TRANSMISSION THROUGH WIRE GRIDS

J.N. Chengalur & R.K. Kher



Summary

The main reflecting surface of the Giant Metrewavelength Radio Telescope is likely to be a wire mesh with wire diameter ~ 0.06 cm and wire spacing ~ 1 to 2 cm. The mesh was chosen to have a square iron section so that it reflects all polarization equally. The desired transmission loss (ratio of power transmitted to power incident) is ~ 13 dB or lower at a minimum wavelength of 21 cm.

Various techniques for calculating the transmission loss were studied, viz. those by Booker [3], the average boundary condition used by Kontrovitch [4] and Astrakhan [1], and the method used by Kaplun [2].

Since none of the published papers included calculations in the desired range of r/λ and d/λ (r = radius of mesh wire, d = grid size and λ = wavelength), fresh calculations were made. For normal incidence on a square mesh made up of perfectly conducting wires, the treatment given by Booker and Astrakhan are identical. Both break down for d/r < 10.

The method used by Kaplunet al is valid even for d/r < 10 and experimental verification of the calculated results is claimed by the authors. However, in the range of r/λ , d/λ examined, they deviated from the experimental results of Wilson and Cottony by as much as 20%.

Introduction

Wire meshes have been widely used as the reflecting surface for large antennae. At long enough wave lengths the reflectivity of a wire mesh is almost as good as that of a solid surface. In addition it has the advantage of being much cheaper, less heavy and offers considerably less surface for wind loading.

A mesh which consists of a set of parallel wires will reflect only one polarization (that with the E field parallel to the wire axis), the perpendicular polarization passing through virtually unhindered. A rectangular mesh can hence be treated as two independent parallel wire meshes, with their axes perpendicular. Some amount of interaction does take place, however, because rectangular meshes are known to produce cross polarized components which this simple theory does not predict. However, these cross polarized components can be shown to be zero for a perfectly soldered mesh.

Booker [3] showed that a plane wave propogating in free space can be treated as a wave in a transmission line with characteristic impedance $Z_0 = 377 \ \Omega$. A wire mesh placed in the path of such a plane wave is equivalent to a lumped shunt impedance placed in the transmission line analogy. Mac Farlane [5] has worked out the surface impedance of such a mesh by expanding the scattered field in a set of modes, each mode being associated with a possible angle of scatter. Kontrovitch [4] has worked out the reflection coefficient by averaging the electric field over one cell of the mesh before applying the boundary conditions. Both these approaches are considered in more detail in the following pages.

 $^{^{\}dagger}$ A high reflectivity is desired not only to ensure that incoming signals are not much attenuated but also to ensure rejection of signals from behind the screen, particularly contribution to system temperature from the ground radiation. For a transmission coefficient of -15dB = 1/30, about 10° K is contributed to the system temperature; for -13dB about 15° K is contributed.

Reflection Coefficient and Transmission loss

As explained earlier, the problem of reflection of a plane wave at an infinite plane wave grid is equivalent to reflection at a lumped shunt impedance at a transmission line with characteristic impedance $Z_0 = 377~\Omega$.

Consider the following situation:

Let the incident wave be $[V^{i}_{(z)}, I^{i}_{(z)}]$ the reflected wave $[V^{r}_{(z)}, I^{r}_{(z)}]$ and the transmitted wave $[V^{t}_{(z)}, I^{t}_{(z)}]$

 $T \stackrel{\triangle}{=} \frac{V^{\tau}(0)}{V^{i}(0)}$

Then the reflection coefficient

$$R \stackrel{\Delta}{=} \frac{V^{r}(0)}{V^{i}(0)}$$
 and the transmission coefficient

The appropriate boundary conditions are :

$$V^{r}_{(0)} + V^{i}_{(0)} = V^{t}_{(0)}$$

$$I^{r}_{(0)} + I^{i}_{(0)} = V^{t}_{(0)}/Zs + I^{t}_{(0)}$$

which give T = 1 + R and $1 - R = T + (1 + R) Z_0/Zs$

which can be solved to give
$$R = \frac{-Z_0}{Z_0 + 2Z_s}$$
, $T = \frac{1}{1 + Z_0/2Z_s}$

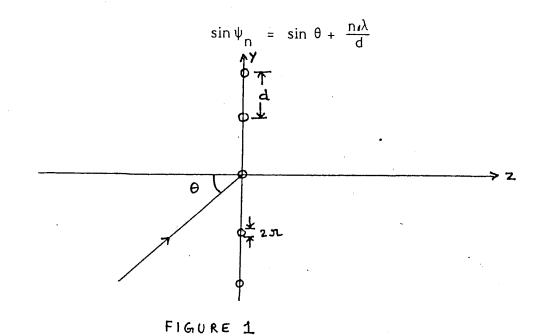
The ratio of the power transmitted to the power incident, (henceforth called the transmission loss, L) is

$$L = |T|^2 = \frac{1}{|1 + Z_0|/2Zs|^2}$$

Surface impedance of an infinite parallel wire grid

Mac Farlane considers a linearly polarized plane wave incident or a set of perfectly conducting parallel (wires with the wire axis parallel to the field of the incident wave. The angle of incidence is θ (Fig. 1).

The angles of scatter i.e. the angles at which the waves re-radiated by the wires arrive in phase are given by ψ_{n} where



The scattered field can then be expressed as

$$E^{S} = \sum_{n=0}^{\infty} A_{n} Z_{0}e^{-jk(y \sin \psi_{n} + z \cos \psi_{n})}$$

Modes with $|\sin \psi_n| < 1.0$ represent plane waves which carry away net power,

those with $|\sin \psi_n| > 1.0$ represent evanescent waves which carry zero net power.

For $d < \lambda$, the only travelling waves are at $\psi = 0$ and $\psi = (\pi - \theta)$ i.e. transmitted and reflected waves. Then assuming the wires to be perfectly conducting, the shunt impedance presented by the grid is purely reactive and for r << d is given by

$$Z_s = jX_s = jZ_o(\frac{d}{\lambda})\cos\theta [F(\frac{d}{\lambda},\theta) + \ln(\frac{d}{2\pi}r)]$$

where

$$F\left(\frac{d}{\lambda},\theta\right) = \sum_{n=-\infty}^{\infty} \frac{1}{2n} \left[\left(\frac{n\lambda}{d}\right) \left\{ (\sin^2 \psi_n^+ - 1)^{-1/2} + (\sin^2 \psi_n^- - 1)^{-1/2} \right\} \right]$$

$$\sin \psi_n^+ = \left[\sin \theta + \frac{n\lambda}{d} \right]; \quad \sin \psi_n^- = \left[\frac{n\lambda}{d} - \sin \theta \right]$$

V.A. Kaplun et al [2] using the same approach derived a slightly more accurate formula for the transmission loss viz.

$$L = \frac{\left[\left(\frac{2d}{\lambda}\right)\left\{F(r/\lambda;\frac{d}{\lambda}) - \ln(1 - e^{-\frac{2\pi r}{d}})\right\}\right]^{2}}{\left[1 + \left(\frac{2d}{\lambda}\right)\left\{F(k/\lambda;\frac{d}{\lambda}) - \ln(1 - e^{-\frac{2\pi r}{d}})\right\}\right]^{2}}$$

where

F(r/
$$\lambda$$
; $\frac{d}{\lambda}$) = $\sum_{m=1}^{\infty} \frac{e^{-2\pi (r/d) \cdot \sqrt{m^2 - \frac{d^2}{\lambda^2}}}}{e^{-2\pi (r/d) \cdot \sqrt{m^2 - \frac{d^2}{\lambda^2}}}} - \frac{e^{-2\pi (r/d)m}}{e^{-2\pi (r/d)m}}$

where only normal incidence has been considered. Calculations using this formula have been made, the results are displayed in Fig. 4. and table I and table II.

The analysis is for a mesh of parallel wires but as explained earlier it can be extended to a square mesh provided the mesh nodes are perfectly soldered.

The average boundary condition method

In the average boundary method (Kontorovitch [4]) the actual field sacttered by the wires is approximated by an averaged or 'smoothered' field, which also obeys Maxwells equations. The grating surface is replaced by a plane at which certain 'averaged' boundary conditions which depend upon the geometry of the grating are fulfilled.

Using the method of averaged boundary conditions, Astrakhan [1] has shown that the reflection coefficient of an infinite rectangular mesh is given by

shown that the reflection coefficient of an infinite rectangular mesh is given by
$$R_{\mathbf{u}}^{1} = K \cos\theta I_{0}^{-1} \{ 1 - K \cos\theta [v_{2} \cos^{2}\phi + (\delta_{2} - \delta_{1})\sin\phi\cos\phi - v_{1} \sin^{2}\phi] \}$$

$$R_{\mathbf{u}}^{2} = -K^{2}\cos\theta I_{0}^{-1} \{ \delta_{1} \sin^{2}\phi - (v_{1} + v_{2})\sin\phi\cos\phi + \delta_{2}\cos^{2}\phi] \}$$

$$R_{ii}^{h} = -K^{2} \cos \theta \ I_{o}^{-1} \{ \delta_{1} \cos^{2} \phi + (\nu_{1} + \nu_{2}) \sin \phi \cos \phi + \delta_{2} \sin^{2} \phi \}$$

$$R_{\bullet}^{h} = -K I_{o}^{-1} \left\{ \cos \theta + K \left[v_{1} \cos^{2} \phi + \left(\delta_{2} - \delta_{1} \right) \sin \phi \cos \phi - v_{2} \sin^{2} \phi \right] \right\}$$
where $K = 2\pi / \lambda$

$$I_{o}/K = \cos \theta [1+K^{2}(\delta_{1} \delta_{2} - v_{1} v_{2})] + K \sin^{2}\theta [v_{2} \cos^{2}\phi + (\delta_{2} - \delta_{1}) \sin \phi \cos \phi - v_{2} \sin^{2}\phi] + K (v_{1} - v_{2})$$

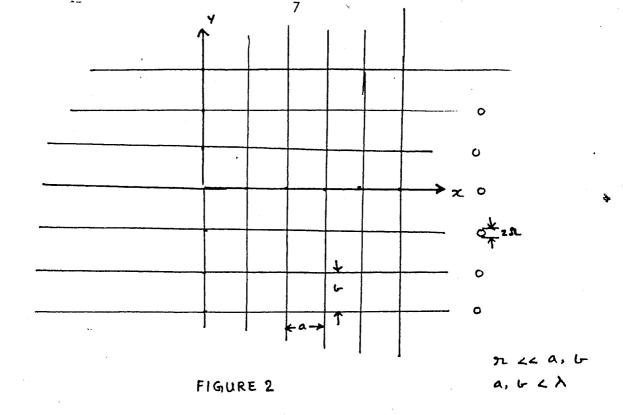
$$\alpha_{1} = \frac{jb}{\pi} \ln \left(\frac{b}{2\pi r} \right); \quad \alpha_{2} = \frac{ja}{\pi} \ln \left(\frac{a}{2\pi r} \right)$$

$$\nu_{1} = \alpha_{1} (1 + F_{x} - \frac{a/b + \kappa_{x}}{1 + a/b + \kappa_{x}} \sin^{2}\theta \cos^{2}\phi)$$

$$\nu_{2} = -\alpha_{2} (1 + F_{y} - \frac{b/a + \kappa_{y}}{1 + \frac{b}{2} \kappa} \sin^{2}\theta \cdot \sin^{2}\phi)$$

$$\delta_{1} = \frac{\alpha_{1} \cdot a/b}{1 + a/b + n} \cdot \sin^{2}\theta \cdot \sin\phi \cos\phi ; \delta_{2} = \frac{-\alpha_{2} \cdot b/a}{1 + b/a + n} \sin^{2}\theta \cdot \sin\phi \cdot c$$

where a and b are the dimensions of the wire grid, a being the spacing along the x axis as shown in Fig. 2. and r is the wire radius.



 F_{χ} and F_{y} allow for finite conductivity and non unity permeability of the grid material and for skin effect in the grid wires.

$$F_x = \frac{\mu[(1-j)/s]}{4 \ln b/2\pi r}$$
 $F_y = \frac{\mu[(1-j)/s]}{4 \ln a/2\pi r}$

where $s = r \sqrt{\lambda \omega \sigma \mu} / \sqrt{2c}$

here as $\sigma \rightarrow \infty$, s $\rightarrow \infty$ and F_x , $F_y \rightarrow 0$

$$\frac{\pi}{x}$$
 and $\frac{\pi}{y}$ take into account the type of contact at the mesh nodes

$$X_{x} = \frac{j\omega bz}{2[\ln{(b/r)} - 1]} \qquad X_{y} = \frac{j\omega az}{2[\ln{(a/r)} - 1]}$$

where z is the impedance between wires at the mesh nodes. For a soldered mesh hence $\mu_{\chi} = \mu_{V} = 0$

$$R_{\parallel} \stackrel{\stackrel{?}{=}}{=} \frac{\stackrel{\stackrel{?}{=}}{\stackrel{r}{=}} \stackrel{\stackrel{?}{=}}{\stackrel{r}{=}} \frac{\stackrel{?}{=}}{\stackrel{r}{=}} \stackrel{\stackrel{?}{=}}{\stackrel{r}{=}} \frac{\stackrel{?}{=}}{\stackrel{r}{=}} \stackrel{\stackrel{?}{=}}{\stackrel{r}{=}} \stackrel{\stackrel{?}{=}}{\stackrel{?}{=}} \stackrel{\stackrel{?}{=}}{\stackrel{r}{=}} \stackrel{\stackrel{?}{=}} \stackrel{\stackrel{?}{=}}$$

$$R_{\parallel} \stackrel{\triangle}{=} \frac{E^{r} \cdot t_{o}^{h}}{|E_{inc}|}; \qquad R_{\perp} \stackrel{\triangle}{=} \frac{E^{r} \cdot n_{o}^{h}}{|E_{inc}|}$$

$$3(a)$$

For $F_x = F_y = 0$ and $x_x = x_y = 0$ and a grid with a square geometr

viz.
$$a = b = d$$
, the reflection coefficients reduce to
$$R_{N}^{Q} = \left[1 + \frac{K\alpha}{\cos\theta} \left(1 - \frac{1}{2}\sin^2\theta\right)^{-1}\right]$$

$$R_{L}^{h} = -\left[1 + K\alpha\cos\theta\right]^{-1}$$

 $R_{\mathbf{n}}^{(\widehat{\mathbf{Q}})} = R_{\mathbf{n}}^{\mathbf{h}} = (\widehat{\boldsymbol{\theta}}) - 0$

where $\alpha = \frac{jd}{\pi} \ln (d/2\pi r)$

for $\theta = 0$ (normal incidence)

$$|R_{II}^{\ell}|^2 = |R_{I}^{h}|^2 = \frac{1}{|1 + K\alpha|^2}$$

since the transmission loss L_{ij}^{e} or L_{ij}^{h} (or L since they are equal) is given by

$$L = 1 - \frac{1}{|1 + K\alpha|^2}$$

Plots for several values of r and d are given in Fig. 5 and Fig. 6. Calculated values are shown in Tables I and II.

Comparison with measured values and conclusion

The average boundary condition method fails for d/a \sim 6 because it contains the term $\ln\left(\frac{d}{2\pi a}\right)$ giving $\alpha \rightarrow 0$ and $L \rightarrow -\infty$ as $d \rightarrow 2\pi a$. Also, it assumes that d/a is of the order of λ /d.

A comparison of the losses predicted by the method of Kaplun et al and that of Astrakhan with those experimentally measured [6] is given in table II. Neither method gives a particularly good fit in the range of r/λ and d/λ examined, both appear to consistently underestimate the transmission loss by a large factor (~15%), the method used by Kaplun giving a marginally better fit than that used by Astrakhan.

As explained earlier wire meshes show cross polarised components unless the mesh nodes are perfectly soldered. We make this assumption thoughout. In addition we have assumed the wires to be made of perfectly conducting material; the errors introduced due to the finite conductivity of the wire has been assumed negligible. Throughout we have concerned ourselves only with the transmission loss: the phase of the reflected wave being of no consequence.

From the measured values Wilson and Cottony either r = 0.793(36SWG) and d = 14 mm or r = 0.295(31 SWG) and d = 20 mm give a 15d8 transmission loss at 21 cm (table III).

References

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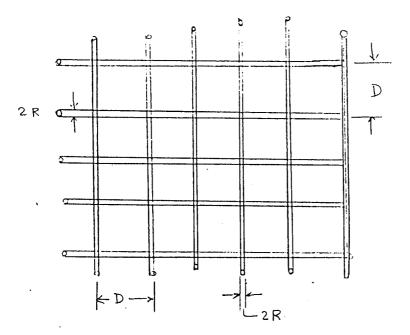
Vol 9, 8, 1964
H.G. Booker: IEE Journ. 93 (III A), 620
M.I. Kontrovitch: Radio <u>Technica</u>-Electronica Vol 8, 9, 1963

G.G. Mac Farlane, Proc. Inst. Elec. Engrs. (London), pt. III A, 9

A.C. Welson, H.Y. Cottony IRE Trans. AP-8, 2, 144, Mar 1960

7.A. Kaplun, N.I.Babkin, B.G. Goryachev: Radio Technic

M.I. Astrakhan, Radio Engg. (Moscow) Vol 23, pp 76-83, 1968



FINITIONS OF MESH USED IN TABLE I

TABLE (I)

LAMBDA (MM) = 210.0

LAMBDA (MM) = 210.0

LOSS (AST

-11.8

-12.1

-12.4

-14.8

-15.3

-15.7

-16.2

-17.0

-17.8

LOSS (KBG)

-13.5

-13.7

-13.9

SOLIDITY

0.0386

0.0426

0.0468

0.0902

0.1016

0.1118

0.1220

0.1422

0.1626

D(IN MM)=10.0

SWG

36

35

34

26

25

24

23

22

21

2R (MM)

0.193

0.213

0.234

0.451

0.508

0.559

0.610

0.711

0.813

33	0.254	-14.1	-12.6	0.0508
32	0.274	-14.3	-12.9	0.0548
31	0.295	-14.5	-13.1	0.0590
30	0.315	-14.7	-13.3	0.0630
29	0.345	-15.0	-13.7	0.0690
28	0.376	-15.2	-14.0	0.0752
27	0.417	-15.6	-14.4	0.0834

-15.8

-16.2

-16.6

-16.9

-17.5

-18.1

LAMBDA = 210.0 mm

D(IN MM)=14.0

SWG	2 R (MM)	LOSS(KBG)	LOSS (AST)	SOLIDITY
36	0.193	-10.6	-8.3	0.0276
35	0.213	-10.7	-8.5	0.0304
34	0.234	-10.9	-8.7	0.0334
33	0.254	-11.1	-9.0	0.0363
32	0.274	-11.3	-9.2	0.0391
31	0.295	-11.4	-9.4	0.0421
30	0.315	-11.6	-9.5	0.0450
29	0.345	-11.8	-9.8	0.0493
28	0.376	-12.0	-10.1	0.0537
27	0.417	-12.3	-10.4	0.0596
26	0.451	-12.5	-10.7	0.0644
25	0.508	-12.8	-11.1	0.0726
24	0.559	-13.1	-11.5	0.0799
23	0.610	-13.3	-11.8	0.0871
22	0.711	-13.8	-12.5	0.1016.
21	0.813	-14.3	-13.1	0.1161 /

LAMBDA = 210.0 mm.

D(IN MM)=20.0

23

22

21

0.610

0.711

0.813

SWG	2 R (MM)	LOSS(KBG)	LOSS (AST)	SOLIDITY
36	0.193	-7.9	-5.1	0.0193
35	0.213	-8.1	-5.3	0.0213
34	0.234	-8.2	-5.5	0.0234
33	0.254	-8.4	-5.6	0.0254
32	0.274	-8.5	-5.8	0.0274
31	0.295	-8.6	-5.9	0.0295
30	0.315	-8.7	-6.1	0.0315
29	0.345	-8.9	-6.3	0.0345
28	0.376	-9.0	-6.5	0.0376
27	0.417	-9.2	-6.7	0.0417
26	0.451	-9.4	-6.9	0.0451
25	0.508	-9.7	-73	0.0508
24	0.559	-9.9	-7.5	0.0559

-10.1

-10.4

-10.8

0.0610

0.0711

0.0813

-7.8

-8.3

- 8.8

$\frac{\text{LAMBDA}(MM) = 500.0}{\text{LAMBDA}(MM)}$

O(IN M	M)=10.0				
SWG	2 R (MM)	LOSS (KBG)	LOSS (AST)	SOLIDITY	
3 5	0.193	-19.8	-19.1	0.0386	
35	0.213	-20.1	-19.4	0.0426	
34	0.234	-20.4	-19.7	0.0468	
33	0.254	-20.6	-19.9	0.0508	
32	0.274	-20.8	-20.2	0.0548	
31	0.295	-21.1	-20.5	0.0590	
30	0.315	-21.3	-20.7	0.0630	
29	0.345	-21.6	-21.2	0.0690	•
28	0.376	-21,9	-21.4	0.0752	
27	0.417	-22.2	-21.8	0.0834	,
26	0.451	-22.5	-22.2	0.0902	
25	0.508	-23.0	-22.7	- 1. -0. 0 016	0.1016
24	0.559	-23.3	-23.2	0.1118	
23	0.610	-23.7	-23.6	<u> </u>	0.1220
22	0.711	-24.4	-24.5	0.1422	
21	0.813	-25.0	-25.3	0.1626	

500.0 mm LAMBDA

LOSS (KBG)

-16.5

LOSS (AST)

-15.2

-18.8

-19.1

-19.8

-20.5

SOLIDITY

0.0276

0.0799

0.0871

0.1016

0.1161

D(IN MM)=14.0

2 R (MM)

0.193

0.559

0.610

0.711

0.813

SWG

36

24

23

22

21

35	0.213	-16.7	-15.5	0.0304
34	0.234	-16.9	-15.8	0.0334
33	0.254	-17.1	-16.0	0.0363
32	0.274	-17.3	-16.2	0.0391
31	0.295	-17.5	-16.5	0.0421
30	0.315	-17.7	-16.7	0.0450
29	0.345	-17.9	-17.0-	0.0493
28	0.376	-18.2	-17.3	0.0537
27	0.417	-18.5	-17.6	0.0596
26	0.451	-18.7	-17.9	0.0644
25	0.508	-19.1	-18.4	0.0726

-19.4

-19.7

€20.3

-20.8

LAMBDA = 500.0mm

D(IN MM)=20.0

21

0.813

SWG	2 R (MM)	LOSS (KBG)	LOSS(AST)	SOLIDITY
36	0.193	-13.2	-11.4	0.0193
35	0.213	-13.4	-11.6	0.0213
34	0.234	-13.6	-11.9	0.0234
33	0,254	-13.7	-12.1	0.0254
32	0.274	-13.9	~12.3	0.0274
31	0.295	-14.0	-12.4	0.0295
30	0.315	-14.2	-12.6	0.0315
29	0.345	-14.4	-12.9	``0.0345
28	0.376	-14.6	-13.1	0.0376
27	0.417	-14.9	-13.4	0.0417
26	0.451	-15.1	-13.7	0.0451
25	9.508	-15.4	-14.1	0.0508
24	0.559	-15.6	'-14.4 ×	0.0559
23	0.610	-15.9	-14.7	0.0610
22	0.711	-16.3	-15.3	0.0711

-16.8

0.0813

-15.8

Table	(II)						~ :				
Compar	isoņ	between	Kaplun	et	al,	Astrakha	n and	measured	values	as	given
Wilson	and	Cottony			٠,						
r/λ	= 0.	001					Loss	(dB)			
<u>d/λ</u>		Kaplu	<u>n</u>		As	strakhan			sured		
0.01		-36.5				110 6					
0.02						-40.6			43.5		
0.02		- 26.1				-26.7			32.0		
		-20.8				-20.6			26.0		
0.04		-17.5				-16.7			22.5		
0.05		-15.1				-13.81		_	19.5		
0.06		-13.3				-11.7		-	17.3		
0.07		-11.8				- 9.8		-	15.5		
0.08		-10.7	•			- 8.5		· -	14.0		
0.09		- 9.7				- 7.3		-	12.5		
0.10		- 8.9				- 6.3		_	11.5		
r/λ	=	0.0005									
<u>d/λ</u>		<u>Kaplur</u>	<u>1</u>		As	trakhan		Mea	sured		
0.01		-31.9				-32.7		_	37.0		
0.02		-22.9				-22.6			27.5		
0.03		-18.3				-17.4			22.5		
0.04		-15.3							18.0		
0.05		-13.2				-11.5			16.6		
0.06		-11.6				- 9.5			14.5		
0.07		-28.3				- 8.0			13.0		
0.08		- 9.3				- 6.7			11.5		
0.09		- 8.4				- 5.7			10.5		
0.10		- 7.7				- 4.9			9.0		
						-					

-32.0

-33.0

-30.5

-32.0

-34.0

-36.0

-37.0

920

-25.0

-27.0

500

-26.5

-28.5

-30.0

-32.0

NΑ

NΑ

NΑ

NA

NΑ

₹300

-23.0

-25.0

NA

NΑ

NA

NA.

NΑ

NΑ

210

-20.

NΑ

NΑ

NA

ΝA

Measured transmission loss from Wilson and Cottony

TABLE III

25

23

SWG

36

31

28

25

23

0.508

0.618

0.193

0.295

0.376

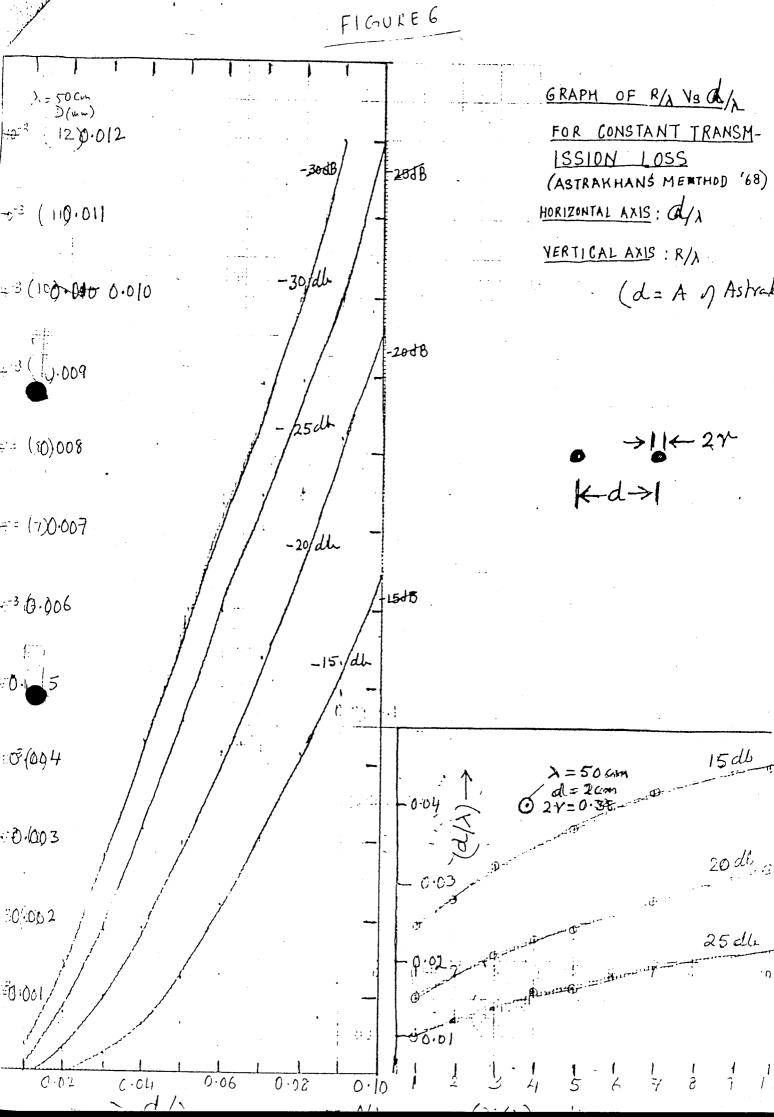
0.508

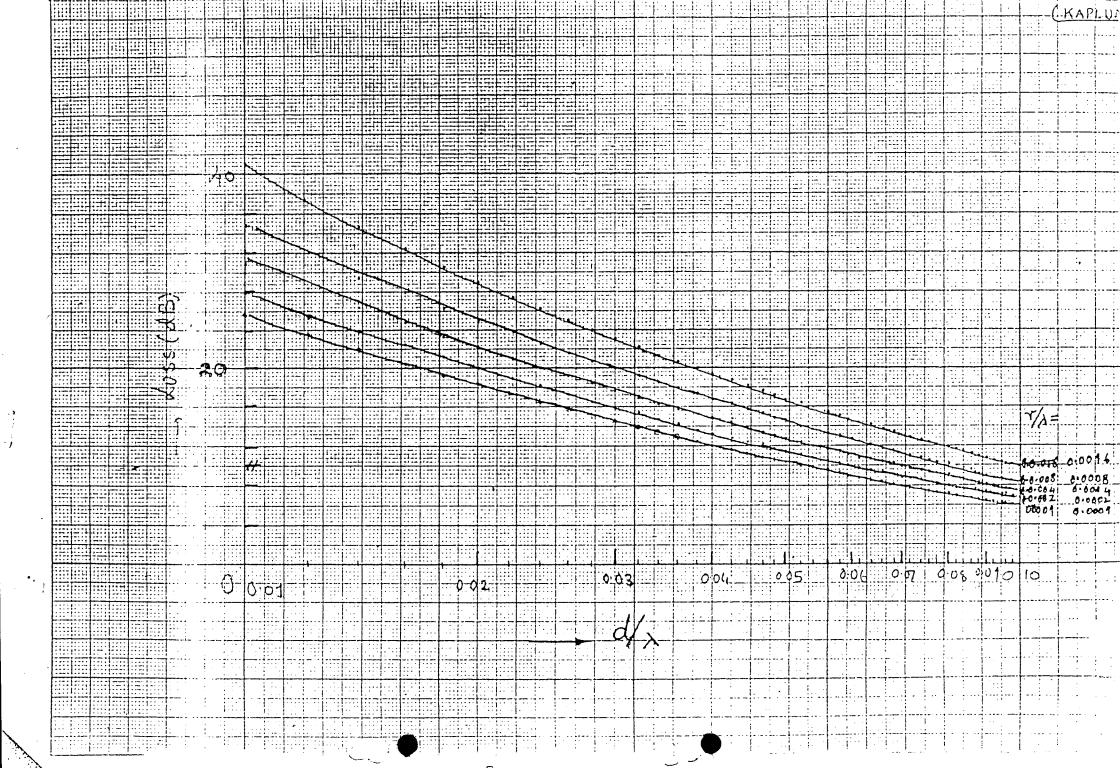
0.610

r

d = 10

(ASTRAKHANS METHOD '48) VERTICAL AXIS: LOSS (4B) HORIZONTAL AXIS: A/A (A= SIDE OF SQUARE)





Title: Fransmishon Through Via Author - A Chengetin and Kler 4987 Leakage Willbe ~ 70 dB $1 + \left\{ \frac{2d}{\lambda} \ln \left(\frac{d}{2\pi a} \right) \right\}^{2}$ Power Ref Coeff . Transmission Loss = 10 log 10 [1- rp] d = 0.158 cm $a = \frac{0.375}{2} = 0.1875$ mm 1+ \$0.003 x lu (0.158 / 6.28 x 0.0188)} 2.

d= 0 1.58 mm = 0.0188 cm $\frac{1}{3} = \frac{0.158}{100}$

285WG= 0375 x10 m 2 a = 0.375 mm 0.1875.mm

25810.0 0-0188

22 mesh 0.115

0,000000 76.

= 0-999.

0,291

=61.16.7