain frequency stability analysis of Rb atomic frequency standard. The main experimental scheme consists of measuring systems, few pre-processing and post-processing steps, analysis tools which are used to get the phase errors as well as stability figure. The specific aims of this project were

1. Verification of the performance of Rb atomic clock by inter-comparison experiments.
2. Evaluation of the performance of the EXISTING oscillator in the observatory using Rb source.
3. Comparison of stability of GPS-disciplined Rb using pulsar observation.
4. A proposed design for Time-Frequency subsystem for GMRT

Chapter 2

Frequency Standards and stability tests

2.1 Historical overview

Historically, astronomical time was based on the movement of earth, moon, sun, solar system and celestial sphere of moving stars and planets. Time was deduced from the dynamical equations of motion of these objects(HP application note).

Prior to 1960, the second was defined as “1/86400 of a mean solar day.” The irregularities in the spin rate of earth led to a new definition of seconds in 1967, when the atomic second, based on hyperfine transition in cesium was introduced. The first atomic clock based on a microwave resonance in the ammonia molecule and using microwave photon absorption particle was introduced to the world by Harold Lynos. Later the Lynos’ group researched the possibility of using a cesium beam as an atomic frequency standard. The new era of atomic time-keeping started in 1967 with International Astronomical Union adopting the atomic time standard after the National Physical Laboratory in UK demonstrated the viability of atomic time keeping. The definition of the second was stated as - “the second is the duration of 9,122,631,770 periods of the radiation corresponding to the two hyperfine levels of the ground state of cesium-133 atom.” In the recent years, the very stable galactic-pulsar clock, with very low time uncertainty, promises to another choice of reference standard. Specially some of the millisceond pulsar timing signals have statistics that approach the best of atomic clock(Allan, Barnes et. al. 1981).

2.2 Mathematical tools for analyzing stability

The output of a frequency source can be modeled as a sine wave of the following form :

$$V(t) = [V_0 + \epsilon(t)] \sin[2\pi\nu_0 t + \phi(t)] \tag{2.1}$$

where

$$\begin{aligned} V_0 &= \text{nominal peak output voltage} \\ \epsilon(t) &= \text{amplitude deviation} \\ \nu_0 &= \text{nominal frequency} \\ \phi(t) &= \text{phase deviation} \end{aligned}$$

For analysis of frequency stability we are primarily concerned with $\phi(t)$. The instantaneous frequency

$$\nu(t) = \nu_0 + \frac{1}{2\pi}\frac{d\phi}{dt} \tag{2.2}$$

and fractional frequency offset

$$y(t) = \frac{\nu(t) - \nu_0}{\nu_0} = \frac{1}{2\pi\nu}\frac{d\phi}{dt} = \frac{dx}{dt} \tag{2.3}$$

Time fluctuation x (t) is related to the phase fluctuation $\phi(t)$ as

$$\phi(t) = x(t) 2\pi\nu_0 \tag{2.4}$$

So it is the noise in the phase (or in frequency, which is the derivative of the phase given by $\sim d\phi/dt$) which is important to characterize the stability of the source. A low value of this noise is desirable in astrophysical situations, as explained in Chapter 1.

This noise can be characterized by its standard variance. However, the normal standard variance does not converge to a single value due to the accumulation of the time errors as the number of measurements increases. Hence, standard deviation is not recommended as a measure of frequency stability. Instead, a second difference of time errors is used a figure of merit as described below.

A good measure of the stability of the frequency source, which is affected by both random as well as systematic errors in phase, is Allan Variance(Allan et. al. 1966), which is the second moment of dispersion of frequency from its nominal value. For a data sequence x_i

$$\sigma_y^2(\tau) = \frac{1}{2(N-2)\tau^2}\sum_{i=1}^{N-2}[x_{i+2} - 2x_{i+1} + x_i] \tag{2.5}$$

where, τ is the nominal sample time each value of y is measured. This is convergent and well-behaved in spite of the apparent walk-off phenomenon exhibit by most clocks and remains bounded. In the subsequent chapter, this number is estimated from the data for each standard tested in this project.

The other two important terminology in the field of time and frequency standard are accuracy and aging. Accuracy refers to the time and frequency offset of the device. As $f_{measured} \rightarrow f_{nominal}$, the device becomes more accurate.

2.3 Clock's noises

A perfect frequency source would have a constant value equivalent to a single spectral line. The instability of most frequency sources can be modeled by a combination of power-law noises. There are five different noise types used to model time and frequency devices : white noise phase modulation(PM), flicker noise PM, white noise(random and uncorrelated) frequency modulation(FM), flicker noise FM and random walk FM.

White noise FM is the classical for atomic frequency standard. Flicker FM is common in the frequency-determining resonator or sustaining amplifier in oscillating loop and random walk indicates the environmental sensitivity. White noise PM is the theoretical limiting noise for measurement systems and for network. Rubidium atomic standard has an inherent white FM noise characteristic that falls with square root of average time on short term until some flicker FM floor is reached(often caused by environmental effect)(Riley et al. 2003).

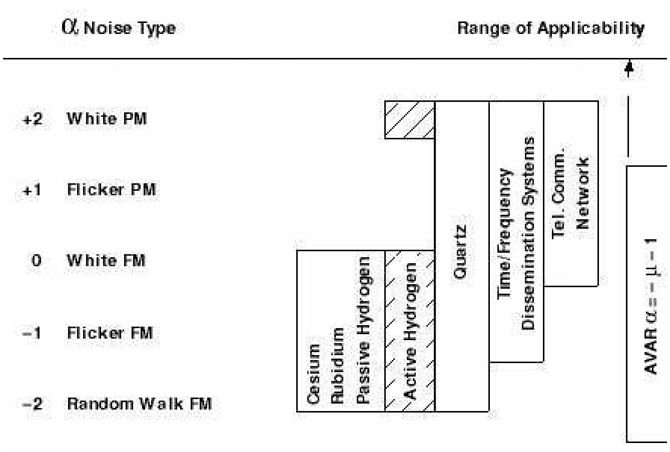

Figure 2.1: Allan Variance (AVA) for different α(HP application note)

The noises have the spectral density of the frequency fluctuations of the form

$$S_y(f) \propto f^{\alpha} \tag{2.6}$$

where, y is the normalized frequency deviation and f is the Fourier frequency.

The value of α is useful in modeling the random fluctuation observed in precise time and frequency systems. The spectral density $S_y(f)$ is a measure of the power present at different Fourier frequencies which are associated residuals in frequency fluctuation plot. The power-law spectral density models is actually represent a performance over a band of Fourier frequencies.

The corresponding instability in time-domain is characterized by Allan variance

$$\sigma_y^2(\tau) \propto \tau^{\mu} \tag{2.7}$$

where, τ is nominal sample time over which each value of y is measured and μ = - α - 1

Figure 2.2 portrays the region of stability for most important kinds of atomic standards.

Now the time-domain measure of stability $\sigma_y(\tau)$ and its frequency domain

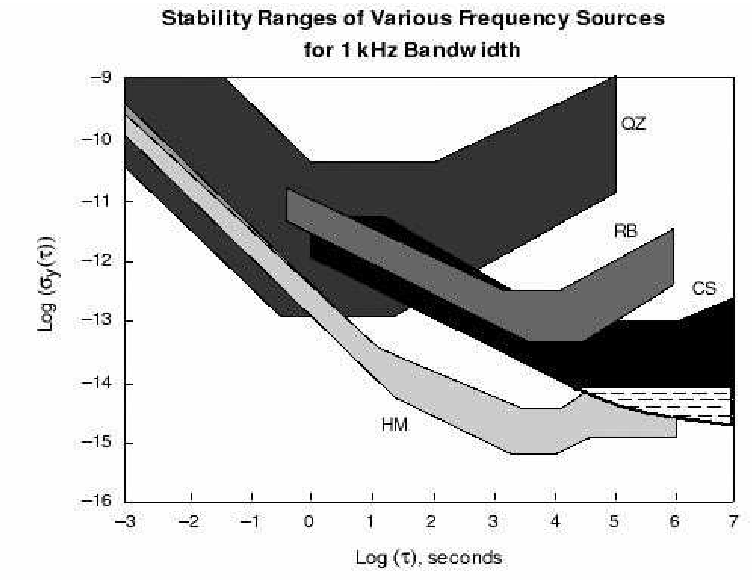

Figure 2.2: Frequency stability diagram for most of the precision clocks. QZ = Quartz Crystal Oscillator, RB = Rubidium Gas-Cell Frequency Standard, CS = Cesium-beam Frequency Standard, HM = Hydrogen Maser Frequency Standard(HP application note).

counterpart $S_y(f)$ is related as

$$\sigma_y^2(\tau) = 2\int_0^{f_h} S_y(f)\frac{\sin^4(\pi f\tau)}{(\pi f\tau)^2}df \tag{2.8}$$

Noise type identification from the slope of the stability plot is important not only for understanding the physical basis of the instability, but also to apply bias corrections and to set confidence limits. So fitting power-law noise lines to stability data is a part of post-processing.

2.4 GPS

The unprecedented navigation is now literally at our fingertips using relatively inexpensive GPS(Global Positioning System) receivers, yet the heart of GPS is an atomic clock synchronized system. The Global Positioning system - a dominant system for the distribution of time and frequency, is a classical example of using precise timing for accurate positioning. GPS features a set of 24 satellites, each with a synchronized on board rubidium or cesium clock. A precise timing device in a GPS receiver is used by its computer to calculate the time of flight of the signal from each of the observable satellite. Receiver's computer can turn the time of flight into a very accurate estimate of the distance to each satellite - accounting for some delay in both the natural and ionized parts of the atmosphere.

Chapter 3

Rubidium Stability checking

3.1 Rubidium atomic clock package

The alkali metal rubidium, has outer shell electron. The red glow of rubidium discharge lamp is due to the transition of this outer electron to the ground state and emits photon of wavelength in the red part of visible spectrum. The ground state of rubidium is split by a small energy due to the relative orientation of the magnetic spin of electron and nucleus. This split corresponds to the energy of photon with f = 6.834GHz.

The basic technique behind Rb frequency standard are optical pumping, microwave interrogation and optical detection. Rb87 lamp has two transitions corresponding a single excited state to the split ground states. The lamp is followed by a Rb85 filter which has one of the two transition levels of Rb87. So it will filter out that particular discharge lines. The remaining photon is absorbed by Rb87 atoms in the resonance cell and moves them to the upper state. The transition from the excited state to any of the ground state have equal probability. So very soon with few transitions there will be a non-equilibrium population difference between two ground states. The photons of energy corresponding to the energy difference between the ground states are emitted. The hyperfine transition frequency can be detected by interrogating the atoms with microwave radiation of 6.834MHz and observing the light transmission through the cell. As the microwave radiation from RF synthesizer matches with the hyperfine transition frequency the detected light emission from the cell will decrease.

In PRS10 unit(10MHz Rb frequency standard which has good short-term

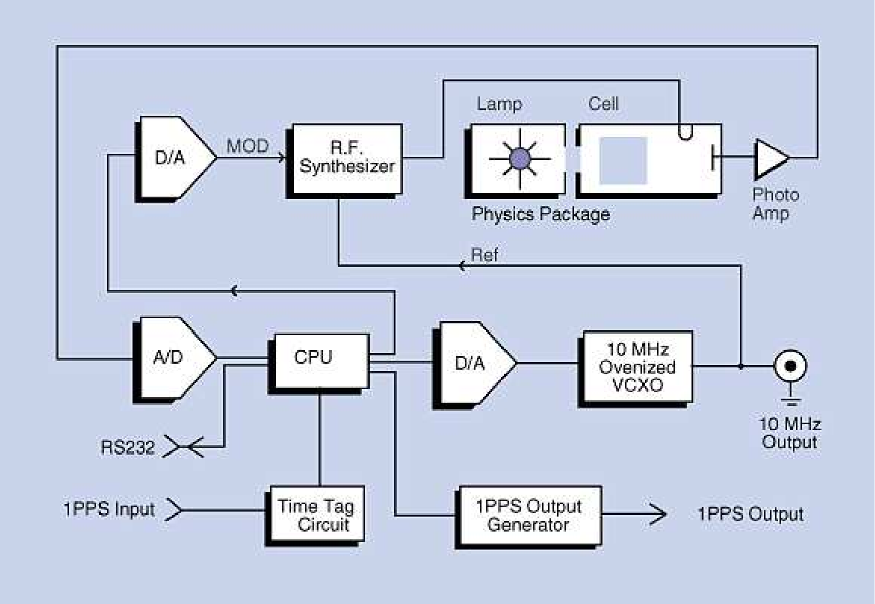

Figure 3.1: Internal block diagram of rubidium atomic clock package

stability and low aging rate), the Rb physical package acts as stable frequency detector of around 6.834GHz. By using a synthesizer which is referenced to the 10MHz VCXO, the 10MHz can be stabilized to the hyperfine transition frequency. The PRS10 can be locked to an external 1PPS source from GPS with very high resolution. The digital PLL is used to adjust the frequency of PRS10 to match the frequency of 1PPS source over long time intervals. The time-tag value is used to phase lock the unit and also is reported back via RS-232, which tells about the drift in time over 1PPS pulse in the resolution of nsecs. The natural time constant of the PLL for following reference phase can be set upto 18hrs which reduce the PLL's sensitivity to the short time jitter of 50-300 nsecs in GPS 1PPS signal and retain the short term stability of rubidium.

3.2 Experimental set-ups and procedures

The overall stability analysis system is consists of

(i) Unit under test and supporting measuring instruments

Test clock, phase monitoring in Vector Voltmeter(VVM)

(ii) Reference standard
GPS disciplined rubidium clock

(iii) Data acquisition and storage
Both phase and time error data with proper time-tagging

(iv) Analysis techniques
Modification of data format, out-lier and drift removal, applying Allan variance statistics

(v) Reporting tools
Plotting, model fitting and interpretation of result

Now the block diagrammatic representation the experimental set-up is given in figure 3.2

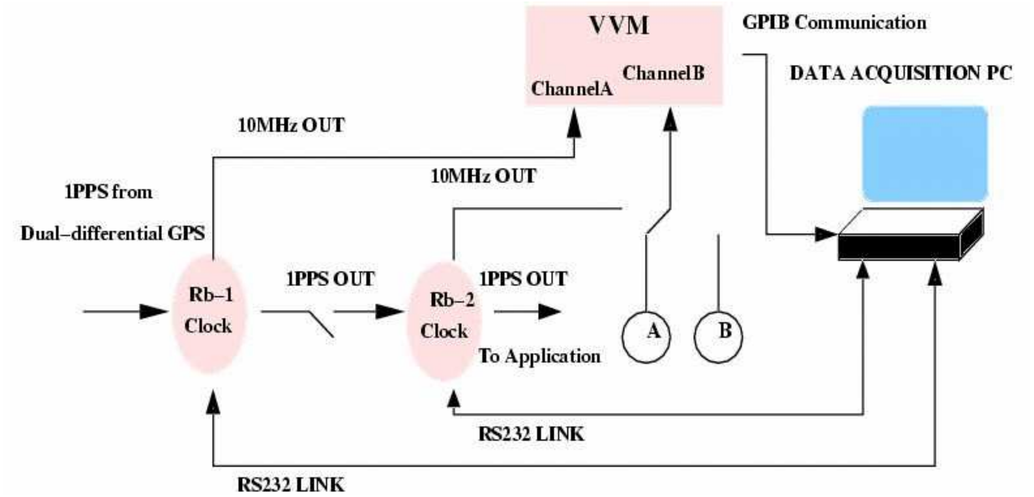

Figure 3.2: Block diagram of the experimental setup, here A is FTS source(Frequency-time standard: existing ovenized crystal oscillator) and B can be any other test unit

The experiments done using this set-up are
(i) Phase data study for GPS disciplined Rubidium Vs existing FTS oscilla-

tor
In this experiment, at the time of initiating data acquisition rubidium and FTS both have crossed the period of frequency retracing with sufficient warm-up time. The phase errors of FTS (connected to the VVM channel B as marked as A in figure 3.2) with respect to the 10MHz GPS disciplined Rb standard (channel A of VVM) was recorded through Vector Voltmeter.

(ii) Phase data and time-tag error study for GPS disciplined Rubidium-1 Vs Free running rubidium-2
In this experiment,also the acquisition was started after the retracing period. The phase error were recorded between VVM channel A(disciplined Rb) and channel B(free Rb) with 1PPS switch(shown in 3.2) open.

(iii) Phase data and time-tag error study for GPS disciplined Rubidium-1 Vs rubidium-2 disciplined by rubidium-1
All experimental conditions are same as in figure3.2 with 1PPS switch closed. Here VVM channel A is connected to the 10MHz output from disciplined Rb and channel B is connected to 10MHz output of Rb2. The time-tag data between the 1PPS pulses of GPS and both of the rubidium were recorded through serial ports.

(iv) Study of the phase stability of GPS disciplined Rubidium using millisecond Pulsar observation.
The main objective of this experiment is to verify the purity of the other blocks in the reference clock chain going to the different subsystems of GMRT receiver. Data on PSR B1937+21 was acquired using the pulsar digital backends and the drift in pulse phase was used as a measure of frequency offset and drift of master oscillator.

Chapter 4

Analysis of data and results

The basic analysis sequences are as follows :

1. The acquired phase data is just a ramp with slope corresponding to the frequency offset, which is eliminated by a linear fit to data.
2. The quadratic shape of residual is due to systematic frequency drift and removing it, phase fluctuation will start show.
3. The residuals are made up of measurement noise, correlated effects driven by a common temperature for both the oscillators and long-term random variations of them. By taking the first order difference between the time errors we have removed the correlated deviations and most of the systematics and we are left with the random uncorrelated deviations.
4. The Allan variance for the system under test is then estimated from the residuals.

4.1 Study of existing FTS with reference of Rb

The phase errors in the 10MHz outputs of FTS and GPS disciplined Rubidium were acquired using the Vector voltmeter. Here GPS disciplined Rubidium acts as an standard to figure out the stability of FTS oscillator(OCXO). The figure 4.1 is showing the offset in the FTS's frequency from the Rb's 10MHz. But after removing the offset, the resultant figure 4.2 is showing that the FTS frequency is drifting with time also. With this slower drift, there is a higher order faster drift (figure 4.3). The quadratic shape in figure 4.3 represents this higher order drift in the selected area in figure 4.2 (marked with dashed line). Subtracting this quadratic drift the residuals phase errors begin to show (figure 4.4). The abrupt jumps evident in figure 4.3 and this unusual 2nd order drift (with random occurrence) in FTS frequency reduces its suitability for use as frequency standard in GMRT. These drifts produced random jumps in arrival times of pulses during pulsar observations, introducing large timing error in pulsar timing.

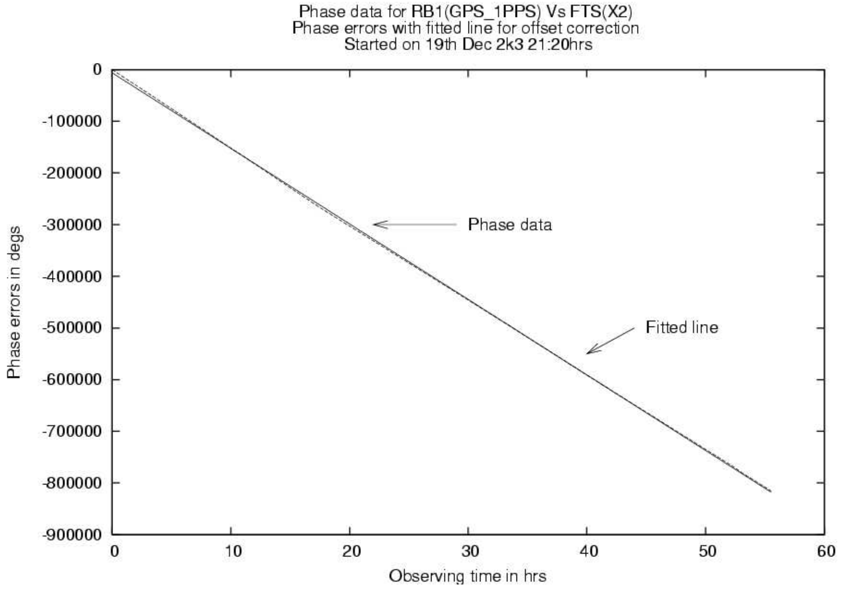

Figure 4.1: Phase errors in FTS oscillator data from GPS disciplined Rb standard

4.2 Study of free running Rb with the reference of disciplined Rb

The phase errors in the 10MHz outputs of free running Rb and GPS disciplined Rubidium were acquired using the Vector voltmeter. In this experiments, the GPS disciplined Rubidium acts as an standard. The figure 4.5 is showing the offset in the free rubidium frequency from the disciplined Rb's 10MHz. After removing the offset, the resultant figure 4.6 is giving the residual phase errors. In figure 4.7, the allan variance is computed from these phase residuals is shown.

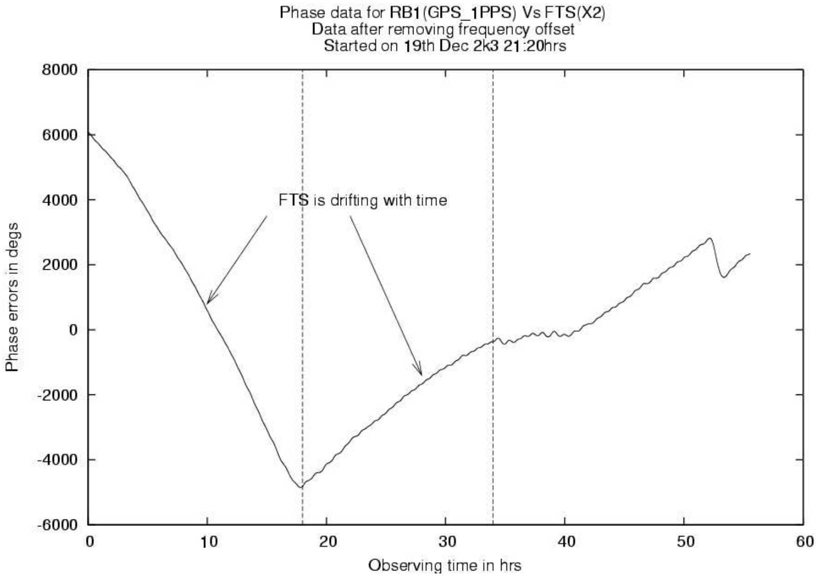

Figure 4.2: Phase errors in FTS oscillator data from GPS disciplined Rb standard after removing offset

4.3 Study the stability of GPS disciplined Rb1 and Rb2 disciplined by Rb1

The phase errors of the 10MHz outputs of GPS disciplined Rubidium and of 2nd rubidium which is disciplined by the first one were acquired using the Vector voltmeter(figure 4.8). The allan variance of Rb2(disciplined by Rb1) from these phase errors is shown in figure4.7. The curve is indicating the short-term stability of disciplined Rb2(which is cascaded connection of two Rbs) and the free running Rb. On short-time scale any disciplined Rb follows the inherent stability of Rb only, ignoring the reference standard. So the free rubidium is actually representing the stability of Rb1(disciplined by GPS) on short time and on the averaging of 1000 secs is arriving on the level of 1×10^{-12}. As the figure 4.7 shows on 10 secs averaging interval the stability of disciplined Rb can have a value between 3×10^{-12} to 7×10^{-12}.

Figure4.9 shows the time-tag error of 1PPS signals of GPS with both of the rubidiums.In this case the data acquisition were started just after connecting the PPS signal to both of the rubidiums and it is indicating that the rubidium will take 14 hrs time constant to get disciplined.

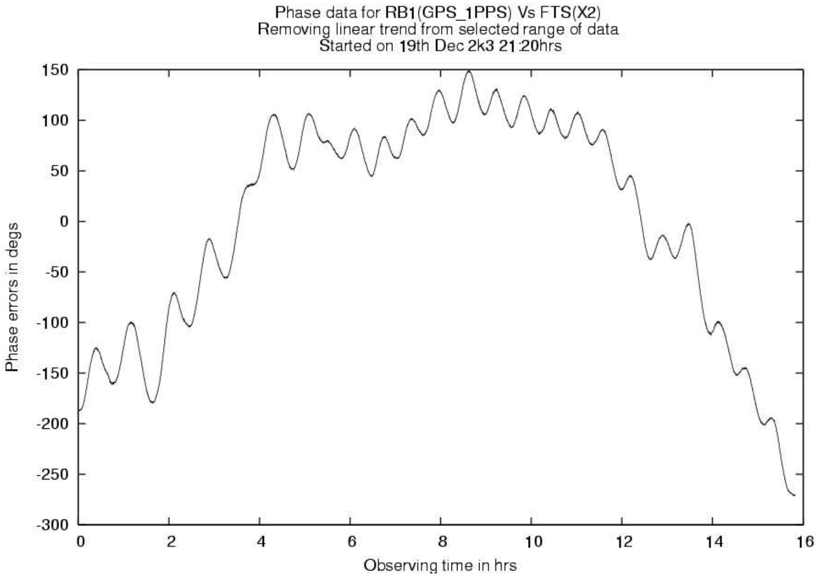

Figure 4.3: Phase errors in FTS oscillator data from GPS disciplined Rb standard over the selecting region(marked with dashed line in figure4.2)

Simultaneous to the phase error data recording between 10MHz output of GPS disciplined Rb1 and of Rb2 (disciplined by Rb1), the time-tag error between 1PPS signal of GPS and the first rubidium and the error between the first rubidium and the second rubidium were also acquired through serial ports (figure 4.10). The short term jitter in GPS signal makes the "GPS Vs RB1" curve much worst than "RB1 Vs RB2" curve. The first rubidium was disciplined for long time, but the data acquisition were started just after connecting the PPS signal to the 2nd rubidium. So for analyzing the stability the first 15hrs of data is considered as outlier where the errors are more due to the undergoing disciplining condition(as mentioned in figure 4.9). In figure 4.11 the stability of GPS and Rb1 time errors and Rb1-Rb2 time errors were compared in terms of Allan Variance. The curve is showing that the Rb1-Rb2 errors is 8 times lesser than the GPS-Rb1 errors on short-term basis and on the averaging of 1000 secs both are arriving at the same level. This is because the short time jitters in GPS pulses are washed out on longer averaging.

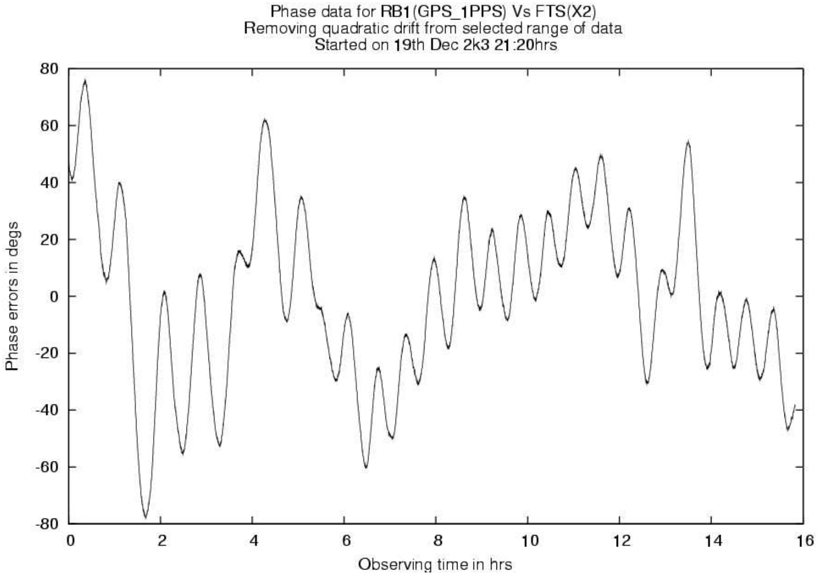

Figure 4.4: Phase errors in FTS oscillator data from GPS disciplined Rb standard after removing drift over the selected region in figure4.2

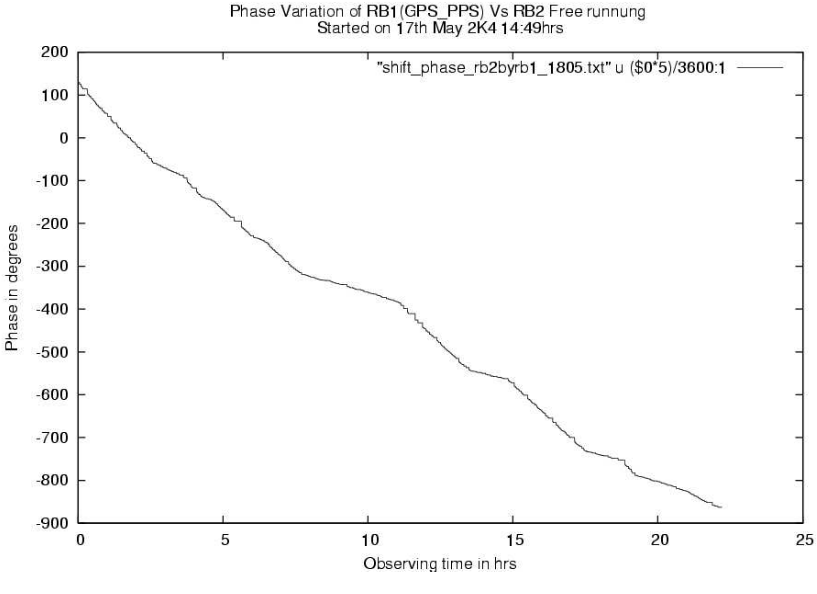

Figure 4.5: Phase errors in free running Rb data from GPS disciplined Rb

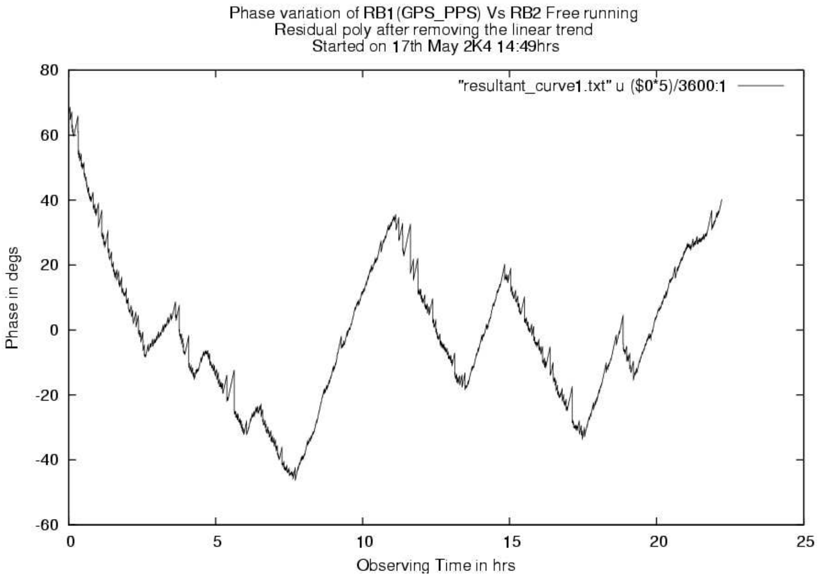

Figure 4.6: Phase errors in free running Rb data from GPS disciplined Rb after removing the offset

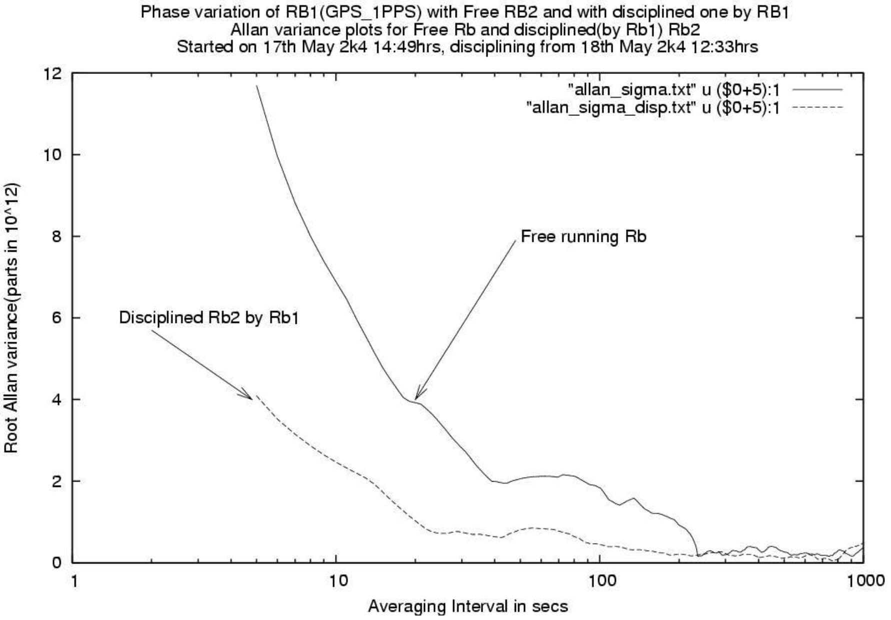

Figure 4.7: Phase stability of free running Rb oscillator and disciplined Rb2(by Rb1)in terms of Allan Variance

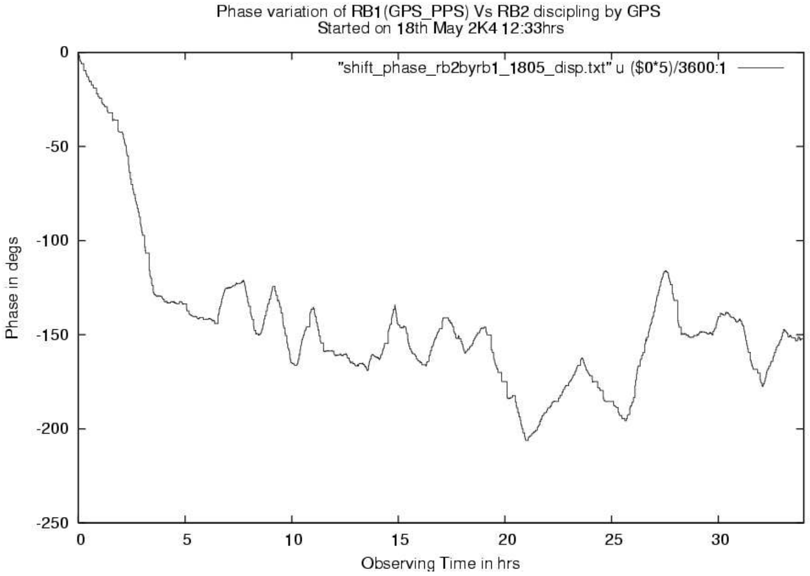

Figure 4.8: Phase errors between GPS disciplined Rb1 and Rb2(disciplined by Rb1)

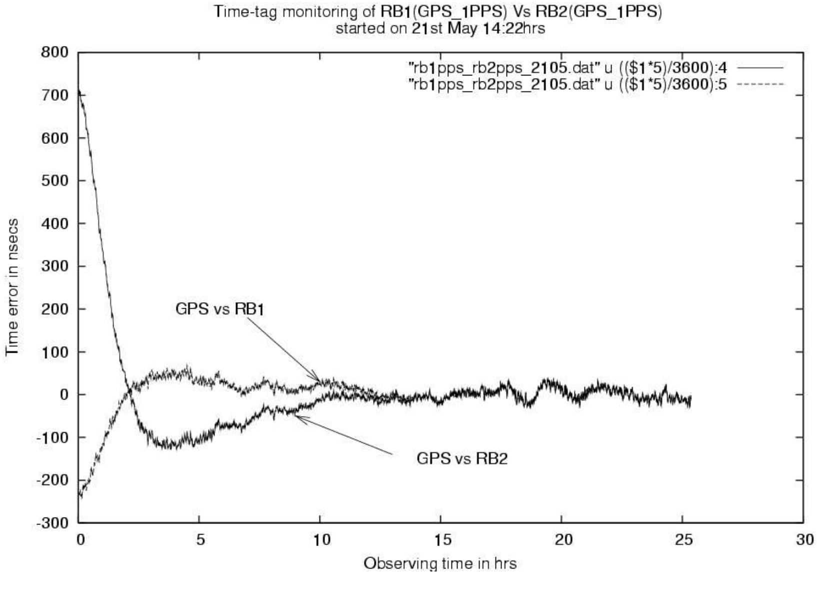

Figure 4.9: Time-tag errors of GPS Vs Rb1 PPS and GPS Vs RB2 PPS

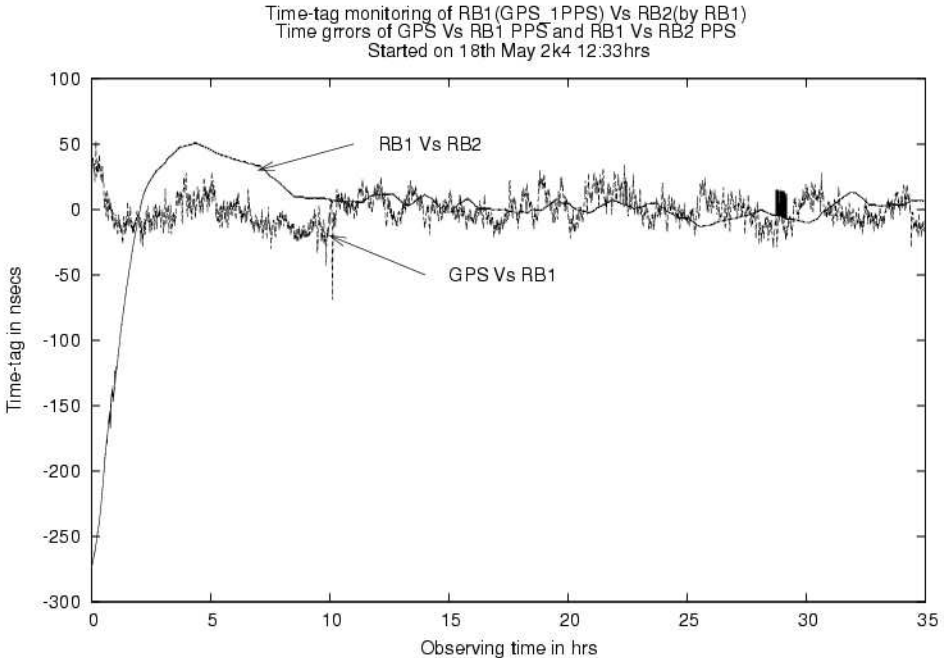

Figure 4.10: Time-tag errors of GPS Vs Rb1 PPS and RB1 Vs RB2 PPS

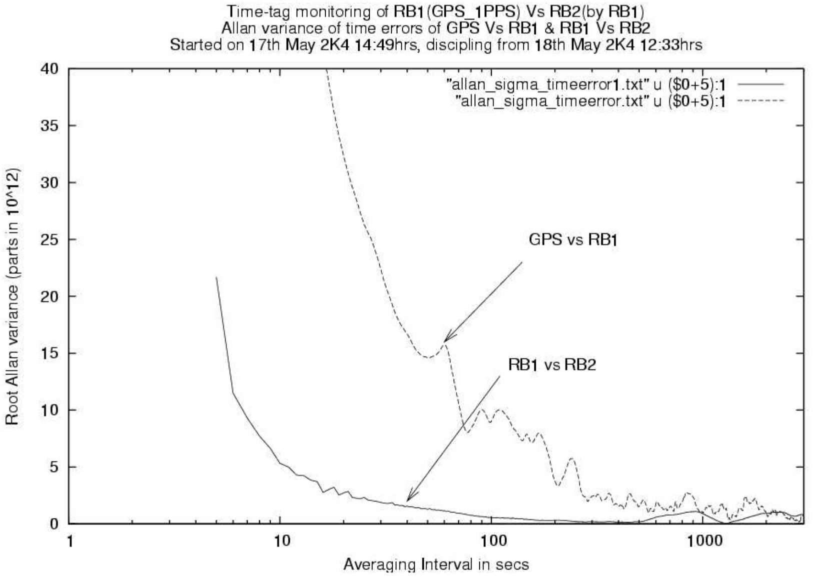

Figure 4.11: Phase stability of Rb1 oscillator and Rb2 oscillator in terms of Allan Variance

Chapter 5

Next Generation Time and Frequency Block

5.1 Time standard generation block

Proposed time standard generation block of GMRT consists of different selectable GPS receivers depending upon the requirement.The PPS and PPM pulses are coming from the receivers are used as reference standards at different parts of the clock chain.The monitoring scheme will provide the options for inter-comparison the different PPS or PPM pulses to assure the health of the given standard.

5.2 Frequency standard generation block

Proposed frequency standard generation block of GMRT is consisted of different rubidium atomic clock along with the existing old FTS oscillator. The rubidium clocks are combined with the PPS standard from any of the GPS receivers for achieving the good long-term stability. The monitoring schemes are also included to make the users aware about the running health of the frequency standards.

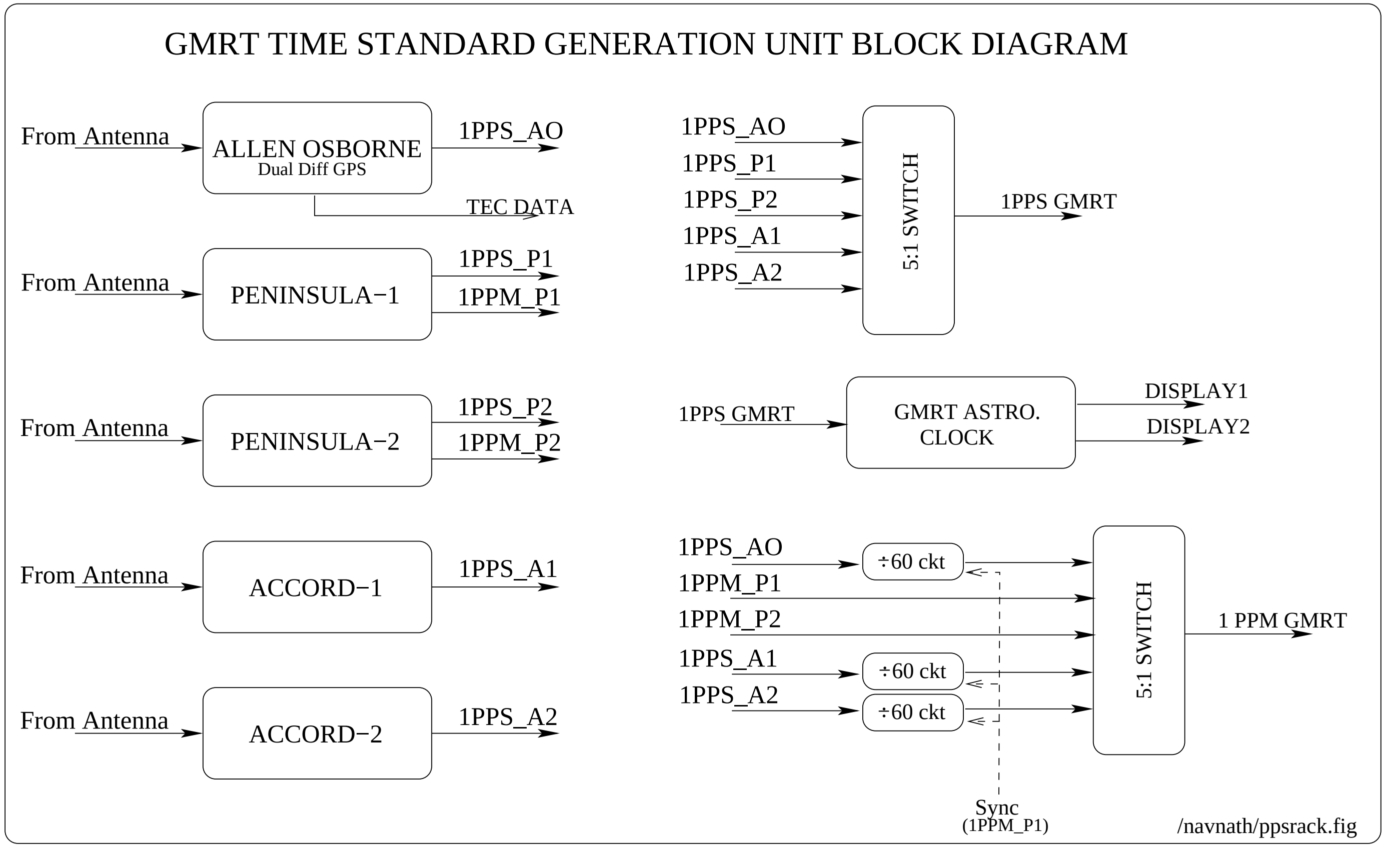

Figure 5.1: Proposed time standard generation block of GMRT

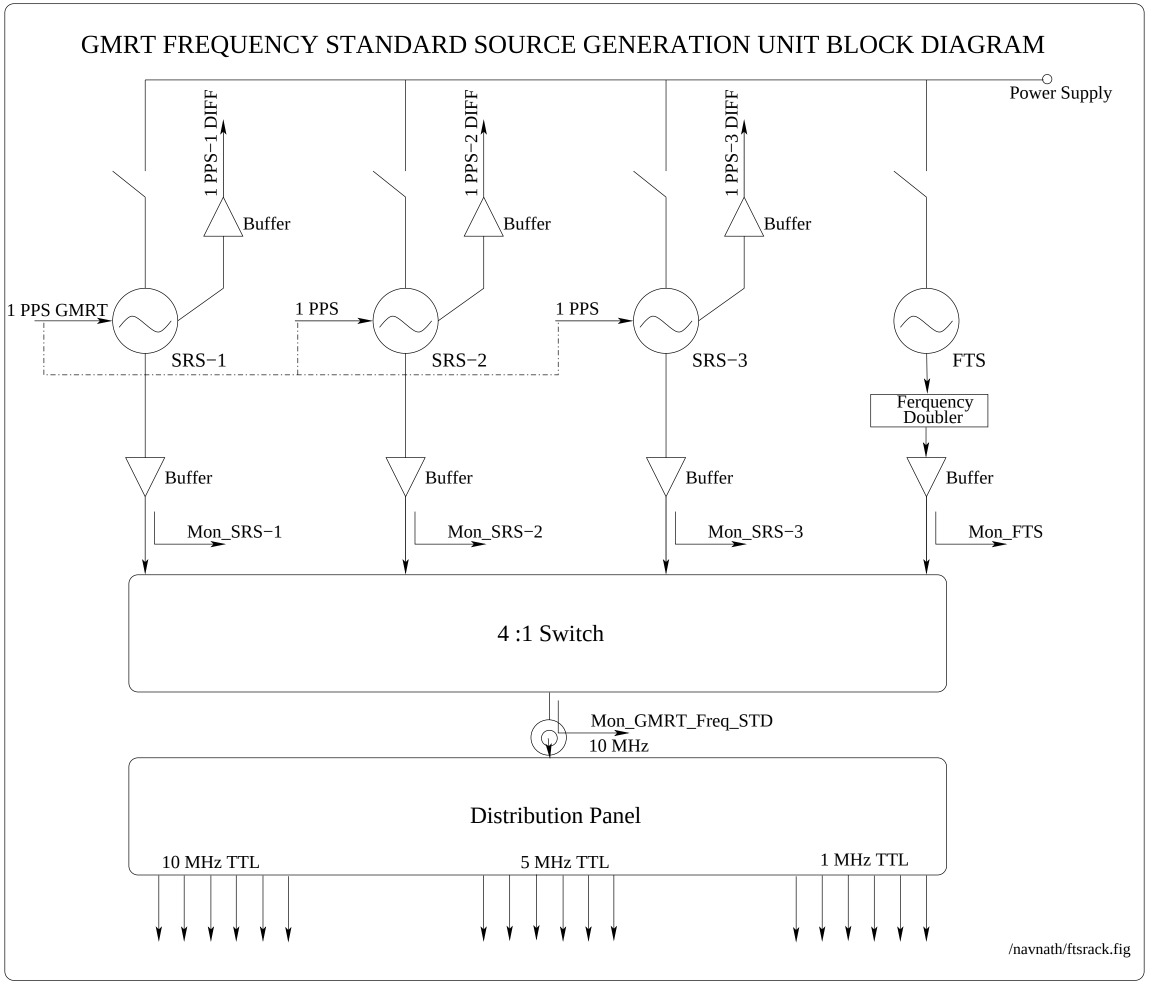

Figure 5.2: Proposed frequency standard generation block of GMRT

Chapter 6

Conclusions and Future Scopes

6.1 Conclusions

The series of experiments show that the errors between GPS and Rb are quite large on short-times because of the short-time jitters (50-300 nsecs) of GPS, which are seen in "GPS Vs Rb1" curve (Figure 4.10). So the disciplined Rb is optimised to follow the inherent stability of Rb by the manufacturer in that time-scale. In this case the stability of GPS disciplined Rb is assumed to be that of free Rb as per the manufacturer's optimisations algorithim. The short-time jitter of GPS is then averaged out in 1000 secs by the disciplined Rb.

The stability of free and disciplined Rb are evaluated against the GPS disciplined Rb. The experimental results are indicating the stability figure in the order of $\times 10^{-12}$. To get a still better estimate one require to have more stable standard like Cesium, Hydrogen Maser, Pulsar.

6.2 Future scopes

The functionality of the clock distribution system of GMRT to the different subsystems of the receiver depends on the purity of the other blocks sitting in that chain, such as generation of 105MHz and 200MHz using PLLs for 2nd and 3rd LO, 70MHz for baseband LO, correlator sampling clock from DDS. Some of these can pollute the achieved stability of the atomic standard. These contributions from other blocks in the clock distribution system can be verified by observing a strong millisecond pulsar with higher time resolution.

The very good long-term stability of the pulsar can be used to figure out the corresponding number for GMRT's frequency standard (GPS disciplined Rb).

The proposed time and frequency standard generation blocks will integrate the all different available standards with monitoring facilities. The implementation of this proposed scheme will help to analyze the clock related problems by the regular monitoring of the health status of the references.

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